HYDRAULICS

WITH

WORKING TABLES

BY

E. S. BELLASIS, M.Inst.C.E.

EXECUTIVE ENGINEER IN THE IRRIGATION BRANCH
OF THE PUBLIC WORKS DEPARTMENT OF INDIA

RIVINGTONS
34 *KING STREET, COVENT GARDEN*
LONDON
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PREFACE

A NEW Treatise on Hydraulics is required for more than one reason It is required in order to include the very considerable advances which have been made of late years in nearly all branches of the science advances due to such investigations as those made by Mons Bazin on Weirs, by Mr Kennedy of the Indian Public Works Department, on the Power of Streams to carry Suspended Matter, and by engineers in America on Pipes and Apertures It is also required in order to develop and expand the branch of Hydraulics which relates to Flow in Open Channels In this branch there has always been an excessive number of matters regarding which information has been obtainable only in a scattered highly condensed, or otherwise defective form or has been altogether non existent[1] so that although they can easily be reduced to general principles and although they have often a direct practical bearing on his work the engineer has had to find them out gradually for himself

The above remarks refer chiefly to the laws and principles of Hydraulics. Not less important is the matter of Co efficients The very large errors (carrying with them waste of time and money when works are designed) in the co efficients given by the older writers are now fully admitted

[1] Owing to these causes and to the consequent want of opportunity for studying matters and referring them to underlying principles fallacies are not uncommon Several are mentioned in the text and one in particular in chap vii art 9

by engineers and the advantage of using modern figures is recognised. Much light has been thrown on co efficients by the researches above mentioned and others equally recent

It has further seemed desirable to have a book which besides being a text book should include practical Examples and full Working Tables

In the present volume an attempt is made to deal with the above matters. The book has been compiled from notes which have extended over a long period and embody the results of twenty five years' practical experience and study both of observed facts and of current literature It contains a very large proportion of new matter especially in chapters ii iv vi vii, and ix It is hoped that it may be found to meet the requirements both of the student and of the engineer

In every branch considerable detail is gone into, but the reader who does not require details will have no difficulty in passing over them some being in small print and some forming special articles

For kindly supplying remarks or information on points which seemed doubtful I beg to thank Messrs Bazin Stearns Schafer and Kennedy Professors Unwin Williams and Bovey, and Dr Brightmore

I also thank Messrs Rivington for the attention which they have given to the printing and issue of the book

L S B

April 18[illegible]

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ERRATA.

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9	5	f and v	F and V
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12	14	c and d	C and D
15	5, 6 & 11 from bottom	sharp-edged weirs, &c.	weirs in thin walls, &c.
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38	4 " "	, appreciably	appreciably,
85	1 & 6 " "	g	$d\ N$
97	10 " "	$\frac{C}{C_w}$	$\frac{C_w}{C}$
115	Table XIII	1058 &c.	1·058 &c
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CHAPTER I

INTRODUCTION

SECTION I.—PRELIMINARY REMARKS AND DEFINITIONS

1. **Hydraulics.**—Hydraulics is the science in which the flow of water, occurring under the conditions ordinarily met with in Engineering practice, is dealt with. Based on the exact sciences of hydrostatics and dynamics, it is itself a practical, not an exact, science. Its principal laws are founded on theory, but owing to imperfections in theoretical knowledge, the algebraic formulæ employed to embody these laws are somewhat imperfect and contain elements which are empirical, that is, derived from observation and not from theory. The science of Hydraulics is concerned with the discussion of laws, principles, and formulæ, of such observed phenomena as are connected with them, and of their practical application. The quantities dealt with are generally velocities and discharges, but sometimes they are pressures or energies. It is frequently necessary in Hydraulics to refer to particular works or machines, but this is done to afford practical illustrations of the application of the laws and principles. Descriptions of works or machines form part of Hydraulic Engineering and not of Hydraulics, and the same remark applies to statistical information on subjects such as Rainfall. Some description of Hydraulic Fieldwork is included in this work for reasons given below (chap. ii. art. 25). The laws governing the power of a stream to move solids by rolling or carrying them are intimately connected with the laws of flow and are naturally included.

2. **Fluids, Streams, and Channels.**—A 'fluid' is a substance which offers no resistance to distortion or change of form. Fluids are divided into 'compressible fluids' or 'gases,' such as air, and 'incompressible fluids' or 'liquids,' such as water. Perfect fluids are not met with, all being more or less 'viscous,' that is, offering some resistance, though it may be very small, to change of form. A 'stream' is a mass of fluid having a general movement of

translation It is generally bounded laterally by solid substances which form its 'channel' If the channel completely encloses the stream, and is in contact with it all round, as in a pipe running full, it is called a 'closed channel', but if the upper surface of the stream is 'free,' as in a river or in a pipe running partly full, it is an 'open channel' An 'eddy' is a portion of fluid whose particles have movements which are irregular and generally more or less rotatory, it may be either stationary or moving with respect to other objects The 'axis' of a stream or channel is a line centrally situated and parallel to the direction of flow In an open channel its exact position need not be fixed but in a pipe it is supposed to pass through the centre of gravity of each cross section

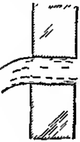


Fig 1

An '*orifice*' or '*short tube*' (*Fig* 1) *is a short* closed channel expanding abruptly, or at least very rapidly, at both its upstream and downstream ends A short open channel similarly circumstanced (Fig 2) is called a 'weir,' provided the expansions are wholly or partly in a vertical direction When they are wholly lateral it is called a 'contracted channel' All these short channels will collectively be termed 'apertures,' and 'channel' will be used for channels of considerable length

Fig 2

The stream issuing from an orifice or pipe is called a 'jet,' that falling from a weir a 'sheet' Except in the case of a jet issuing under water a stream bounded by other fluid of the same kind is called a 'current'

3 Velocity and Discharge —The direction of the flow of a stream is in general parallel to the axis, but it is not always so at each individual point If at any point the flow is not parallel to the axis the velocity at that point may be resolved into two components, one of which is parallel to the axis and the other at right angles to it The component parallel to the axis is termed the 'forward velocity' A 'cross section' of a stream is a section at right angles to the axis The velocities at all points *in the cross* section of a stream are not equal A curve whose abscissas represent distances along a line in the plane of the cross section and whose ordinates represent forward velocities is called a 'velocity curve' The 'discharge' of a stream at *any cross* section is the volume of water passing the cross section in the

unit of time The 'mean velocity' at any cross section is the mean of the different forward velocities It is the discharge of the stream divided by the area of the cross section Thus

$$V = \frac{Q}{A} \text{ or } Q = AV \qquad (1)$$

This is the first elementary formula of Hydraulics Except when velocities at individual points are under consideration, the term 'velocity' is generally used instead of 'mean velocity'

As long as the conditions under which flow takes place at any given cross-section of a stream remain constant, the velocity and discharge are constant, that is, they are the same in succeeding equal intervals of time In this case the flow is said to be 'steady' As soon as the conditions change, the velocity and discharge usually change, and the flow is then said to be unsteady Owing to the introduction or abstraction of water by subsidiary channels, leakage, or evaporation, the discharges at successive cross sections of a stream may be unequal, but the flow may still be steady Flow is unsteady only when the discharge varies with the time, and not when it merely varies with the place In Hydraulics, flow is always assumed to be steady unless the contrary is expressly stated For instance, in the statement that a rise of surface level gives an increase in velocity, it must be understood that this refers to the period after the surface has risen, and not to that while it is rising In any length of stream in which the flow is steady, and in which no water is lost or gained, the discharges at all cross-sections are equal, or

$$Q = A_1 V_1 = A_2 V_2 = \text{etc.}, \qquad (2)$$

where A_1, A_2, etc, are the areas of the cross sections, and V_1, V_2, etc, the mean velocities In other words, the mean velocity at any cross section is inversely as the sectional area

Section II —Phenomena observed in Flowing Water

4 Irregular Character of Motion—In flowing water the free surface oscillates, especially in large and rapid streams The oscillation is probably greater near the sides than at the centre The motion of the water is also irregular Except under peculiar conditions the fluid particles do not move in parallel lines, or 'stream lines' but their paths continually cross each other, and the velocity and direction of motion at any point vary every instant The stream is, in fact, a mass of small eddies The

irregularities of motion increase with the roughness of the channel and with the velocity of the stream They are especially great in open channels Eddies produced at the bed are constantly rising to the surface Floats dropped in at one point in quick succession move neither along the same paths nor with the same velocities In experiments made by Francis, whitewash discharged into a stream four inches above the bed came to the surface in a length which was equal to ten to thirty times the depth, and was less, the rougher the channel The eddies are strongest where they originate, namely, at the border of the stream To compensate for the upward eddies there must, of course, be downward currents, but they are diffused and hardly noticeable The resistance to flow caused by all these irregular movements is enormously greater than that which would exist in stream line motion

Although the velocity and water level at any point fluctuate every moment as above described, the average values obtained in successive periods of time of longer duration are more or less constant The velocities obtained at any point in successive seconds will, perhaps, vary by 20 per cent, those obtained in successive minutes will vary much less, and those in successive periods of five minutes each probably scarcely at all The same is true of the direction of the flow For the water level the averages of several observations obtained in periods of a minute each will probably agree very closely A velocity curve obtained from a few observations is generally irregular, but one obtained from a large number is regular If the flow is not steady, the average velocities and water levels obtained in successive long periods of time may, of course, vary, but they will exhibit a regular change When velocity and water level are spoken of, the average values and not the momentary values are meant, and this remark applies to the foregoing definition of steady flow The discharge at any cross section, if considered in its momentary aspect, is *probably never steady* *The irregularity of the motion* of water renders the theoretical investigation of flow extremely difficult, and no complete theory has yet been propounded

5 **Contraction and Expansion** —Except under an infinite force, a body cannot, without either coming to rest or describing a curve, change its direction of motion Acting in obedience to this law, water cannot turn sharp round a corner Wherever any sharp salient angle *A* or *B* (Fig 3) occurs in a channel, or at the entrance of an aperture the water travelling along the lines *GA*, *HB* cannot turn suddenly and follow the lines *AC*, *BD* It follows

the lines AE, BF, which are curves. At A and B the radii of the curves may be very small, but the curves doubtless touch the lines GA, HB. This phenomenon is known as 'contraction.' The stream contracts from AB to EF. If the channel or aperture extends far enough, the stream expands again and fills it at MN, the spaces AME, BNF containing eddies. These have, however, little or no forward movement, and are not part of the stream. There are also eddies at K, L. In a case of abrupt enlargement (Fig. 4) the stream expands gradually, and there are eddies in the corners. Similar phenomena occur at abrupt bends, bifurcations, and junctions. For a closed channel or an orifice, Fig. 3 represents any longitudinal section. For an open channel or a weir, it represents a plan or a horizontal section, and its lower part—from PQR downwards—a vertical section. And similarly with Fig. 4. Sometimes still or 'dead' water may replace part of an eddy. The term eddy will be used to include it.

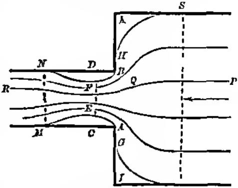


Fig. 3

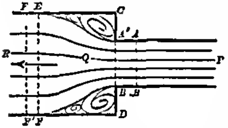


Fig. 4

Section III.—Useful Figures

6. **Weights and Measures.**—The following table[1] gives the weight of distilled water for various temperatures. The weights of clear river and spring water are practically the same as the above. For all ordinary practical purposes the weight of fresh water may be taken to be 62.4 lbs. per cubic foot when clear, and 62.5 lbs. or 1000 ounces when containing sediment. Water is compressed by about one twenty-thousandth part of its bulk by a pressure of one atmosphere. Sea water weighs about 64 lbs. per cubic foot. Water usually contains a small quantity of air in solution.

[1] Smith's *Hydraulics*, chap. i.

Temperature (Fahrenheit)	Pounds per Cubic Foot	Temperature (Fahrenheit)	Pounds per Cubic Foot	Temperature (Fahrenheit)	Pounds per Cubic Foot
32°	62 42	95°	62 06	160°	61 01
35°	62 42	100°	62 00	165°	60 90
39 3°	62 424	105°	61 93	170°	60 80
45°	62 42	110°	61 86	175°	60 69
50°	62 41	115°	61 79	180°	60 59
55	62 39	120°	61 72	185°	60 48
60°	62 37	125°	61 64	190°	60 36
65°	62 34	130	61 55	195°	60 25
70°	62 30	135°	61 47	200°	60 14
75°	62 26	140°	61 39	205°	60 02
80°	62 22	145°	61 30	210°	59 89
85°	62 17	150°	61 20	212°	59 84
90°	62 12	155°	61 11		

An Imperial gallon of water contains $\frac{1}{6.23}$ cubic feet, and weighs almost exactly 10 lbs A United States gallon is five sixths of an Imperial gallon A metre is 3 2809 feet, a cubic metre 35 317 cubic feet, a kilogram 2 2055 pounds avoirdupois, and a litre 61 027 cubic inches or 2201 gallons A cubic metre of water weighs 1000 kilograms The metric system being that chiefly employed on the continent of Europe, these figures may be useful in the conversion of figures given in reports of foreign experiments or investigations A French inch is 02707 of a metre or 0888 of an English foot

The units employed in this work are the foot, the second, and the pound Thus velocities and discharges are in feet or cubic feet per second, weights in pounds per cubic foot

7 **Gravity and Air Pressure**—The force of gravity, denoted by g, is generally assumed to be 32 2, that is, it is supposed to increase the velocity of a falling body by 32 2 feet per second, and $\sqrt{2g}$, a quantity very frequently occurring in hydraulics, is then 8 025 These figures are suitable for Great Britain and Canada, but the force of gravity varies with the locality, increasing with the latitude and decreasing with the height above sea level At the Equator at the sea level g is 32 09, and at the Pole at the sea level it is 32 26 The mean values of g and $\sqrt{2g}$ for ordinary elevations and for latitudes up to 70° are 32 16 and 8 02 respectively These are suitable for the United States, India, and Australia, and are adopted in this work They, however, differ by only 12 per cent and 06 per cent respectively from the values given above, and ordinarily this difference is of no account what ever An increase of elevation of 5000 feet decreases g by only 016 and $\sqrt{2g}$ by 002

The pressure of the atmosphere near the sea level is about 14 7 lbs. per square inch, and is equivalent to about 30 inches of mercury or 34 feet of water According to the 'English system' of computation by 'atmospheres,' one atmosphere is equivalent to 29 905 inches of mercury in London at a temperature of 32° Fahrenheit The French system gives a pressure which is greater in the ratio of 1 to 9997 For elevations above the sea level the atmospheric pressure decreases Up to a height of 6000 feet the reduction for every thousand feet is about 5 lb per square inch, or 1 inch of mercury, or 1 13 feet of water Above 6000 feet the reduction is less rapid, amounting to 1 9 lbs per square inch in rising from 6000 to 11,000 feet

SECTION IV—HISTORY AND REMARKS

8 **Historical Summary**—A historical sketch of Hydraulics given in the *Encyclopædia Britannica*[1] comprises the names of Castelli, Torricelli, Pascal, Mariotte, Newton, Pitot, Bernoulli, D'Alembert, Dubuat, Bossut, Prony, Eytelwein, Mallet, Vici, Hachette, and Bidone To these may be added Michelotti, D'Aubuisson, Castel, and Borda

Coming to specific branches of Hydraulics and recent periods, flow in pipes has been made the subject of experiment and investigation by Weisbach, Coulomb, Venturi, Couplet, Darcy, Lampe, Hagen, Poiseuille, Reynolds, Smith, and Stearns, and flow through apertures by Poncelet, Lesbros, Weisbach, Rennie, Blackwell, Boileau, Ellis, Bornemann, Thompson, Francis,[2] Unwin, Fteley and Stearns,[3] Herschel, Steckel, Fanning, and Smith[4] All the chief experiments on pipes and apertures have been discussed and summarised by Fanning[5] and Smith,[4] both of whom have compiled tables of co-efficients for pipes and apertures Smith's discussions show great care, and his figures and conclusions will be largely utilised in this work, but since the publication of his and Fanning's works further important experiments have been made on weirs by Bazin,[6] and on weirs and pipes by various American engineers[7]

[1] *Encyclopædia Britannica* 9th Edition Article 'Hydromechanics'

[2] *Lowell Hydraulic Experiments*

[3] *Transactions of the American Society of Civil Engineers*, vol xii

[4] *Hydraulics*

[5] *Treatise on Water Supply Engineering*

[6] *Annales des Ponts et Chaussées* 6th Series, Tomes 16 and 19 and 7th Series, Tomes 2, 7 12, and 15 A *résumé* is given in *L'Écoulement en Déversoir*

[7] *Transactions of the American Society of Civil Engineers*, vols xix.,

Regarding flow in open channels, extensive observations and investigations have been made by Darcy and Bazin[1] on small channels, by Humphreys and Abbott[2] on the Mississippi, and by Cunningham[3] on large canals. Many observations have also been made by German engineers and some by Revy[4] on the great South American rivers. In this branch of Hydraulics the Swiss engineers Ganguillet and Kutter have analysed most of the chief experiments,[5] including some made by themselves, and arrived at a series of *co-efficients for mean velocity*. Their writings have been translated and commented on by Jackson,[6] who has framed tables of co-efficients[7] based on their researches. Finally Bazin has reviewed the whole subject[8] and arrived at some fresh co-efficients. Investigations have been made by Francis[9] on rod floats, by Stearns[10] on current-meters, and by Kennedy[11] *on the* silt-transporting power of streams.

9. **Remarks.**—The different branches of Hydraulics are shown by the headings of chapters III to X of this work. In the following chapter the whole subject is considered in a general manner. This enables us to dispose once for all of many points which would otherwise have had to be mentioned in more than one of the subsequent chapters. Moreover, the different branches are not always divided by such hard and fast lines as might appear; there are many points common to two branches, and the preliminary consideration of the various branches of the subject in connection with one another instead of separately will be advantageous.

xxii, xxvi, xxviii, xxxii, xxxiv, xxxv, xxxvi, xxxviii, xl, xli, xlii, xliv, xlvi, xlvii.

[1] *Recherches Hydrauliques.*

[2] *Report on the Physics and Hydraulics of the Mississippi River.*

[3] *Roorkee Hydraulic Experiments.*

[4] *Hydraulics of Great Rivers.*

[5] *A General Formula for the Uniform Flow of Water in Rivers and other Channels.* Translated by Hering and Trautwine.

[6] *The New Formula for Mean Velocity in Rivers and Canals.* Translated by Jackson.

[7] *Canal and Culvert Tables.*

[8] *Étude d'une Nouvelle Formule pour Canaux Découverts.*

[9] *Lowell Hydraulic Experiments.*

[10] *Transactions of the American Society of Civil Engineers,* vol. xii.

[11] *Minutes of Proceedings, Institution of Civil Engineers,* vol. cxix.

CHAPTER II

GENERAL PRINCIPLES AND FORMULÆ

SECTION I—FIRST PRINCIPLES

1 **Bernoullli's Theorem**—Let Fig 5 represent a body of still water, the openings at f and i being supposed to be closed The

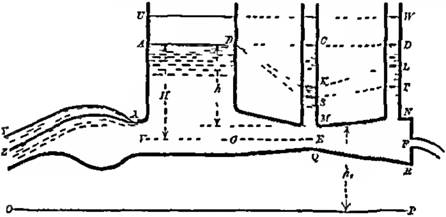


FIG 5

water in the tubes at C, D stands at the same level as AB The 'head' or 'hydrostatic head' over any point is its depth below the plane AB This plane is sometimes called the 'plane of charge' The pressure is as the head If P is the pressure per square foot at the depth H, and W the weight of one cubic foot of water, then $P=WH$ or $H=\frac{P}{W}$ The head H is said to be that 'due to' the pressure P

Every particle of water in the reservoir possesses the same degree of potential energy Comparing a particle at the depth H with one at the surface, the one possesses energy in virtue of its pressure, the other in virtue of its elevation

Let an orifice be opened at F so that water flows along the pipe GFF, and let the reservoir be large, so that the water in it has no velocity and the surface AB is unaltered The pressure in the water flowing in the pipe is reduced, and the water levels in the

tubes fall to K, L. The heights KM, LN are as the pressures at M and N, and they are called the 'hydraulic heads' or 'pressure heads.' The tubes are called 'pressure columns' and the line BKL the line of 'hydraulic gradient.' Let p be the pressure at M, and h_p the pressure head. Then $h_p=\frac{p}{w}$. Let V be the velocity in the pipe at M and let $h_v=\frac{V^2}{2g}$. Then h_v is the 'velocity head.' It is the height through which a body falls under the influence of gravity in an unresisting medium in acquiring the velocity V, or the height to which it could be made to rise by parting with its velocity. Let it be supposed that there are no resistances to the motion of the water, so that no energy is consumed in overcoming them. Then by the law of the conservation of energy the total energy of any moving particle of water remains as before. Whatever is lost as pressure is gained as velocity. The head ck lost in pressure is the velocity head h_v. Thus

$$h=h_p+h_v \qquad (3)$$

or the pressure head added to the velocity head is the hydrostatic head. This equation, due to Bernoulli,[1] is the basis of all theoretical hydraulic formulæ. It obviously applies to any point in the pipe.

It has been seen that the pressure at M is as the height KM. Assume that the velocities at all points in the cross section MO are equal. Let H_p and H_v be the pressure head and velocity head at P, then $\qquad H=H_p+H_v,\ h=h_p+h_v$.
But since the velocities are equal, $H_v=h_v$, therefore $H_p-h_p=H-h$, or the change in pressure in passing from M to P is the same as it was when there was no flow. The pressure head at P is KP, and the pressure at any point in the cross section is as its depth below K.

Let OP be a datum line and let h_e be the 'head of elevation' of any point M above OP. Then $h+h_e$ is constant for all points in the system, and therefore

$$h_p+h_v+h_e=K \qquad (4)$$

where K is constant. This is Bernoulli's theorem more fully stated. The total energy possessed by a particle of water is the sum of the energies due to its pressure, velocity, and elevation.

If instead of a pipe we consider an open channel XY, the results obtained will be the same as before. If pressure columns were used the water in them would not rise above the surface XY. At each point in the surface the pressure head is zero and the velocity

[1] The simple method of proof just given is not Bernoulli's, but is taken from Merriman's *Hydraulics*, chap. iii.

head is equal to the hydrostatic head. If the velocities at all points in a cross section are assumed to be equal, the law of change of pressure with depth is the same as before.

Since the area NR is greater than MQ, the velocity is less and the pressure greater. Thus from K to L there is a rise in the hydraulic gradient. Similarly, in the open stream there is a rise where the sectional area is increasing.

The pressure in a body of flowing water can never be negative, as the continuity of the liquid would be broken.

2. **Loss of Head from Resistances.**—Practically a certain amount of head h' is always expended in overcoming resistances, due to the friction of the water on its channel and to the internal movements of the water, so that the total head diminishes in going along the stream in the direction of the flow. In other words, the pressure head and velocity head do not together equal the hydrostatic head. The difference is the 'head lost.' The actual water-levels would in practice be S, T, and CS, DT would be the total losses of pressure head up to the points M and N. As head is lost, the work which the water is capable of doing in virtue of its elevation, pressure, and velocity is diminished. If h is the head lost by resistance between two cross sections, then

$$h = k - \frac{V_2^2 - V_1^2}{2g} \qquad (5),$$

or the head lost is equal to the fall in the surface or line of gradient less the increase in the velocity head. The same is true of the open channel. The surface would be AZ instead of AY.

3. **Atmospheric and other Pressures.**—Generally a body of water is subjected to the atmospheric pressure P_a. The head due to this pressure is $\frac{P_a}{W}$, and this has to be added in order to obtain the total head over any point. The case is the same as if the water surface at each point were raised from AD to UW by a height $\frac{P_a}{W}$. But usually—as in the preceding demonstrations—the relative heads over two or more points are considered, the pressure of the atmosphere affects all parts equally and is left out of consideration. If, however, different portions of the water are subjected to pressures of different intensities caused, say, by partly exhausted air, by steam, or by a weighted piston, the water surface of each portion of the system must be considered as being raised by a height $\frac{P}{W}$, where P is the intensity of the special pressure acting on it.

Section II.—Flow through Apertures

4. **Definitions.**—An aperture is said to be 'in a thin wall' when its upstream edge is sharp (Figs. 6 and 7), and the 'wall' or structure containing the aperture is thin, or is bevelled or stepped, so that the stream after passing the edge springs clear and does not touch it again. An aperture like that shown in Fig. 1 or Fig. 2, page 2, may have its upstream edge sharp, but it does not come within the definition.[1] A rounded or 'bell mouthed' orifice (Fig. 8) is one in which the sides are curved so that the tangents at *c* and *d* are parallel, and the stream after passing *CD* does not contract. A weir of analogous shape may be formed by rounding the angle between the top and the upstream side or 'face,' and by prolonging the side walls upstream.

Fig. 6.

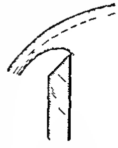


Fig. 7.

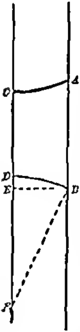


Fig. 8.

The upstream surface of the wall surrounding an aperture will be called the 'margin.' The margin is said to be 'clear' when it is free from projections, leakages, or anything which would interfere with the free flow of water along the wall towards the aperture. The clear margin, if not otherwise limited, is bounded by the sides of the reservoir or channel, or by any other aperture existing in the same wall. When an aperture has sharp edges an increase in the clear margin, up to a certain limit, increases the degree of contraction. When this limit has been reached the contraction is said to be 'complete.'

An aperture with sharp edges is 'normal' when the margin is a plane and the axis of the aperture is perpendicular to the plane. Any other aperture is normal when its sides and approaches are symmetrical with regard to any plane (in the case of a weir any vertical plane) through the axis. For a weir it is a further condition that the 'crest' or highest portion must be straight and horizontal from one side wall to the other, and, in the case of a weir in a thin wall, that the wall must be

[1] For this reason the expression 'sharp edged,' used by some recent writers in preference to the old one of 'in a thin wall' is not suitable.

vertical Every aperture is assumed to be normal unless the contrary is expressly stated

5 **Flow through Orifices**—Let H be the height of the free surface (Fig 9) above the centre of gravity of the small orifice C, D, or E, and let V be the velocity of the issuing jet Both the jet and the free surface AB are supposed to be subject to the atmospheric pressure P_a The total head over the orifice is $H+\frac{P_a}{W}$, and the pressure in and upon the issuing jet is P_a Then from equation 3 (page 10), supposing no head to be lost in overcoming resistances,

$$H+\frac{P_a}{W}=\frac{P_a}{W}+\frac{V^2}{2g},$$

or

$$V=\sqrt{2gH} \quad (6)$$

All formulæ for flow from apertures are modifications of this The velocity $\sqrt{2gH}$ is called the 'theoretical velocity' It is the same as would be acquired by a body falling from rest in a vacuum through a height H If the jet issues vertically upwards it will, in the absence of all resistance except gravity, rise to the level of AB The velocity depends only on H and not on the direction in which the jet issues If AGR is a parabola with axis vertical and parameter $2g$, the theoretical velocities of jets issuing at F, M, N are as the ordinates FG, MK, NR Practically owing to resistances caused by friction and internal movements of the water, the velocity of efflux is less than the theoretical velocity, and is given by the formula

$$V=c_v\sqrt{2gH} \quad (7),$$

where c_v is a 'co-efficient of velocity' whose mean value for the two kinds of orifices under consideration is about 97

Instead of assuming the water in the reservoir to have no appreciable motion, let it be supposed that it is moving with a velocity v directly towards the orifice This velocity is called 'velocity of approach' and the discharge through the orifice is increased The energy possessed by the water can, theoretically, raise it to a height $\frac{v^2}{2g}$ or h This is called the head due to the velocity of approach, and it must be added to the hydrostatic head Practically, for reasons which will be given below, a head nh has to be added, n being 1 0 or less The formula thus becomes

$$V=c_v\sqrt{2g(H+nh)} \quad (8)$$

If the fluid moved without resistance, a velocity v in any direction, and not only toward the orifice, could be utilised in increasing the

head and the discharge, but practically the only useful component of the velocity is that parallel to the axis of the orifice

In the case of an orifice in a thin wall (Fig 6), the jet attains a minimum cross section at AB, whose distance from the edge of the orifice is about half the diameter of the orifice, or half the least diameter if the orifice is of elongated form This minimum section is called the 'vena contracta' The ratio of its sectional area a to the area a of the orifice is called the 'co-efficient of contraction,' and is denoted by c_c thus $a=c_c a$ The mean value of c_c is about 63 A vena contracta occurs with any kind of orifice having sharp edges, and c_c is probably about the same For a bell mouth $c_c=1\ 0$

The discharge of an orifice is

$$Q=a\,v=ac_c c_v\sqrt{2gH}$$

Let $c_c c_v=c$ Then c is the 'co efficient of discharge' and

$$Q=ac\sqrt{2gH} \qquad (9)$$

Or when there is velocity of approach

$$Q=ac\sqrt{2g(H+nh)} \ . \qquad (10)$$

The value of c for orifices in thin walls averages about 61, and for bell mouthed orifices 97 It does not usually vary much with the head Generally the values of c_v, c_c, and c are not very greatly affected by the shape and size of an orifice nor by the amount of head Generally c is better known than c_v or c_c, and it is also of far more importance

When an orifice has a head of water on both sides it is said to be 'submerged' or 'drowned,' and H in the formula is the difference between the two heads Thus for any orifice Q or O (Fig 9), the head is BW It has nothing to do with the actual depth of the orifice below AB If an orifice is partly submerged it must be divided into two parts and only the lower part treated as submerged If the water level at Y is higher than at X, as it may be when XUY is a stream whose size is not very great relatively to that of the orifice, the head is IX and not IW[1] It is the pressure at X and not at Y

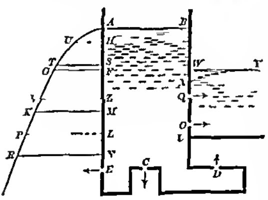


Fig 9

[1] Smith's *Hydraulics*, chap iii

that affects the discharge from the orifice The rise from A to Y is owing to the stream being in 'variable flow' (art 10)

When an orifice is in a horizontal plane, or when it is submerged, formulæ 7 to 10 apply, no matter what the size of the orifice may be When an orifice is in a vertical or inclined plane the theoretical velocity of each horizontal layer of water is $\sqrt{2gH}$, where H is the head over that layer When the vertical height between the upper and lower edges of the orifice is small compared to the head, the mean velocity in the orifice is practically that at its centre of gravity If an orifice extends from M to N (Fig 9), its centre being L, it is clear that, the curve KR being nearly straight, LP is practically the mean of all ordinates from M to N But with an orifice HZ, whose centre is F, the protuberance of the curve UV causes the mean ordinate to fall short of that at F, and a correction has to be applied depending on the shape of the orifice and the ratio of its depth to the head over its centre

6 **Flow over Weirs**—Unless the contrary is stated, it will be assumed that all weirs have vertical side walls, such forming in practice the vast majority The remarks just made regarding the protuberance of the curve apply *a fortiori* to a weir Let M (Fig 9) be the level of the crest of a weir Let $AM=H$ and $AS=\frac{4H}{9}$ The mean of all the velocities from A to M is represented by ST[1] Thus the theoretical velocity V is $\sqrt{2g\frac{4H}{9}}$ or $\frac{2}{3}\sqrt{2gH}$ The practical formula is

$$Q=\frac{2}{3}cl\sqrt{2g}\,H^{\frac{3}{2}} \qquad (11)$$

where l is the length of the crest, H the head on the crest, and c is a co-efficient of discharge whose value for sharp-edged weirs averages about 62, and for others varies greatly according to the form of the weir With increase of head the co-efficient increases in some cases and decreases in others It is not usual to give a separate formula for finding v or to divide c into c_v and c_c but roughly these are about the same for sharp-edged weirs as for sharp edged orifices If there is velocity of approach the formula is

$$Q=\frac{2}{3}cl\sqrt{2g}\,(H+nh)^{\frac{3}{2}} \qquad (12)$$

where n is 1 0 or more, and h, as for orifices is $\frac{v^2}{2g}$, v being the velocity of approach

[1] For proof see chap. iii art. 19

When the water on the downstream side of the weir or 'tail water' rises above its crest (Fig 10), the weir is said to be 'submerged' or 'drowned' instead of being 'free' The discharge of AB is found by the ordinary weir formulæ, equations 11 and 12 The discharge of BC is considered as being that of a submerged orifice BC under a head AB, and is found by equation 9 or 10

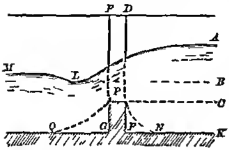


Fig 10

The same procedure is adopted when a sudden fall occurs in the surface of a stream owing, not to a weir, but to a lateral contraction of the channel The length l in equation 11 or 12 and the area a in equation 9 or 10 are measured downstream of DE where the contraction occurs The question whether the stream expands again at FG or continues contracted for an indefinite distance may affect the co-efficient to be used, but does not affect the formulæ When the fall AB is small compared to LC in the case of a weir, or to BK in the case of a contracted channel, equation 9 or 10 alone is often used The tail water level, which should theoretically be measured at L (see remarks regarding submerged orifices in the preceding article) is measured at M The co efficients for most such cases are imperfectly known, and refinements as to details are unnecessary See also article 19

7 Concerning both Orifices and Weirs —With all kinds of apertures small heads are troublesome, not only because of the difficulty in measuring them exactly, but because complications occur, and the co efficients are not properly known

At a weir the water surface always begins to fall at a point A (Fig 11) situated a short distance upstream of the weir Hence, whatever the crest and end contractions may be, there is always surface contraction The angular spaces between the wall and the bed and sides of the channel are occupied by eddies The fall in the surface begins where the eddies begin From this point the section of the stream proper or forward moving water diminishes, its velocity and momentum increase, and the increased surface fall is necessary to give the increased momentum (art 10) A similar fall occurs upstream of an orifice, though it may only be perceptible when the orifice is near the surface

The section where the eddies begin will be termed the 'approach

section. It is here that the head should be measured and the velocity of approach observed or calculated, but when, as often happens with a weir, and generally with an orifice, the surface upstream of A is nearly level, the head may be observed either at A or up stream of it. It must not be observed down stream of A. In some of the older observations on weirs the head

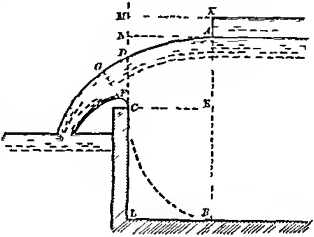


FIG. 11

was measured from D to C instead of from A to E_1 but the co-efficients thus obtained are more variable, and it is very difficult in practice to observe the water level at D with accuracy. The section for velocity of approach may be shifted either way from AB provided its area is not appreciably altered.

The velocity of approach v, is the discharge, Q, of the aperture divided by the area, A, of the approach section. If water enters a reservoir in such a manner as to cause a defined local current towards the aperture, the sectional area of the current may be estimated or observed, and this area not that of the whole cross section of the reservoir, used for determining the velocity of approach. If the axis of an aperture is oblique to the direction of the approaching water the component of the velocity of the latter parallel to the axis of the aperture may be taken to be the *velocity of approach*. *Equations 8, 10 and 12 cannot be solved* directly because, until Q or V is known v and h are unknown. It is impossible to find v by direct observation in the case of a proposed structure or unless the water is actually flowing and even then it is not a convenient process. The usual procedure is to estimate a value for v, calculate h, solve equation 10 or 12, divide by A, and thus find a corrected value for v. If this differs much from the value first assumed it can be substituted and Q calculated afresh. Velocity of approach has very little effect when the area of the approach section is about fifteen times that of the smallest

section of the stream issuing from the aperture, that is for a sharp edged aperture nine or ten times the area of the aperture, and for a bell mouthed orifice fifteen times the area of the orifice. In a weir the height of the aperture is to be considered AE, not DC.

In order that the contraction may be complete the margin must be clear for a distance from the aperture extending in all directions to about three times the least dimension of the aperture. Any further extension has no effect. If the ratio of the width of the clear margin to the least dimension of the aperture is reduced to 2.67 and 2.0, the discharge is *increased* by only about 1.6 and 5.0 per cent respectively, so that practically a ratio of 2.75 is sufficient and will be so regarded. In a weir the length of crest is usually the greater dimension and the least dimension is then the head AE and not DC. Another condition essential for complete crest contraction is that air shall have free access to the space under the issuing stream. In an aperture in a thin wall with complete contraction air usually has free access unless the tail water rises very nearly to the crest or lower edge when its surging may shut out the air. In a weir with no end contractions the width of the channel, both upstream and downstream of the weir, is very likely, the same as the length of the crest, and air will be excluded unless openings in the sides of the downstream channel are provided to admit it. Any want of free admission of air causes the sheet of water to be pressed down by the air above it, the contraction is reduced and various complications may occur. It is also necessary for complete contraction that the edges be perfectly sharp. Any rounding increases the discharge.

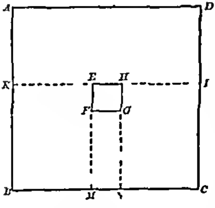


Fig 12

In Figs 12 and 13 $ABCD$ is the boundary of the minimum clear margin necessary to give full contraction, supposing $EFGH$ to be an orifice, $KICL$ the boundary supposing it to be a weir, and $FMNG$ supposing it to be a weir with no end contractions. In Fig 13 $EH = EF \times 20$. The ratios of the areas within these boundaries to those of the apertures are 42.25, 24.38, and 3.75 in Fig 12 and 8.29, 4.78, and 3.75 in Fig 13. It is thus clear that of the two conditions namely, sufficiency of the marginal area to give full

contraction and sufficiency of the area of the approach section to give a negligible velocity of approach, one does not necessarily imply the other. The two matters must be kept distinct. An elongated aperture, especially a weir, is most likely to have a high velocity of approach and a square aperture, especially an orifice, to have incomplete contraction. Even when the area of the approach

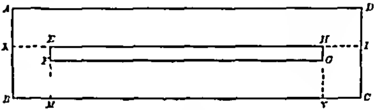


Fig. 13

section is very large, it may allow of incomplete contraction in a portion of an aperture if unsymmetrically situated.

The co-efficients for apertures in thin walls are known with more exactness than for others, but they are best known for orifices when the contraction is complete, and for weirs either when it is complete on all three sides or complete at the crest and absent at the sides. The co-efficient n for velocity of approach is not very accurately known. Hence very high velocities of approach are objectionable where Q has to be accurately computed from assumed co-efficients, but when v is not very high, that is, when the area A is more than three times that of the smallest section of the issuing stream, Q depends very little on n.

The fall in the surface upstream of an aperture, the rise CF due to crest contraction in a sharp-edged weir, and the effect of velocity of approach greatly complicate the theoretical discussion of weir formulæ.

Section III.—Flow in Channels

8. **Definitions.**—The 'border,' or 'wet border,' B, of a stream is the perimeter of its cross section, omitting in the case of an open stream, the surface width. The 'hydraulic radius,' R, also called in the case of an open stream the 'hydraulic mean depth,' is the sectional area A divided by the border. Thus $R=\frac{A}{B}$. The flow of a stream is 'uniform' when the mean velocities at successive cross sections are equal, that is, when the areas of the cross sections are equal. Otherwise the flow is 'variable.' A pipe is

uniform when all its cross-sections are of equal area The flow in such a channel must be uniform when it is flowing full An open channel is uniform when it has a constant bed slope and a uniform cross section The flow in such a channel is uniform when the water surface is parallel to the bed, but otherwise it is variable The 'inclination' or 'surface slope' of an open stream is the 'fall' or difference between the water levels at any two points divided by the horizontal distance between them The 'virtual slope' or 'virtual inclination' of a pipe is the difference between the levels of two points in the hydraulic gradient divided by the horizontal distance between them

9 **Uniform Flow in Channels**—When a stream flows over a solid surface the frictional resistance is independent of the pressure, and approximately proportional to the area of the surface, and to the square of the velocity Thus if f is the resistance for an area of one square foot at a velocity of one foot per second the resistance for an area A and a velocity V is nearly fAV^2 The value of f increases with the roughness of the surface

In the case of a uniform stream open or closed, $ACDB$ (Fig 14) the second term on the right in equation 5 (p 11) vanishes and the loss of head h in a length L is equal to the fall in the surface or in the hydraulic gradient In an open stream the pressures on the ends AC, BD of the mass of water are equal and the accelerating force is that component of its weight which acts parallel to its axis or $WAL\frac{h}{L}$ On the assumption that the resistance is entirely due to friction between the stream and its channel, the resistance is approximately $fPLV^2$ Since the motion is uniform this is equal to the accelerating force or

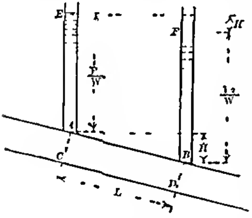


Fig 14

$$V^2=\frac{W}{f}\frac{A}{P}\frac{h}{L}$$

But $\frac{A}{P}=R$ and $\frac{h}{L}=S$ the surface slope of the stream Let

$\frac{W}{f}=C^2$ Then $$h=\frac{V^2L}{C^2R} \qquad (13),$$

or $$V=C\sqrt{RS} \qquad (14)$$

where C is a co-efficient In the case of a uniform pipe the pressures on the ends have to be taken into consideration, but the resulting equation is the same, S being the hydraulic gradient EF For if P_1 and P_2 are the pressures at A and B, the resultant pressure on the mass $ACDB$, resolved parallel to its axis, is $A(P_1-P_2)$ or $WA\left(\frac{P_1}{W}-\frac{P_2}{W}\right)$ or $WA(h-h)$ The component of the weight parallel to the axis is as before WAh These two together are WAh Equation 14 is the usual formula for uniform flow in streams It is known as the 'Chezy' formula Obviously the co efficient C is greater the smoother the channel The formula for the discharge is

$$Q=AC\sqrt{RS} \qquad (15)$$

The theoretical proof just given takes no account of the resistances due to the internal motions of the fluid, nor of the facts that the velocities at all the different points in the cross section differ from one another, that the mean velocity V of the whole is greater than the mean velocity v of the portions in contact with the border, and that the frictional resistance may not be exactly as V^2, nor even as v^2 Practically, it is found that the co-efficient C depends not only on the nature of the channel, but on R and S The co-efficient increases with R, that is, generally with the size of the stream It depends also to some extent on S, and perhaps on other factors which will be mentioned It increases with S in pipes of the sizes met with in practice, and in open streams of small hydraulic radius The value of C varies generally between 40 and 120 for earthen channels, and between 80 and 160 for clean pipes The chief difficulty with all kinds of channels consists in forming a correct estimate of the value of C The difficulty is the greater because the roughness of a particular channel may be altered by deposits or other changes

Let an open stream of rectangular cross section have a depth of water D, width W, and velocity V Let W be great relatively to D, then R is practically equal to D and the fall in a length L is $\frac{V^2L}{C^2D}$ Let other reaches of the same stream have equal lengths, but widths $2W$, $3W$, etc, the longitudinal slopes being flatter, so that D is the same in all The velocities will be

$\frac{V}{2}$, $\frac{V}{3}$, etc, and the losses of head will be $\frac{V^2L}{4C^2D}$, $\frac{V^2L}{9C^2D}$, etc The total loss of head in two reaches of widths W and $3W$ is $\frac{V^2L}{C^2D}(1+\frac{1}{9})$ The loss of head in two reaches, each of width $2W$, will be $\frac{V^2L}{C^2D}(\frac{1}{4}+\frac{1}{4})$ Thus, the loss of head in a reach of length $2L$ and width $2W$ is less than half the loss in an equal length of the same mean width, but in which the width is W for half the length and $3W$ for the other half If the streams compared have circular or semicircular sections the difference is still greater Thus, in conveying a given discharge to a given distance, the advantage as regards fall is on the side of uniformity in velocity

10 Variable Flow in Channels.—When the flow is variable the loss of head from resistances is the same as in a uniform stream, that is $\frac{V^2L}{C^2R}$, provided the change of section is gradual and the length L short, so that the velocity and hydraulic radius change only a little, say by 10 per cent, V and R being their mean values Then, from equation 5 (p 11) the fall in the surface or hydraulic gradient in the length L is

$$h=\frac{V^2L}{C^2R}-\frac{V_1^2-V_2^2}{2g} \qquad (16)$$

where V_1 and V_2 are the velocities at the beginning and end of the length L The equation may be written

$$V=C\sqrt{R}\sqrt{\frac{h+h_v}{L}} \qquad (17)$$

where $h_v=\frac{V_1^2-V_2^2}{2g}$ This is the equation for variable flow in streams It is the same as equation 14 $\left(\text{since } S=\frac{h}{L}\right)$ with the addition of the quantity h_v, which is introduced because of the change in the *vis viva* of the water The quantity V_1^2 is the square of the means of all the different velocities in the cross section It ought strictly to be the mean of the squares In a case which was worked out, it was found to be 3 3 per cent in excess But a nearly equal error occurs with V_2 The quantity h_v thus represents the change of *vis viva* without appreciable error

If the section of the stream is decreasing, V_1 is less than V_2, h_v is negative, and V is less than it would be in a uniform stream with

the same values of R and S. Or, V being the same, the fall h in the surface, or in the hydraulic gradient, is greater than in a uniform stream. This is because work is being 'stored' in the water as its velocity increases. If the section is increasing V_1 is greater than V_2, h_2 is positive, and V is greater than in a uniform stream, or V being the same, h is less. Work is being 'restored' by the water. There may even be a rise in the surface or line of hydraulic gradient instead of a fall.

Consider any stream AE (Fig 15) in which the sectional areas A and E are equal and the velocities therefore equal, and let the area D be not more than 10 per cent greater than C. Make C' and C'' each equal to C. Evidently the quantities h_2 for the lengths AC', $C''E$ will be equal, but of opposite signs, and the total fall in the surface in $AC'+C''E$ will be the same as if the flow were uniform and the section of the stream were an average between the sections at A and C'. The same is true of the length $C'C$ and of CC''. It does not matter whether the fluctuations in section are due to changes in the width or in the depth, or both. The formula $V=C\sqrt{RS}$ therefore applies to a variable stream AE if the velocities at both ends of it are equal and the fluctuations moderate, but evidently it does not apply any the better to a short length of such a stream in which the velocities at the ends are not equal. Evidently in such a stream S varies from point to point. It is greater as A is less. S in the formula must be deduced from the total fall.

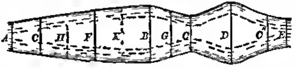


Fig 15

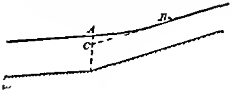
Now let the fluctuations be so great that the reaches must be subdivided before the equation can be applied to them. Make F equal to G. The fall in $CF+GC$ is the same as in a uniform stream of section H. The fall in $FB+BC$ is the same as in a uniform stream of section K. The total fall in CG is the same as the sum of the falls in two uniform streams of sections H and K. This total fall is (art 9) greater than that in a uniform stream, having a section equal to the mean of H and K. It will also be seen in section v that if there are any abrupt changes the falls at the contractions are by no means counterbalanced by the rises at the expansions. Thus a variable stream is less efficient than a uniform stream of the same mean section, or in other words, it must have a greater total fall in order to carry the same discharge.

This and the result arrived at in article 9 are analogous to other mechanical law. Uniformity in speed is best, slight fluctuations are unimportant, but great, and especially abrupt, fluctuations give reduced efficiency.

It is clear that the formula $V=C\sqrt{RS}$ applies to the case last considered if a suitable value is given to C and S is the slope deduced from the total fall. It even applies approximately to a stream in which the two end velocities are not equal, provided the length is considerable, so that h_a is small relatively to h. It applies to such a case still more nearly if the value assigned to C is such as to take account of the change in the end velocity, C being greater than for uniform flow if V increases and less if it decreases. It may not always be easy to say how much C should be altered in such a case, but it may still be highly convenient to use the formula in generalising regarding such a stream, for instance in comparing the discharges for two different water levels or stages of supply in an open stream. Thus the formula for uniform flow applies either exactly or nearly to a vast number of cases met with in practice in which more or less approximate uniformity of flow exists.

11 Concerning both Uniform and Variable Flow.—Pipes are nearly always of approximately uniform section, and the flow in them nearly uniform, but the sections are seldom exactly equal. Open channels are sometimes nearly uniform and, if there is no disturbing cause, the flow is nearly uniform. But in both cases much confusion and error have been caused by applying the formula for uniform flow to variable streams of short lengths or, supposing the short length to be uniform, by carrying the slope or hydraulic gradient observations into variable reaches.

Owing to a change, for instance a change of slope, or of section, or a weir, in a uniform open stream, the water may be 'headed up' (Fig 16) or 'drawn down' (Fig 17) for a great distance, AB, upstream of the point of change. In these cases the surface slope AB differs from the bed slope and the *flow is variable*



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although the channel is uniform. Heading up is also known as 'afflux' or 'back water.' In all such cases the water surface AB, which would, if the upstream reach had continued without any change, have followed the line AC, has to accommodate itself to

the downstream level at A, and assumes a curve such that the surface-slope changes in the opposite manner to the sectional area. Downstream of A the flow is uniform. In uniform closed channels the section of the stream cannot vary, and if from any cause the gradient level at any point is altered, the change of slope runs back to the commencement of the pipe.

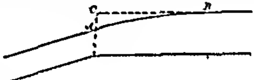

Fig 17

In the absence of any disturbing cause, that is when the flow is uniform throughout, it is obvious from equations 14 and 15 that in an open stream an increase of discharge is accompanied by a rise of water level and *vice versa*. The same is the case in a variable stream. In uniform flow in an open stream, the dimensions and slope of the channel being known, the discharge can be found if the water level is given and *vice versa*. The surface slope is the same as the bed slope. In variable flow the surface slope may be very different from the bed slope, and it is necessary to know the water levels at two points in order to find the discharge, or to know the discharge and the water level at one point in order to find the water level at the other point.

A large stream, whether in an open or closed channel, has an advantage over a small one both in sectional area and in velocity. For as A increases R usually increases, and with it C. If the slopes are equal Q is much greater for the larger stream. If Q is the same for both, S is much less, that is the loss of head is less, for the larger stream. This applies to variable as well as to uniform streams. A fire hose of diameter D is fitted at its end with a tapering 'nozzle' whose least diameter d is perhaps $\frac{D}{3}$, so that the velocity of the issuing jet is nine times the velocity in the hose. If the hose were made of diameter d the loss of head in it would be greatly increased, and more pressure would be required to drive the water through it. The size is limited by convenience in handling. If part of the hose stretches under pressure, so that the flow is variable, there is a gain all the same. Again, let Fig 16 represent an irrigation distributary with discharge Q, the bed slope downstream of A being the same as upstream, so that BC is the water level. To supply water to high ground near A a dam may be made, raising the surface to JA, and enabling a discharge q to be drawn off at A, whereas a small

branch made for this purpose from B, with a slope such as BA, might discharge hardly any water

The theoretical proof (art 1) regarding the variation of pressure with depth depended on the assumption that the velocities at all points in a cross section were equal Though they are not equal, it is found in practice that the law holds good

12 **Relative Velocities in Cross section** —The velocity at any point in a straight uniform stream flowing in a channel is, generally speaking, greater the further the point is removed from the border The border retards the motion of the water next to it, and the retardation is thus communicated to the rest of the stream In a pipe of square or circular section the velocity is greatest at the axis, and thence decreases gradually to the border In an open channel the form of cross section varies greatly in different streams, and the distribution of the velocities varies with it The distribution of velocities in the cross section of a variable stream, provided the section of the channel changes gradually, is practically the same as if the *flow* were uniform *The distribution* depends on the form of the section, and is not likely to be appreciably affected by the fact that the whole velocity is slowly changing In all cases the velocity changes more rapidly near the border (probably very rapidly quite close to the border, but observations cannot be made there) and less rapidly towards the centre of the stream Thus all velocity curves are convex downstream Nothing in this article relates to the velocities at or near to abrupt changes of any kind

13 **Bends** —In flow round a bend the distribution of velocities is modified, the line of greatest velocity being shifted, by reason of the centrifugal force, towards the outer side of the bend, and all the velocities on the outer side being increased while those on the inner side are reduced The loss of head from resistance in a bend is greater than in the same length of straight channel The additional resistance is chiefly caused by work done in redistributing the velocities consequent on the transfer of the maximum line from its normal to its new position, and in the fresh redistribution after the bend is passed This fresh redistribution cannot be effected instantaneously, so that the normal distribution is not restored till some distance below the termination of the bend Besides these resistances it is probable that wherever the distribution is abnormal, no matter whether any redistribution is in actual *progress* or not, the *resistance is greater*, owing to the high velocities near the border on the outer side of the bend

For a given channel and given radius of bend the total resistance or loss of head caused by the bend is not proportional to its length because, however long it may be, the redistribution has to be effected only twice. If the lower half of a bend is reversed in position, thus forming two curves, the loss of head in the whole bend is greater than before, because the redistribution of velocities has now to be effected in the opposite direction, doubling the work of this kind done before. No abnormal distribution of velocities occurs upstream of a bend unless, as in the case of an earthen channel, the section of the stream is also abnormal a little upstream of the bend. The laws regarding bends, both in pipes and open channels, are imperfectly known. Recent experiments on large pipes show that, for a given angle subtended by a bend, a small radius of bend is, down to a certain limit, preferable to a large radius. This is contrary to what has hitherto been believed. Flow round a bend may be either uniform or variable. If the section of the stream is the same as in the straight reaches, the slope of the surface or gradient must be greater, and there will be heading up in the upstream reach.

SECTION IV.—CONCERNING BOTH APERTURES AND CHANNELS

14. **Comparisons of different cases.**—The difference between the case of an aperture and that of a channel depends on the nature of the work done. It is a difference of degree and not of kind. In flow through a small orifice in the side of a large reservoir a mass of water which is at rest has a velocity impressed on it. The motive power is the pressure of the water due to the head, and the work done consists almost entirely in imparting momentum to the water, friction and resistance being unimportant. In uniform flow in a channel a mass of water slides, under the influence of gravity, with a constant velocity. The motive power is that component of the weight of the water which acts parallel to the surface or line of gradient, and the work done consists in overcoming friction and the resistance caused by internal movements. No fresh momentum is imparted. These are the two extreme cases. In flow through some kinds of apertures there are considerable resistances and in variable flow in channels much of the work may consist in the imparting of momentum. The two extreme cases thus merge one into the other.[1] Most cases of

[1] Fig. 10, p. 16, may be regarded as a case of variable flow.

abrupt changes in channels, dealt with in articles 17 to 21, occupy an intermediate position

Comparing channels or apertures which entirely surround the flowing stream with those which leave the water-surface free, it will be found that the latter are far more elastic than the former In the case of the pipe GEF (Fig 5, p 9) and the orifice C (Fig 9, p 14), if it is desired to double the discharge, it is necessary to quadruple the head or the hydraulic gradient In either case a very great rise in the water level AB is required But for a weir, since Q is roughly as $H^{\frac{3}{2}}$, in order to double Q it is only necessary to increase H by some 60 per cent For an open channel with vertical sides the discharge—recollecting that C increases with R—is doubled by increasing the depth about 50 per cent The above comparisons do not of course take exact account of variations in the co efficients For an open channel with sloping sides the discharging power may vary very greatly for a quite moderate change of water level When the changes in the conditions governing the flow are slight, so that the co efficient is practically unaltered, the changes in the discharge are as follows a change of 1 per cent in the head over an orifice or in the slope of a channel changes the discharge 5 per cent, a change of 1 per cent in the head on a weir or in the sectional area of a stream changes the discharge 1 5 per cent

A 'module' is an arrangement by which it is sought to ensure a constant discharge of water from a fluctuating source of supply Generally it is a machine which automatically alters the size or position of an aperture as the water level varies Some modules are imperfect, and in such cases, having regard to the preceding paragraph, it is clearly best that the water to be delivered should pass through an orifice or pipe, and the surplus over a weir or through an open channel In Foote's module (Fig 18) a gate E, regulated at intervals by hand, causes the water level in the canal at C to be nearly constant, and higher than at D By an orifice F water flows into the tank F A, and on to the branch AB, the surplus passing over a

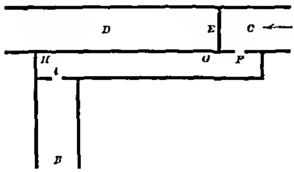

Fig. 18

weir *GH*. The regulation is better the longer the weir, but it would be improved by so arranging the gate *I* that the water would flow over it instead of under it.

Even if the water in a canal is steady, an outlet consisting of an orifice of fixed size will not, if submerged, give a constant discharge if the branch channel is liable to be altered. If it is enlarged, its water-level falls, and thus the head at the outlet is increased. The limit is not reached until there is a free fall.

15. **Special Conditions affecting Flow.**—The condition of water, as for instance its temperature or the amount of suspended matter which it contains, has in some cases an effect on the flow. A rise in the temperature of water probably causes an increase in the discharge, while an increase in the suspended matter causes, for flow in channels, a decrease. But it seems that appreciable changes in the discharge are caused only by great changes in the conditions, and scarcely even then unless the channels or apertures are small and the velocities also low.

For velocities under six inches per second the frictional resistance of water flowing over a solid is not as V^2. For velocities of one inch per second and less it is nearly as V. At very low velocities the nature of flow in pipes is essentially different from that at ordinary velocities. For any given pipe there is a certain 'critical velocity.' For velocities lower than this the motion is in parallel filaments, V varies nearly as S and as R^2 and increases with the temperature of the water. With pipes whose diameter was ·03 inch or less, V, when below the critical velocity, was found to be trebled as the temperature rose from 0° to 45° Centigrade. With larger pipes some increase occurs. When the velocity in a pipe rises to the critical amount, a very rapid or even sudden change occurs, the motion becoming first sinuous and then eddying. Reynolds, who made investigations with very small pipes, concluded that the critical velocity was higher the smaller the pipe. Thrupp[1] states that with pipes having a hydraulic radius of two inches and more the critical velocity increases with the hydraulic radius, and that there is a similar law for open streams, but no details of his observations have been published. It is not known how flow through apertures is affected, if at all. Experiments made by Shaw[2] with very small bodies of water—he used films whose thickness did not exceed $\frac{1}{40}$ of an inch—tend to show that the water immediately adjoining the

[1] *Engineering*, vol. lxxii. p. 834 and *Min. Proc. Inst. C.E.*, vol. cxlvii.
[2] *Engineering*, vol. lxiv. p. 00; vol. lxv. p. 444; vol. lxvii. p. 28.

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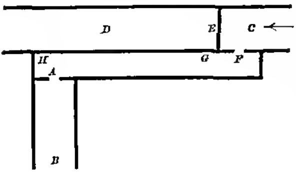

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[1] *Engineering* vol. lxxii p 834 and *Min Proc Inst C E* vol cxlvii

[2] *Engineering*, vol lxiv p 90 vol lxv p 444 vol lxvii p 28

channel moves in parallel lines, and that in going further away from the border sinuous or eddying motion takes place suddenly

16 Remarks.—The solution of a numerical question in Hydraulics by means of formulæ may be either direct or indirect. When the conditions are given and the discharge, say, is to be found, it is only necessary to look out the proper co-efficient and apply the formula. But frequently the problem is inverted and consists in finding a suitable set of conditions to give a particular result. This is especially the case when channels or structures have to be designed. In many cases a direct solution cannot be obtained by inverting the formula, either because its form is unsuitable—an instance of this has been given in article 7—or because the co-efficients are not known until the conditions are determined. It is often necessary to obtain an indirect solution by assuming a certain set of conditions, calculating the discharge or other quantity sought, and, if it is not what is desired, making alterations in the assumed conditions and calculating afresh. In order to facilitate calculations which would otherwise become very tedious, numerous working tables are given. By their use work is vastly reduced.

Both in apertures and channels the co-efficients in the formulæ vary more or less as above stated. Various attempts have been made to modify the formulæ (putting for instance H^m, R^n, S^p, instead of $H^{\frac{1}{2}}$, $R^{\frac{1}{2}}$, $S^{\frac{1}{2}}$) in such a way as to make the co-efficient constant. Such formulæ either have a restricted range or else the functions of H, R, and S involved are very inconvenient. It is far better to adhere to the simple indices in common use and to accept the variations in the co-efficients.

Although for discharge computation one should avoid complex conditions such as incomplete contraction, small heads, high velocity of approach, or variability of flow, yet in practice an engineer is frequently compelled to accept such conditions, and some attention will be given to methods of dealing with them.

In many of the more complicated cases (such as some considered in the following section and in chap. VIII.) it may be difficult to arrive at any exact results by calculation but it may still be most useful to recognise the existence of the phenomena referred to and to take note of their general effects.

SECTION V.—ABRUPT AND OTHER CHANGES IN A CHANNEL

17 **Abrupt Changes**.—Any change in a channel whether of sectional area or direction, and whether or not there is a bifurcation or junction, which is so *sudden as to cause contraction* or eddies is called an abrupt change At an abrupt change the first term on the right in equation 5 (p 11) is omitted It would be small because of the small length of stream considered, and owing to the stream being bounded partly by eddies and changing rapidly in form, it would be difficult to assign values to the quantities I and C The second term only is used Thus the formulæ are analogous to, or identical with, those for apertures. In fact abrupt changes include submerged weirs and (in certain respects which will be specially noted) other apertures

At abrupt changes there are special losses of head, owing to work being expended on eddies The length and violence of the eddies at an enlargement are much greater than at a corresponding contraction (Figs 3 and 4, p. 5) and the loss of head is consequently much greater At a contraction the pressure at K, L is slightly greater, and in the case of an open stream the water level slightly higher than in the flowing stream These remarks apply also to orifices and weirs with which there is velocity of approach At an expansion the conditions are the reverse The loss of head at an abrupt change of any kind is most important when the velocity is high, it can seldom be calculated with exactness, and often can only be roughly estimated

18 **Abrupt Enlargement**.—At an abrupt enlargement (Fig 4) the loss of head due to the enlargement can be found theoretically by assuming that the intensity of pressure on A C, FD is the same as at A B Let V_1, A_1, be the velocity and sectional area at AB, P_1 the pressure on its centre of gravity, and V_2, A_2 P_2, similar quantities at EF The force $A_2(P_2-P_1)$ causes the velocity to be reduced from V_1 to V_2 In a short time, t, the fluid $ABFE$ comes to $A'B'F'E'$ Since the momentum of $A'B'FE$ is unchanged the change of momentum in the whole mass is the difference between that of $ABB'A'$ and that of $EFF'E'$, and that is

$$WQt\left(\frac{V_1}{g}-\frac{V_2}{g}\right)$$

where W is the weight of a cubic foot of water and Q is the discharge per second This change of momentum is equal to the impulse $A_2(P_2-P_1)\,t$, therefore

$$A\,(P_2-P_1)=\frac{WA_2V_2(V_1-V_2)}{g}$$

or
$$\frac{P_2-P_1}{W}=\frac{V_2(V_1-V_2)}{g}$$

But $\frac{P_1-P}{W}$ is the fall h in the surface or line of gradient, therefore from equation 5 (p 11)

$$h+\frac{P_2-P_1}{W}=\frac{V_1^2-V_2^2}{2g},$$

subtracting the preceding equation from this

$$h=\frac{V_1^2-V_2^2-2V_1V_2+2V_2^2}{2g}=\frac{(V_1-V_2)^2}{2g} \qquad (18)$$

or the loss of head is the head due to the relative velocity of the two streams In order to simplify the calculation it has been assumed that the stream flows horizontally, that is that the centres of gravity of the sections AB, EF are at one level but the loss of head due to the enlargement is the same in any case The pressure in the eddy has been found to be really less than in the jet, so that the assumption made is incorrect, and the formula has been found in practice to give incorrect results for small pressures and velocities, but for other cases it is fairly accurate

Equation 18 is of the same form as the equation giving the loss by shock, in a case of impact of inelastic solid bodies, and the loss of head due to an abrupt enlargement is often called 'loss by shock though there is not really any shock, the stream always expanding gradually

If there were no loss of head in the length AE there would be a rise of $\frac{V_1^2-V_2^2}{2g}$ in the surface or hydraulic gradient In a pipe the loss of head $\frac{(V_1-V_2)^2}{2g}$ is always much less than $\frac{V_1^2-V_2^2}{2g}$, and there is actually a rise whose amount is approximately

$$\frac{V_2(V_1-V_2)}{g} \qquad (18\text{A})$$

This proof is usually given only for a pipe but it clearly applies to an open stream if there is no rise in the surface If there is a rise the pressure on the wave QI, supposing Fig 4 to be a vertical section, is not P but P_a (the atmospheric pressure) and the loss of head is greater than $\frac{(V_1-V_2)^2}{2g}$ Moreover, the section usually changes not only in size but in form and the redistribution of

the velocities absorbs more work The rise in the water level is thus generally slight, and it cannot usually be calculated accurately

When an enlargement is immediately succeeded by a contraction so as to cause a deep recess the water in the recess has little or no forward motion, and the flow is practically the same as if the recess did not exist

19 **Abrupt Contraction**—At an abrupt contraction in a pipe (Fig 3) it is necessary, if exact results are required, to calculate the sectional area at the vena contracta EF and find the velocity V_1 at that section Then, V_1 being the velocity at ST, the fall in the hydraulic gradient, due to increase in the velocity head from ST to EF, is $\frac{V_2^2 - V_1^2}{2g}$, but some head is lost owing to friction and to the eddies at K, L The expansion of the stream from EF to MN causes loss of head, which may be calculated as explained in the preceding article The case of an open stream is analogous, but the whole fall due to loss of head and increase of velocity head is considered together (art 6) and equation 10 (p 14) is used

A particular case of abrupt contraction occurs when a stream issues from a reservoir There is a fall in the surface or hydraulic gradient Most likely the velocity of approach is negligible If so the fall, in the case of a pipe can be calculated without finding the area EF (chap v art 1), and, if not the above procedure can be adopted For an open stream equation 10 is to be used

At a local contraction the channel contracts and expands again, but not necessarily to the same size For an open channel equation 10 is used For a pipe there are various empirical formulæ for local narrowings, all involving the factor $\frac{V^2}{2g}$ (chap v art 6)

20 **Abrupt Bends, Bifurcations, and Junctions**—An abrupt bend (Fig 19) is called an 'elbow' The contraction causes a local narrowing of the stream It has been found in small pipes that, with an elbow of 90°, the head lost is very nearly $\frac{V^2}{2g}$

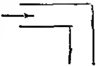


Fig 19

Judging from analogy and from observation it is probable that this is nearly true for any pipe and also for an open stream For elbows of other angles the relative loss of head is known for small pipes (chap v art 6) and it may be assumed that for other channels it is roughly the same

At a bifurcation (Figs 20 and 21) the stream entering the branch may be regarded as flowing round a bend whose outer boundary is shown by the dotted lines In the main channel below the branch there is an enlargement (art 18) Let θ be the angle made by the centre lines of the branch and of the main channel upstream of it When θ is 90° or thereabouts the whole head due to the velocity is lost, and there is a fall in the surface or hydraulic gradient of the branch of about the same amount as there would be if it issued from a reservoir But if V is high the absence of contraction at A does not compensate for the excessive contraction at B, and the fall is increased, or the discharge of the branch diminished When θ exceeds 90° the component of V resolved parallel to the axis of the branch may be regarded as velocity of approach, the discharge being increased accordingly It is not known for what angle the velocity of approach compensates for the greater contraction as compared with that in the case of a reservoir The angle differs with the velocity and probably with the width of the branch, and is perhaps generally not much greater than 90° By the arrangement shown in Figs 22 and 23, the losses of head both in the branch and in the main stream are reduced and that in the branch is not relatively altered by a high velocity If the branch is 'bell mouthed' (Figs 24 and 25) the loss of head in it is somewhat reduced, and it is further reduced by filling in the portions shown in dotted lines, thus doing away with eddies

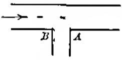


Fig 20

Fig 21

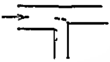


Fig 22

Fig 23

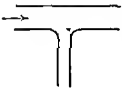


Fig 24

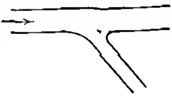


Fig 25

Figs 20 to 25 represent junctions if the stream is supposed to flow in the directions opposite to those of the arrows The losses of head are very much the same as in the corresponding cases of bifurcations

21 Concerning all Abrupt Changes—The 'limits' of an abrupt

change are those of the peculiar local flow caused by it The upstream limit is, in Fig 4, at *AB*, in Fig 3, just as with a weir and certain kinds of orifices (art 7), at *ST*. In the other cases it is where the eddying or curvature begins In all cases eddies exist in the stream itself for some distance downstream of an abrupt change The downstream limit is where these eddies have become reduced They may not cease altogether for a long distance

In the reach downstream of an abrupt change the flow, except for eddying and probably disturbance of the relation to one another of the various velocities in the cross section, is normal, and the water surface or hydraulic gradient takes the level suited to the discharge just as if no abrupt change existed Within the limits of the abrupt change there occurs the fall or rise discussed in the three preceding articles Thus the level of the surface or hydraulic gradient at the downstream limit of the abrupt change governs that at the upstream limit, and this again affects the slope in the upstream reach in the manner indicated above (art 11) But the distribution of the velocities in the upstream reach is normal There is nothing to affect it until the abrupt change actually begins (Cf also Bends, art 13) Thus, at all changes, whether of sectional area or direction of flow, and whether strictly abrupt or not, the effect on the hydraulic gradient or slope is wholly upstream, but eddies and disturbance of the velocity relations are wholly downstream

It follows that discharge observations in which the mean velocity of the whole stream is to be deduced from observations taken, say, in the centre only, should not be made within a considerable distance downstream of an abrupt change, but may be made a short distance upstream of it

Any alteration which makes a change less abrupt reduces the loss of head This has been seen in considering bends, elbows, and bifurcations Regarding changes of section an instance would be the rounding of the edges of the weir in Fig 10 But in all cases, if the eddies are replaced by solid matter, the flow is very much as before Though rounding is caused, the size of the aperture is reduced The friction on the solid is added, but the maintenance of the eddy is subtracted Wherever eddies are referred to the term may be considered to apply also to a solid of the form of the eddy

SECTION VI.—MOVEMENT OF SOLIDS BY A STREAM

22. **Definitions.**—When flowing water transports solid substances by carrying them in suspension, they are known as 'silt,' when by rolling them along the channel they are termed 'drift.' The weight of silt present in each cubic foot of water is called the 'charge' of silt. Silt consists chiefly of clay, mud, and fine sand, drift, of sand, gravel, shingle, and boulders. When a stream obtains material by eroding its channel, it is said to 'scour.' When it deposits material in its channel, it is said to 'silt.' Both terms are used irrespective of whether the material is silt or drift. The difference between silt and drift is one of degree and not of kind. Material of one kind may be rolled and carried alternately.

23. **General Laws.**—If a number of bodies have similar shapes, and if D is the diameter of one of them and V the velocity of the water relatively to it, the supporting or rolling force is theoretically as V^2D^2, and the resisting force or weight as D^3. If these are just balanced D varies as V^2, or the diameters of similarly shaped bodies which can just be supported or rolled are as V^2 and their weights as V^6. From practical observations, it seems that the diameters do not vary quite so rapidly as they would by the above law, the weights being more nearly as V^5.

If a stream has power to scour any particular material from its channel, it has power to transport it, but the converse is not always true. If the material is hard and compact the stream may have far more difficulty in eroding it than in retaining it.

It has long been known that the scouring and transporting power of a stream increases with its velocity. Recent observations made by Kennedy have shown that its power to carry silt decreases as the depth of water increases.[1] The power is probably derived from the eddies which are produced at the bed. Every suspended *particle tends to sink, if its specific gravity is greater than unity.* It is prevented from sinking by the upward components of the eddies. If V is the velocity of the stream and D its depth, the force exerted by the eddies generated on one square foot of the bed is greater as the velocity is greater, and is, say, as V^n. But, given the average charge of silt, the weight of silt in a vertical column of water whose base is one square foot is as D. Therefore the power of a stream to support silt is as V^n (say as V^2) and inversely as D.

The power of water to move drift is probably as V^2, and the

[1] *Min. Proc. Inst. C.E.*, vol. cxix.

depth does not affect it. It has sometimes been said that increased depth gives increased scouring power, because of the increased pressure but this is not so. The increased pressure due to depth acts on both the upstream and downstream sides of a body. It is moved only by the pressure due to the velocity. It is impossible to construct an equation which shall include both suspended and rolling matter, because the proportions in which they exist are not known.

It is sometimes supposed that the inclination of the bed of a stream, when high, facilitates scour, the material rolling more easily down a steep inclined plane. The inclination is nearly always too small to have any appreciable direct effect on the rolling force. In fact the bed is generally more or less undulating, and the drift may be moving either uphill or downhill. The inclination of the surface of the stream of course affects its velocity, and this is the only real factor in the case.

A stream of given velocity and depth can only carry a certain charge of silt. When it is carrying this it is said to be 'fully charged.' In this case, if there is any reduction in velocity, or if any additional silt is by any means brought into the stream, a deposit will occur (unless there is also a reduction of depth) until the charge of silt is reduced again to the full charge for the stream. The deposit may, however, occur slowly, and extend over a considerable length of channel. If a stream is not fully charged, it tends to become so by scouring its bed. A stream fully charged with silt cannot scour silt from its channel, but its power to move drift is, perhaps, unaffected by its being charged with silt.

It is not known how the full charge is affected by the nature of the silt. The specific gravity of fine mud is not much greater than that of water, while that of sand is about 1·5 times as great. If two streams of equal depths and velocities are fully charged, one with particles of mud and the other with equally sized particles of sand, the latter will sink more rapidly and will have to be more frequently thrown up. They will probably be fewer in number, but in what proportion is not known.

In the 'Inundation Canals,' so called because they flow only when the rivers are in flood, fed from the rivers of Northern India, the silt entering a canal usually consists of sand and mud. The sandy portion, or most of it, is deposited in the head reach of the canal, forming a wedge-shaped mass, with a depth of perhaps two or three feet at the head of the canal, dwindling to zero at a point a few miles from the head. Beyond this point the water,

charged with mud and perhaps a little sand, usually flows for many miles without any deposit occurring, although there are frequent reductions in the velocity caused by the diminutions in the size of the stream as the distributaries are taken off, and sometimes also by reductions in the gradient The absence of further deposits inexplicable till the discovery of Kennedy's law, is due to the fact that the depth of water diminishes as well as the velocity Many of the channels were constructed long ago by the natives, and they seem to have learned from experience to give the channels such widths that the depth of water decreases at the proper rate

It is a common practice to so reduce the velocity of a stream that silting must take place The object may be either to 'warp up' certain localities by silt deposit or to free the water from silt, and thus reduce the deposit in places further down When the velocity of a stream is arrested altogether, as it practically is when a stream flows through a large reservoir, the whole of the silt will deposit if it has time to do so, that is, if the reservoir is large enough Low lying and marshy plots of ground may be silted up, and rendered healthy and culturable by turning a silt-bearing stream through them In order to prevent deposit in the head of a canal the water may be made to pass through a 'silt-trap' or large natural or artificial basin, where the velocity is small, or the supply may be drawn from the upper layers of the river water (art 24)

Silting and scouring are generally regular or irregular in their action according as the flow is regular or irregular, that is, according as the channel is free or not from abrupt changes and eddies In a uniform canal fed from a river the deposit in the head of the canal forms a wedge shaped mass, as above stated, the depth of the deposit decreasing with a fair approach to uniformity Salient angles are most liable to scour, and deep hollows or recesses to silt Eddies have a strong scouring power Immediately downstream of an abrupt change scour is often severe

Most streams vary greatly at different times both in volume and velocity and in the quantity of material brought into them Hence the action is not constant A stream may silt at one season and scour at another, maintaining a steady average When this happens, or when the stream never silts or scours, appreciably it is said to be in 'permanent régime

Waves, whether due to wind or other agency, may cause scour, especially of the banks Their effect on the bed becomes less as

the depth of water increases, but it does not cease altogether at a depth of 23 feet, as has been supposed. Salt water possesses a power of precipitating silt.

24 **Distribution of Silt Charge.**—Since the eddies are strongest near the bed, the charge of silt must generally increase towards the bed, but the rate of increase varies greatly. Fine mud having a low specific gravity, the charge is probably nearly as great near the surface as elsewhere. Sand is heavy, and is oftener rolled than carried. When carried it is usually in much greater proportion near the bed. Materials such as boulders, do not generally rise much above the bed. A perfectly clear stream may be moving drift. The ratio of the silt charge at the surface to that at the bed thus varies from 0 to 1. For a given kind of silt the rate of variation from surface to bed probably increases with the depth and decreases with the velocity. The distribution in any particular stream can only be ascertained by observation, or by experience of similar streams. It is a matter of great practical importance, as affecting the best bed level for a branch taking off from the stream. The results of observations show considerable discrepancies, even when averaged, and individual observations very great discrepancies. In some rivers 10 to 17 feet deep the silt charge has been found to increase at the rate of about 10 per cent. for each foot in depth below the surface. In others, with depths ranging up to 16 feet, the silt charge at about three fourths or four fifths of the full depth has been found to bear to that near the surface a ratio varying from 1½ to 2.

Section VII.—Hydraulic Observations and Coefficients

25 **Hydraulic Observations.**—It is frequently necessary in Hydraulic Engineering to observe water levels, dimensions of streams, and velocities, and from these to compute discharges. The object of a set of observations may be either simply to ascertain, say, the discharge in a particular instance, or to find and record the co-efficients applicable to the case, so as to enable other discharges under similar conditions to be calculated. Observations of the latter class, when extensive, are usually termed 'Hydraulic Experiments.' A consideration of the instruments and methods adopted in Hydraulic Observations may be strictly a matter of Hydraulic Engineering, but it is necessary to include it in a general manner in a Treatise on Hydraulics, both because

the principles involved in such work are closely connected with the laws of flow, and also in order that proper estimates may be formed of the errors which are possible and of the reliability of the results which have been arrived at by various observers [1]

In making observations accurate measurements of lineal dimensions, depth, and water levels are necessary, as well as accurate timing The number and duration of the observations should be sufficient to eliminate the effects of the irregular motion of the water, and bring out the true average values of the quantities sought for Owing to imperfections in these matters, or in the instruments used, errors of various kinds may occur These are known as 'observation errors' They may balance one another more or less, but are liable to accumulate in one direction in a remarkable manner Care in observing, as well as sufficiency in the number of observations, are therefore essential points An error in measuring length or time has, of course, a greater relative effect when the amount measured is small In a channel the fall in the surface or hydraulic gradient is often a small quantity, and thus in slope observations the error is often large With an aperture under a small head the error in observing it may be serious It has been shown by Smith [2] that, even in the careful experiments made by Lesbros on orifices, the co efficients were probably affected by such causes as the expansion and contraction of the long iron handles attached to the movable 'gates,' and to the bending, under great pressure, of the plates forming the orifices Besides quantities which can be actually measured there are conditions which can be observed but may be overlooked, such as a slight rounding of a sharp edge, the clinging of some portion of the water to an aperture when it is supposed to be springing clear, or the occurrence of a deposit in a channel Such matters not always very perceptible may have considerable effects on the flow

Again, there are conditions which cannot be ascertained, and assumptions are made regarding them It has, for instance, been assumed that a local surface slope too small to be observed is the same as the observed slope in a great length, or that the diameter of a pipe, measured at only a few places, is constant throughout Lastly, there are some things very difficult to describe, such as the degree of sharpness of an edge, or of roughness of a channel Thus there is often, in accounts of experiments, a defective or erroneous description of the conditions which existed This may be termed 'descriptive error' In some cases it has been

[1] Details will be given in chap VIII

[2] *Hydraulics*, chap III

very great Its effect is similar to that of observation error, and the line between the two cannot easily be drawn

When the quantity whose law of variation is sought depends on several conditions which vary together, it is often difficult to determine the effect of the variation of any one condition alone As far as possible observations should be made with only one condition varying at a time Generally, observations at one site are kept distinct from those at other sites, but if the conditions of different sites are nearly similar, it is legitimate to combine observations at different sites In such a case, care should be taken that the effect of any slight or accidental dissimilarity in the sites will not affect any one set of values, but will be distributed throughout all It would, for instance, be undesirable to have all the low water observations at one site and the high water observations at another

A series of observations containing a source of error may show results quite consistent with one another, and may be of great use in bringing out certain laws The well known weir experiments of Francis and of Fteley and Stearns give results which are consistent, and have long been accepted as practically correct, but when they are compared with the later results of Bazin certain discrepancies appear, and it is clear that one or the other set of experiments contains some error

Detailed accounts of Hydraulic Experiments do not of course, find a place in a textbook References to the chief works on such experiments have already been given (p 7) but special points will be noticed whenever necessary

26 Co efficients —From the causes above stated the co efficients, or other figures, arrived at by various observers frequently show grave discrepancies This is especially the case with the older experiments In the more recent ones the discrepancies have been reduced

The 'probable errors' of co efficients have in some cases been estimated by those who have investigated them The meaning of this may be explained by an example which will be made to include all kinds of errors Let a weir have a crest 1 foot wide, sharp edges, and a head of 1 foot Suppose the co-efficient arrived at is 600, and that it is estimated that the observation error may probably be 1 per cent either way Then 1 per cent is the probable error, and the value of the co efficient is as likely to be between 606 and 594 as to be outside of these limits But there may also have been descriptive errors connected with, say,

the width of the crest or sharpness of the edges and the real probable error may be much greater than 1 per cent Finally, if the co efficient is applied to a weir, over which water is actually flowing, there may be again observation error in measuring the head Sometimes these different errors balance one another, but sometimes, as before remarked they all accumulate in one direction

The co efficients for different cases contain probable errors of very different amounts For sharp edged apertures under favourable circumstances, the probable error is only about 50 per cent For channels and especially for pipes owing chiefly to the causes above indicated (arts 9 and 11) it may easily be 5 or 10 per cent

Although in the above instance the final operation of observation introduces an additional error, complete observation is much better than calculation If no co efficient had been assumed at all, but the discharge of the stream carefully observed, as well as the head on the weir, then both the discharge and the co efficient for that particular case would have been obtained in the best possible manner

The results of individual experiments nearly always show irregularities, that is when plotted they do not give regular curves The usual method is to draw a regular curve in such a manner as to average the discrepancies and correct the original observations Most published co efficients have been obtained in this manner

When an experimenter obtains a series of co efficients for any particular case, he often connects them by an empirical formula involving one or two constants This has been done by Bazin and Kutter for open channels, and by Fteley and Stearns, Francis and Bazin for certain kinds of weirs What the engineer really needs and uses is a table of the co efficients, but the formulæ may be useful in finding a co efficient when a table is not at hand or in finding its value for cases intermediate between those given in the tables or outside the range of the observations This last practice must, however, be adopted with caution and within narrow limits

Further experiments are required in all branches of hydraulics A feature in future experiments will no doubt be the increased use of automatic and self recording methods, electric communications, and photography.

The co-efficients given, except for conical tubes, are approximate and average values, further details being given in the succeeding articles. The length of a tube must not exceed three times the diameter, otherwise the co-efficient is reduced, owing to friction and the tube becomes a pipe. A tube generally has its axis horizontal, but may have it in any direction. If the lengths of the cylindrical tubes (Figs. 28 and 29) are reduced till the jet springs clear from the upstream edge, the co-efficients change to the values shown for Figs. 25A and 30. The length at which the change takes place may for a very great head be two diameters or more, but is generally less than one diameter. The cross sections of all the tubes are supposed to be circular, but the co-efficients apply nearly to square sections and to others differing not greatly from circles and squares. Thus 'cylindrical' includes 'prismatic,' and similarly with the others. In the case of an elongated section, 'diameter' is to be understood as 'least diameter.'

For orifices up to a foot in diameter, metal edges filed sharp should be used, if full contraction is required. For larger orifices wooden edges can be made sufficiently sharp. These remarks apply to all kinds of orifices in which the edges are supposed to be sharp, that is to all except bell mouths, though with a convergent conical tube the effect of want of sharpness is probably small, the final contraction occurring outside the tube.

In experiments made by Mair and Simpson[1] with circular orifices 1 to 3 inches in diameter in thin metal plates it was found that a hardly perceptible rounding of the edge caused in one instance (the diameter is not stated) an increase in the discharge of about twenty per cent., but this increase seems excessive, even if the diameter was only an inch.

The co-efficient of discharge does not generally alter much as the head varies, so that, neglecting the effect of velocity of approach, *the discharge through a given orifice under different heads is nearly as* $H^{\frac{1}{2}}$. In order to double the discharge H must be quadrupled. If the head is doubled the discharge is increased in the ratio of about 1.4 to 1.

To facilitate the working out of problems, the theoretical velocities corresponding to various heads are given in table 1. V can be found from H or H from V.

2. **Measurement of Head.**—Upstream of an orifice there may be a vortex in the water, or, when the velocity of approach is high,

[1] *Minutes of Proceedings of the Institution of Civil Engineers*, vol. lxxxiv.

a wave or heaping of water where it strikes the wall, and the head should be measured a short distance upstream from such vortex or wave. If the part of a reservoir adjoining an orifice is closed (Fig. 33) the head may be measured at *B*, but if the length of the closed portion is more than thrice its least diameter, it is necessary to find the loss of head in it, treating it as a pipe.

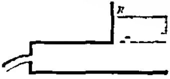


Fig. 33.

Smith states that for an orifice in a thin wall the head should probably be measured to the centre of gravity of the vena contracta. The matter seems to admit of no doubt, and the rule should apply to all kinds of orifices in which there is contraction. It is at the vena contracta and not elsewhere that the theoretical velocity is $\sqrt{2gH}$. In a bell mouthed orifice in a horizontal wall the head would be measured to the 'discharging side' of the orifice, and the jet from an orifice in a thin horizontal wall issues under the same conditions, except that friction against the sides is removed. Under a small head the jet from an orifice in a thin vertical wall may drop appreciably in the distance *PM* (Fig. 34), and the true head, that at *M*, is not the same as at *P*, the centre of the orifice. Nearly all co-efficients have been obtained from orifices in vertical walls under considerable heads, so that it has made no difference how the head has been measured, but in applying these co-efficients to orifices in other positions the head should be measured to the vena contracta.

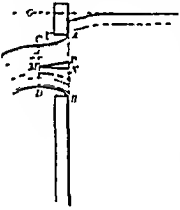


Fig. 34.

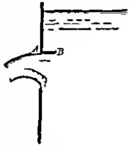


Fig. 35.

3. **Incomplete Contraction.**—The contraction in an orifice with a sharp edge may be partly suppressed by adding an internal projection *AB* (Fig. 35), extending over a portion of the perimeter of the orifice. The contraction is then said to be 'partial.' If the length *AB* is not less than 1.5 times the least diameter of the orifice, the co-efficients for orifices in thin walls are, according to Bidone—

For a rectangular orifice $c_p = c\left(1 + 152\frac{S}{P}\right)$ (20),

For a circular orifice $c_p = c\left(1 + 128\frac{S}{P}\right)$ (21),

where c is the co efficient of discharge for the simple orifice, P its perimeter, and S that of the portion on which the contraction is suppressed Partial suppression may be caused by making one or more of the sides of an orifice flush with those of the reservoir The above formulæ were obtained with small orifices and heads under six feet They are not applicable when $\frac{S}{P}$ is greater than $\frac{3}{4}$ for a rectangle or $\frac{2}{3}$ for a circle They are not quite reliable in any case, and especially when the orifice is elongated With a rectangular orifice of length twenty times its breadth the suppres sion of the contraction on one of the long sides has been found to increase c by 8 to 12 per cent, whereas by the formula the increase should be 7 2 per cent

For a square orifice in a thin wall c is, say, 62 with full con traction and 1 0 when all contraction is suppressed Therefore, if the contraction is suppressed on half the perimeter, that on the other half remaining unchanged, c will be about 81 But by equation 20 it is $62 \times 1\ 076$ or 667 It is clear, therefore, that if the contraction is suppressed on one part of the perimeter that on the remaining part increases, and this is what would be ex pected The increase is, no doubt, most pronounced on the side opposite to the suppressed part, because the contracting filaments of water are no longer directly opposed by others

In a bell mouthed tube the contraction must be complete, what ever the clear margin may be In all other cases decrease in the clear margin causes the contraction to be 'imperfect' In chapter iv (art 3) some rules are given regarding the allowance to be made for imperfect contraction with weirs in thin walls Con sidering them in connection with the above formulæ for partial contraction the figures shown in table ii are arrived at In this table S is the length of the perimeter on which the clear margin is reduced, G the width of the margin in the reduced part, d the least diameter of the orifice, and c, c_i the co efficients for the orifice with complete and incomplete contraction respectively The table is meant for orifices in thin walls, but even for these it is only approximate It probably applies almost as well to other orifices with sharp edges The above formulæ and figures apply to $c_{\ }$ as well as to c, both probably altering in about the same pro

portion and c_c being constant It may happen that the contraction is suppressed on one part of the perimeter of an orifice and imperfect on another part Example 4, page 74, shows the method which may be adopted for such cases When the contraction is either suppressed or very imperfect on nearly the whole perimeter the approximation becomes very doubtful

When an orifice 30 feet long and ·05 feet high was bisected by vertical brass sheets of various thicknesses, it was found that a very thin sheet had little or no effect either on c or on the jet, but a sheet 01 feet thick increased c nearly 1 per cent, the jets, however, uniting a short distance from the orifice [1]

4 **Changes in Temperature and Condition of Water**—The results of some experiments by Smith, Mair, and Unwin respectively are shown in the following table —[2]

Kind of Orifice	Diameter	Amount by which Temperature of Water was raised	Effect on the Discharge	Head	Remarks
	Inches	Fahr		Feet	
Orifice in thin wall,	24	82°	Decrease of 1½ per cent	56 to 3 2	In all cases the initial temperature of the water was normal namely, 45° to 61° Fahr
	40	144°	Decrease of 1 per cent	1 to 1 5	
	2 5	96°	Increase of ½ per cent	1 75	
Bell mouthed tube,	40	110°	Increase of 3½ per cent	1 to 1 5	
	1 5	115°	Increase of 2 per cent	1 75	

It is clear that it requires a great change of temperature to cause an appreciable change in the discharge, and that the change is greater the smaller the orifice The law governing the change is not clear. Smith considers that with a head of 10 feet a change of 50° in temperature probably has no appreciable effect for orifices of more than 24 inch in diameter [3]

Smith states that for small orifices (05 foot and less in diameter, and with heads less than 1 foot) the discharge fluctuates considerably, and that this is perhaps due to unknown changes in the character of the water With either larger heads or larger orifices

[1] Smith's *Hydraulics*, chap III
[2] *Ibid* and *Min Proc Inst C E*, vol lxxxiv
[3] *Hydraulics*, chap III

the uncertainty disappeared It was not due to experimental error

Smith also states as follows Water containing clayey sediment may have a greater co efficient because of its oiliness Thick oil, though very viscous, has a greater co efficient than water When the water is in a disturbed condition, and approaches the orifice in an irregular manner, the jet may be ragged and twisted, but c is not affected appreciably Greasy matter adhering to the edge of an orifice slightly reduces the discharge, if the diameter is 10 foot or less, the reduction being due to the diminished size of the orifice

5 **Velocity of Approach** —The subject of velocity of approach is of more importance for weirs than for orifices, and a full discussion regarding it is given in chapter IV (art 5) In equations 8 and 10 (pp 13 and 14) n may be taken to be 1 0, when the aperture is opposite that part of the approach section where the velocity is greatest—that is generally the central part and near the surface—and about 80 when it is opposite a part where the velocity is lowest—that is near the side or bottom The method of solving the above equations has been stated in chapter II (art 7) For an orifice with sharp edges, whenever velocity of approach has to be taken into account, there will very likely be imperfect contraction on some part of the perimeter, and c_i must be substituted for c

Another method of procedure is to alter the forms of the equations Since $h=\frac{v^2}{2g}=\frac{a^2}{A^2}\ \frac{V^2}{2g}$ therefore equation 8 may be written $V^2=c_v^2\left(2gH+n\frac{a^2}{A^2}V^2\right)$

Whence $$V^2\left(1-c_v^2\ n\ \frac{a^2}{A^2}\right)=c_v^2\ 2gH.$$

Or $$V=c_v\sqrt{2gH}\sqrt{\frac{1}{1-c_v^2 n\ \frac{a^2}{A^2}}} \quad . (22).$$

And $$Q=c\ a\ \sqrt{2gH}\sqrt{\frac{1}{1-c_v^2 n\frac{a^2}{A^2}}} \quad . (23)$$

These can be solved directly The quantity $\sqrt{\frac{1}{1-c_v^2 n\frac{a^2}{A^2}}}$ is "a co-efficient of correction" for velocity of approach It may be denoted by c_a *Table III shows some values of* $\frac{a^2}{A^2}$ for different

values of $\frac{a}{A}$, and it also shows the value of c_a and of the quantities leading up to it, for $c_v = .97$ and $n = 1.0$. For a bell mouthed tube a is simply the area of the discharging side of the tube and c_v is c. When $\frac{a}{A}$ is less than $\frac{1}{3}$ a change in c_v or in n makes very little difference in c_a, and a mere inspection of the table will enable its proper value to be found. Thus the use of c_a simplifies matters. For other kinds of orifices c must be separated into its factors c_c and c_v, and a found by multiplying a by c_c. But it will be seen from the examples (p. 72 *et seq.*) that the use of c_a may often be convenient. In all cases the use of c_a causes a little inaccuracy when $\frac{A}{a}$ is small. If greater accuracy is required c_a may be used for the first approximation only. Another form of c_a is $\sqrt{\frac{1}{1-c^2 n\frac{a^2}{A^2}}}$, which would be very convenient for sharp-edged orifices, but there are so many values of c that extensive tables would be needed.

Let $cc_a = C$, then C is an "inclusive" co-efficient and

$$Q = Ca\sqrt{2gH} \qquad (24)$$

This formula is not convenient for general use, because it would be difficult to tabulate all the values of C for different kinds of orifices for various velocities of approach. But where it is desired to ascertain by experiment the co-efficients for any orifice so as to frame a discharge table for that orifice alone, then equation 24 is by far the best and simplest to use.

If there are two orifices supplied from the same reservoir and situated not far apart, the discharge of each may be increased by the effect of the other, especially when both are in the same wall. In Bazin's experiments twelve orifices, each 8"×8" nearly, and capable of being closed by gates were placed side by side. The following values of the inclusive co-efficient C were found:—

Number of gates open	1	2	3	4	5 or more
Total co-efficient for all	.633	.642	.646	.649	.650

When one gate was raised two inches and the others were fully opened the co-efficients were as follows:—

Number fully open	1	2	3	4	5 or more
Co-efficient for the one partly open	.650	.657	.660	.662	.663

The contraction was not complete, the twelve orifices being in

the end of a chamber only 18 feet wide In order that two orifices in the same plane may have no effect on one another, it is probable that there should be no overlapping either of the minimum clear margins or of the minimum areas of approach sections requisite for full contraction and for negligible velocity of approach respectively (cf chap v art 2)

6 **Effective Head** —The 'effective head' over an orifice is the head which would produce the actual velocity supposing c_v to be unity If H and H_e are the actual and effective heads

$$V=c_v\sqrt{2gH}=\sqrt{2gH_e} \qquad (25)$$

If $H-H_e=H_r$, then H_r is the head wasted in overcoming resistances Let $\frac{H_r}{H_e}=c_r$, then c_r is the 'co efficient of resistance,' or ratio of the wasted to the effective head

$$\text{Since } 1+c_r=\frac{H_e+H_r}{H_e}=\frac{H}{H_e}$$

$$\text{And from equation 25 } \frac{H}{H_e}=\frac{1}{c_v^2}$$

$$\text{Therefore } c_r=\frac{1}{c_v}-1 \qquad (26)$$

If there is velocity of approach $H+nh$ must be put for H in the foregoing The following table shows the values of c_r for different values of c_v The head wasted is only a small percentage of the effective head, when c_v is high, but it may be more than the effective head when c_v is low

$c_v=$ 995	99	98	97	95	90	
$c_r=$ 010	020	041	063	111	233	
$c_v=$ 85	82	80	75	72	715	70
$c_r=$ 384	489	563	778	929	956	1 019

The equation $V=\sqrt{2gH_e}$ gives the actual velocity for an orifice referred to an imaginary water surface situated H_r feet below the actual surface (Fig 40), but the equation will not apply to another similar orifice in the same reservoir at a different level, because H_r will not have the same value

7 **Jet from an Orifice** —The jet of water from an orifice retains its coherence for some distance and then becomes scattered With an orifice in a thin wall, not circular and not in a horizontal plane, and with a head not very great compared to the size of the orifice, a phenomenon called 'inversion of the jet' occurs The section of the jet is at first nearly of the shape of the orifice,

but afterwards spreads into sheets perpendicular to the sides of the orifice. Those portions of the jet which issue under different heads behave somewhat similarly to separate jets, which, if two of them meet obliquely, spread into a sheet perpendicular to the plane containing them. This expansion into sheets reaches a limit and the jet contracts again to nearly the form of the orifice, but if its coherence is retained it again throws out sheets in directions bisecting the angles between the previous sheets. This is probably due to surface tension or capillarity. The fluid is enclosed in an envelope of constant tension, and the recurrent form of the jet is due to vibrations of the fluid column,[1] as they would be if the orifices were far apart.

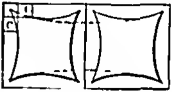


Fig 36

Fig 36 shows the cross sections of jets from two square orifices. At a corner the two streams A and C in contracting interfere with one another, and some fluid is forced towards the corner. The full line in Fig 37 shows the form next assumed, and the dotted line that assumed subsequently. The dotted lines in Fig 36 show the form of jet where the two squares are joined to form a rectangular orifice.

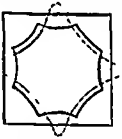


Fig 37

Let H_e be the effective head over an orifice. Then if the jet issues vertically upwards and H is not great, it rises to a height very nearly equal to H_e. It then expands on all sides (Fig 38) and scatters. Let x be the head, measured from the plane AB, over any cross section of the jet, and y the diameter of the jet at the cross section. The velocity of the jet is very nearly $\sqrt{2gx}$ and its sectional area is as y^2. But since the discharges at all cross sections are equal the velocities are inversely as the sectional areas. Therefore if d is the diameter of the jet at the vena contracta where the velocity is $\sqrt{2gH_e}$

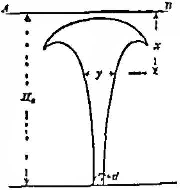


Fig 38

[1] *Encyclopædia Britannica*, ninth edition. Article 'Hydromechanics'.

$$\frac{y}{d}=\frac{\sqrt{2gH_e}}{\sqrt{2gx}}=\left(\frac{H_e}{x}\right)^{\frac{1}{2}}$$

Or $$y=d\left(\frac{H_e}{x}\right)^{\frac{1}{2}} \qquad (27)$$

Theoretically y should be infinite when $x=0$ but practically the jet breaks up and scatters. The velocity of the jet decreases uniformly; that is, decreases by equal amounts in equal periods of time. When the head is great the jet does not retain its coherence long enough to rise to the height H_e.

A body of water issuing from an orifice in a direction not vertical describes like any other projectile, a curve which if the resistance of the air is neglected, is a parabola with a vertical axis and apex upwards. If the jet issues with velocity V, and at an angle θ with the horizon (Fig 39) the equation to the parabola as given in Dynamical Treatises is

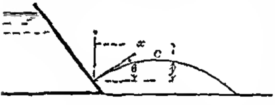


Fig 39

$$y=x\tan\theta-x^2\frac{g\sec^2\theta}{2V^2} \qquad (28)$$

where y is the height of any point above the orifice corresponding to any horizontal distance x. The maximum value of y that is the height of the point C above the orifice is $\frac{V^2}{2g}\sin^2\theta$. If $y=0$

$$x=\frac{2V^2}{g}\frac{\tan\theta}{\sec^2\theta}=\frac{V^2}{g}\sin(2\theta) \qquad (29)$$

This gives the range of the jet on a horizontal plane passing through the orifice. If $\theta=45°$, $x=\frac{V^2}{g}$. This is the maximum range, and in this case the maximum height is $\frac{V^2}{4g}$.

If the jet issues horizontally (Fig 40) equation 28 becomes

$$y=x^2\frac{g}{2V^2}=\frac{x^2}{4H_e} \qquad (30)$$

and the range of the jet on a horizontal plane H feet below the orifice is

$$x=2\sqrt{H_eH} \qquad (31)$$

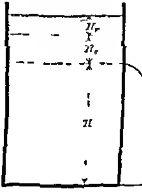


Fig 40

The range is a maximum when $H_e=H$, or, for a plane passing

through the bottom of a reservoir, when the orifice is slightly below mid depth (See also Nozzles, art 16)

Section II—Orifices in Thin Walls

8 **Values of Co efficient**—The co efficient c has been determined for a great variety of cases, and its values for orifices with full contraction are given in tables IV to VII. They are all for orifices in vertical planes, but if the head is measured to the vena contracta they probably apply to orifices in other planes, except when the head is small compared with the height of the orifice. These cases, marked off by horizontal lines in the tables, will be considered in article 19. Tables IV and V contain the figures arrived at by Smith[1] from a discussion of various experiments, including some made by himself. Smith states that the probable error in these co efficients is about 5 per cent. Tables VI and VII contain the results arrived at by Fanning[2] and Bovey[3] respectively, the latter from his own observations and the former by a consideration of various experiments, some of which, however, do not seem to be quite reliable. The table given below contains selections from the above tables. A rectangle with ratio n to 1 means a rectangle having the horizontal side n times the vertical side.

The co-efficient c_v is about the same for orifices in thin walls as for bell mouthed orifices (art 14). It is about 96 for small heads and 99 or more for great heads. By dividing c by c_v the value of c_c may be obtained. It is clear from Fig 36 that the jet from a rectangular orifice formed from two squares is greater relatively to the size of the orifice than for a single square, and that the relative size will go on increasing as the orifice is lengthened. In other words, the effect of the end contractions decreases as the orifice is lengthened.

9 **Laws of Variation of Co-efficient**—The following laws regarding the variation of the co-efficient c are easily traced. For laws 1 to 6 it is only necessary to compare the figures in any horizontal line of a table, for law 7 in any vertical line.

(1) With high heads (relatively to the size of the orifice) c is about the same for a given rectangular orifice whether the longer or the shorter side is horizontal.

[1] *Hydraulics* chap III

[2] *Treatise on Water Supply Engineering*, chap XI

[3] *Hydraulics* chap II

Head over Centre of Orifice	Smith			Fanning				Bovey			
	02′ × 02′ Square	05 × 05′ Square.	1′×1′ Square	1 × 125′ 8 to 1	1 × 25 4 to 1	1×1′ Square.	1×4′ ¼ to 1	056 side Equilateral	037 × 037 Square	073 × 018′ 4 to 1	146 × 0092 16 to 1
				Horizontal side, 1 foot.				Area of each = 1 b sq inches			
(1) Feet.	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
5	660	630	.....	633	632	610	....	.....	.....	.	..
1 0	648	622	598	632	632	605	.	636	627	643	664
3 0	632	612	603	627	627	605	637	625	618	632	646
10	616	606	601	606	603	601	604	618	612	624	633
20	606	603	600	607	604	601	605	616	609	621	629
50	602(*)	601(*)	599	614	607	602	609	... .	.....	.	.....

(2) For a square with sides vertical c is nearly the same as when the diagonal is vertical

(3) Comparing a square with a rectangle having its shorter side equal to the side of the square, c is least for the square and increases with the length of the rectangle The difference is less marked when H is great

(4) For a circle c is about ·005 less than for a square of the same diameter, and for the Bovey's orifices it is about ·007 less than for a square of equal area

(5) For a square c is about 007 less than for a triangle of equal area

(6) For orifices of the same shape, c is greatest for small orifices and decreases for larger orifices, the decrease, however, becoming less and less rapid as the size and head increase This law does not apply to Fanning's rectangles (cf, say, columns 6 and 8 of the table on page 54)

(7) As H decreases c increases, especially for small orifices and small heads There are some exceptions for Fanning's orifices with large heads

Since the values of c_r do not differ much the variations in c must be due chiefly to variations in c_c It will be seen below (art 13) that for Borda's mouthpiece the value of c can be found theoretically, and is about 50 For orifices in thin walls it is clear that c must be more than 50, but theory does not show how much more The main fact, namely, that the general value of c is about 61, and the next most important facts, namely that c usually increases for small heads and small orifices (laws 6 and 7), do not admit of theoretical proof But the laws governing the minor variations of the co-efficients can to some extent be explained Laws 1 and 2 are what might be expected and are proved by Bovey's results because he used the same orifice in different positions, and therefore no error could arise from accidental differences in its size or character The notes to table vii indicate some minor laws which cannot be explained Law 3 is clearly proved by Fanning's results, and also indirectly by Bovey's, although he used rectangles of equal area to the square and not with least side equal The cause of law 3 is clear from what has been said above regarding c_c. Regarding law 4, the higher co efficient of the square is owing to the smaller contraction in the angles Smith states that if this were the case the co efficient for a square would be greater than for a rectangle but he probably did not consider the matter carefully with the

aid of a diagram The cause of law 5 is similar The angles being more acute than in a square the suppression of contraction in them is still greater

The co efficients cannot apparently all be quite correct The difference between columns 2 and 3 of table vi is appreciable only for great heads, while between columns 3 and 4 it is greatest for small heads The co efficients in column 10 of the table on page 54, when compared with columns 2 and 3, agree well except for the head of 20 feet Fanning's figures showing c as increasing for great heads seem to be incorrect The experiments considered by him do not seem to have included heads greater than 23 feet, and only a few of these

The manner in which the co efficient varies for orifices of different sizes and shapes is the opposite to what it would be if the friction of the orifice had any appreciable effect The smaller the orifice, and the greater its deviation from a circle, the greater is the ratio of the border to the sectional area, but the greater the co-efficient It is remarkable that as H increases laws 3 and 6 become less pronounced, and that there is a strong tendency for the co efficients of all orifices of one shape to become equal

10 **Co-efficients for Submerged Orifices**—All the co efficients above mentioned are for cases in which the orifice discharges into air Table viii shows the results found by Smith for drowned orifices, the downstream water being 57 feet to 73 feet above the centre of the orifice The co efficients are less by about 1 per cent, or for small heads 2 per cent, than for similar orifices discharging into air The cause may perhaps be the formation of eddies, and the friction of the jet against the water surrounding it

11 **Remarks**—If an orifice in a thin wall is in a surface not plane, the co efficient will be greater or less than for a plane surface, according as the surface is concave or convex towards the reservoir

In some districts in America, where water is sold for mining purposes, the quantity taken is measured by orifices The 'Miner's Inch' is a term which often means the quantity of water discharged by an orifice 1 inch square, in a vertical thin wall, under a head of $6\frac{1}{2}$ inches In this case, if c is taken at 621, Q is 1 53 c ft per minute, but the head is not always the same, and the orifices used are of many different sizes, generally much larger than a square inch the Miner's Inch is then some fraction of the total discharge, and its value in c ft per minute varies from 1 20 to 1 76 The Miner's Inch is, in fact, a name with local varieties

of meaning The wall containing the orifice is often made of 2 inch plank, and the chief practical point to be noted is, that with a small orifice, or a very long orifice of small height, not only is exactness of size more difficult to attain, but there may be a chance of the orifice acting as a cylindrical tube, and giving a greater discharge than intended Before the discharge of the orifice can be known, the size, shape, head, degree of sharpness, thickness of wall, width of clear margin, and velocity of approach must all be known

SECTION III—SHORT TUBES

12 Cylindrical Tubes—In a cylindrical tube (Fig 41) the jet contracts, but it expands again, fills the tube, and issues 'full bore' The sectional area at GK is, as in a simple orifice in a thin wall, about 63 times the area at LM, but the velocity at GK is greater than $\sqrt{2gH}$, and the discharge through the tube is greater than that from an orifice of area LM When the flow first begins, the air in the spaces NG, KO is at the atmospheric pressure, and the discharge is not greater than that from an orifice LM The action of the water exhausts the air and produces a partial vacuum Let p be the pressure in NG, KO The pressure in the jet GK is also p The pressures at QR and ST are P_a Let V, v be the velocities at GK and QR The loss of head from shock between GK and QR (equation 18, p 32) is $\frac{(V-v)^2}{2g}$ Then from equation 5 p 11, if the tube is horizontal,

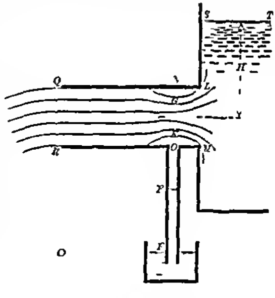


FIG. 41.

$$H+\frac{P_a}{w}=\frac{p}{w}+\frac{V^2}{2g} \qquad (A)$$

And $$H+\frac{P_a}{W}=\frac{P_a}{W}+\frac{v}{2g}+\frac{(V-v)^2}{2g} \qquad \text{(B)}$$

But $$v=.63V \text{ and } V-v=.37V$$

Therefore from (B) $$H=\frac{V^2}{2g}\left\{(.63)^2+(.37)^2\right\}=\frac{.534V^2}{2g}$$

Or $$V=\sqrt{\frac{2gH}{.534}}=\sqrt{\frac{2gH}{.73}}=1.37\sqrt{2gH}$$

Practically there is some loss of head between LM and GK and actually

$$V=1.30\sqrt{2gH} \qquad (32),$$
$$v=.63V=.82\sqrt{2gH} \qquad (33)$$

Also from (A) $$H+\frac{P_a}{W}=\frac{p}{W}+\frac{V^2}{2g}$$
$$=\frac{p}{W}+(1.30)^2H$$

Therefore $$\frac{P_a}{W}-\frac{p}{W}=.69H \qquad (34),$$

Or the pressure at GK is less than the atmospheric pressure by $.69WH$ The result is nearly the same if the tube is not horizontal, provided H is large relatively to the length of the tube If c_c is not exactly .63, or if the actual loss of head differs from that assumed, the above results are somewhat altered With a great head the vacuum becomes more perfect, the contraction, owing to the diminished pressure on the jet, less complete and the figures 1.30 and .69 are reduced For moderate heads they are found to be about 1.32 and .75

If holes are made at N, O, water does not flow out but air enters, and the discharge of the tube is reduced If a sufficient number of holes are made, or if the whole tube and reservoir are in a vacuum, or if the tube is greased inside, so that water cannot adhere to it, the discharge is no greater than for a simple orifice If the holes are made at a greater distance from LM than about 1½ diameters the discharge is unaffected If a tube is added communicating with a reservoir F, the water for ordinary heads rises to a height $FF=.75H$, and if the height EO is less than this, water will be drawn up the tube and discharged with the jet This is the crudest form of the 'jet pump' The height to which water can be pumped, even if the vacuum is perfect, is limited to 34 feet The discharge of the tube is reduced by the pumping With a great head the quantity $.75H$ may exceed 34 feet, but in no case can the difference of pressures exceed that due to 34 feet

The co-efficient of discharge for a cylindrical tube, like that for a simple orifice, increases as the head and diameter decrease. The approximate values are given in table IX, but the number of observations made has not been great.

The co-efficient for a tube ACG or $ACGE$ (Fig. 42), CD being $AB \times .79$, has been found to be the same as for a simple cylinder.

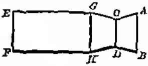


Fig. 42.

13. Special forms of Cylindrical Tubes.—If the tube projects inwards (Fig. 43) the contraction and loss of head by shock are greater than in the preceding case, and if the edge of the tube is sharp the co-efficients c_v and c are reduced to about .72. This is because some of the water comes from the directions AB and CD.

When the length AC (Fig. 44) is so short that the jet does not again touch the tube, it is known as Borda's mouthpiece. For

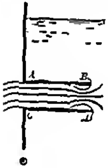


Fig. 43.

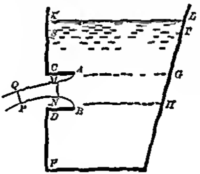


Fig. 44.

small heads AC is about half of AB. The co-efficient c_v is about the same as for a simple orifice, but the contraction is greater. It is the greatest that can be obtained by any means. The value of c_c is .52 to .54. That of c is .51 to .53, and it does not vary much. The jet also retains its coherence longer than those from other kinds of orifices.

The co-efficient for Borda's mouthpiece can be found theoretically. The velocity of the fluid along the sides of the reservoir FD, SC, which in most orifices is considerable, is here negligible. Thus the pressures on all parts of the reservoir are taken to be the simple hydrostatic pressures, and they all balance one another except the pressure on GH, which, resolved horizontally, is $Wa\left(H+\frac{P_a}{W}\right)$. The horizontal pressure on $AMNP$ is $P_a a$. The difference between the two is WaH. In a short time T let the

water between KL and MN come to $STQP$. Its change of horizontal momentum is the difference between the horizontal momenta of $KSTL$ and of $MNQP$, and that is the horizontal momentum of $MNQP$, since $KSTL$ has no horizontal momentum. This change of momentum is caused by the force WaH. Equating the impulse and momentum,

$$WaHt = WQt\frac{V}{g} = Wc_c aVt\frac{V}{g}$$

Therefore $$H = c_c\frac{V^2}{g}$$

Let $$V^2 = 2gH$$

Then $$H = \frac{V^2}{2g} = c_c\frac{V^2}{g}$$

Or $$c_c = \frac{1}{2}$$

When a tube is placed obliquely to the side of the reservoir (Fig 45) the co efficient is about $c - \cdot 0016\theta$ where θ is the number of degrees in the angle made by the axis of the tube with a line perpendicular to the side of the reservoir, and c is the co efficient for the tube when θ is 90 (Neville)

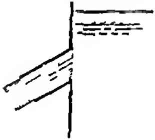


FIG 45

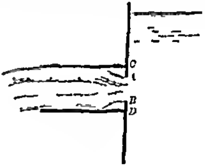


FIG 46

For a cylinder with a thin diaphragm at its entrance (FIG 46) the following co efficients are given by Neville. They apply only when the tube is filled, which it will be if not too long nor too short

Ratio of Area AB to area CD	Coefficient of Discharge for AB
0	000
1	066
2	139
3	219
4	307
	399
6	493
7	587
8	675
9	763
1·0	821

14 Bell mouthed Tubes—A simple bell mouthed tube (Fig 8, page 12) is made of the shape of the jet issuing from an orifice in a thin wall The length EE is half the diameter AC, and the curves AC, BD have a radius of 1 30 times AB This makes $CD = 80 \times AB$ The edges at A and B must be rounded and not left sharp Weisbach found the following co-efficients for small bell mouthed tubes —

Head in feet	61	1 64	11 48	55 77	337 93
Co-efficients (c_v and c)	959	967	975	994	994

This form of tube is often used as a mouthpiece for pipes to prevent loss of head by contraction If the tube is not carefully made according to the above description c will probably not exceed 95 For tubes of square cross-section 1 foot in diameter resembling bell mouths co-efficients of 94 and 95 have been found

15 Conical Converging Tubes—In a conical converging tube (Fig 47) the stream contracts on entering and again on leaving the tube The co-efficients vary with the angle of the cone, but c_v is always greater than for a cylinder The following table shows the co efficients found by Castel for a tube whose smaller diameter was 61 inch, and its length 2 6 times the smaller diameter The co efficients have reference to the smaller end of the tube As the angle of the cone increases c_c diminishes and c_v increases Their product c is a maximum for an angle of 13° 24 The co-efficients were found to be independent of the head

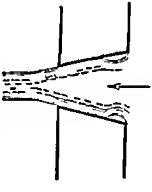


FIG 47

Angle of con[illegible]	[illegible]	16 36	21 0	9 [illegible]	40 [illegible]0′	48 50′
[illegible]	[illegible]	969	945	[illegible]19	·8[illegible]	[illegible]61
[illegible]	[illegible]	9[illegible]1	9[illegible]1	9[illegible]	930	[illegible]54
[illegible]	[illegible]	938	918	·896	809	·847

The following have also been found —

Cross sect on of Tube	Head in Feet	Smaller end of Tube	Larger end of Tube	Length of Tube	Angle of Convergence	c
Circle	300	1 20 in diam	4 20 in diam	10 ins	17°	1 00
Circle	2 7	1 21 „ „	1 50 „ „	92 „		934
Circle	1 8	2 17 „ „	2 75 „ „	7 67 „	4° 20	903
			3 50 „ „		10°	898
			5 0 „ „		20°	888
			9 83 „ „		45°	864
Rect angle	9 6	44 ft × 62 ft	2 4 ft × 3 2 ft	9 59 ft	11° 38 and 15° 18	976 to 987

Conical converging tubes are used to obtain a high velocity, but the above tables show that the velocity is not generally greater than for a bell mouthed tube The angle is usually 10° to 20°. A cylindrical tip is sometimes added, its length being about $2\frac{1}{2}$ times its diameter In the case shown above, with a head of 300 feet, the jet did not touch the cylinder If the tube projects inwards into the reservoir the co efficient is reduced, but is greater than for an inwardly projecting cylinder Conical tubes (Fig 48) are used in India at canal falls for delivering streams of water on to wheels for driving mill stones There is loss of head both at the entrance and at the bend The loss would be reduced by using a bell mouth and a curve

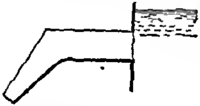


Fig 48

16 Nozzles —In order to give a high velocity to the stream

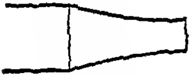


Fig 49

Fig 50

Fig 51

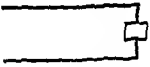


Fig 52

issuing from a hose pipe a nozzle is applied to its extremity Figs 49 and 50 show 'smooth nozzles,' and Figs 51 and 52

two forms of 'ring nozzle' The diameter, d, of the orifice is usually about one third of the diameter, D, of the pipe, and the length of the nozzle six to ten times d Experiments with nozzles have been made by Ellis, Freeman, and others [1] The pressure, p, at the entrance to the nozzle being measured by a pressure gauge, the head on the nozzle is $\frac{p}{W}$ The following co efficients have been found for the smooth nozzles, the pressure being 15 to 80 lbs per square inch

Diameter of orifice	$= \frac{3}{4}$ in	$\frac{7}{8}$ in	1 in	$1\frac{1}{8}$ in	$1\frac{1}{4}$ in
c_v	$=$ 983	982	972	976	971

For the ring nozzle c is for Fig 51 about 74, and for Fig 52, where a Borda's mouthpiece is added, about 52 In both cases c_v is about the same as for smooth nozzles

To allow for velocity of approach since $\frac{D}{d} = 3$, therefore $\frac{A}{a} = \frac{D^2}{d^2} = 9\ 0$

From table ii, noting that c_v is greater than 97, it is clear that c_u is about 1 01, and the true co efficient c must be increased 1 per cent to give the inclusive co efficient C

The following table shows the vertical heights attained by jets from nozzles in experiments made by Ellis It will be seen that the height of the jet is greater for the smooth nozzle than for the ring It is also greater the larger the diameter of the nozzle, and this may be due to the jet longer retaining its coherence

VERTICAL HEIGHTS OF JETS FROM NOZZLES

Pressure in pounds per square inch	Pressure head in feet	1 inch Nozzle		$1\frac{1}{4}$ inch Nozzle		$\frac{3}{4}$ inch Nozzle
		Smooth	Ring	Smooth	Ring	Smooth
10	23	22	22	23	22	
20	46	43	42	43	43	
30	69	62	61	63	63	59
50	115	94	92	99	95	92
70	161	121	115	129	123	113
100	230	148	136	164	155	133

The total height to which the jet remains serviceable as a fire stream is less than that to which the scattered drops rise, the former height being about 80 per cent of the latter for small

[1] *Transactions American Society of Civil Engineers* vol xxi

heads and 60 or 70 per cent for greater heads, but it is difficult to say exactly to what height the stream is serviceable The heights given in the above table are the total heights Many kinds of nozzles have been tried, but with none of them does the stream remain clear, polished, and free from spraying up to the end of the first quarter of its course Such a stream can be obtained for a pressure of 5 or 10 lbs per square inch, but not for a good working pressure

17 Diverging Tubes —With a conical diverging tube (Fig 53) the jet contracts on entering and expands again With a tube having an angle of 5°, smaller diameter 1 inch, and *length* $3\frac{1}{2}$ *inches, the co* efficient of discharge for the smaller end was 948, but with a tube having an angle of 5° 6 and a length of nine times the smaller diameter, a co efficient of 1 46 was found The case is similar to a cylindrical tube If the angle exceeds 7° or 8° the jet may not fill the tube, and the co efficient is then reduced If the angle is further increased, the jet does not touch the tube, and the case becomes an orifice in a thin wall

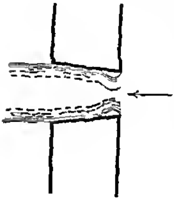


Fig 53

If the tube projects inwards into the reservoir the co efficient is reduced, but is *greater than for an inwardly projecting cylinder* If the length of the tube is now reduced so that the jet does not touch the tube, the co efficient is greater than 51, the value for Borda s mouthpiece, and becomes about 61 if the taper is increased till the case becomes a simple orifice

A compound diverging tube (Figs 54 to 60) consists of a converging or bell mouthed tube with an additional length in which the tube expands again If there are no angularities no head is lost by shock The case is similar to that of a cylindrical tube The air in the neck is partially removed by the water and the pressure reduced

The following table contains information regarding various diverging tubes It is clear that the co efficient increases with the ratio of expansion (column 5) and decreases as the taper (column 6) increases, the highest co-efficients being obtained with high ratios of expansion and gentle taper With a mean taper of 1 in 13 7 the limit seems to be reached when the ratio of expansion is 3 15, but with a taper of 1 in 5 33, not till the ratio is 5 0

A negative pressure in the neck is impossible (chap ii art 1), but if the vacuum there were perfect the pressure would be zero and the velocity would be $\sqrt{2g\left(H+\frac{P_a}{W}\right)}$ or $\sqrt{2g(H+34)}$ By making H small the discharge could be increased enormously, but practically the vacuum is always imperfect, and at a certain point the water ceases to fill the tube at the discharging end The maximum co-efficient ever obtained is 2 43

The remarks regarding pumping action made under cylindrical tubes apply equally to diverging tubes In a vacuum or with a greased tube the discharge from a diverging tube is no greater than from the mouthpiece alone, and the same may be the case with a great head, the stream passing the expanding portion without touching it

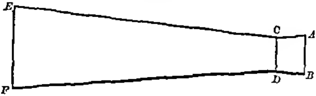


Fig 54.

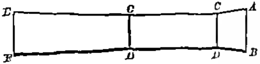


Fig 55

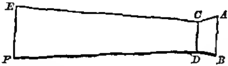


Fig 56

In Figs 54, 55, and 56 $AB=1\ 5$ in, $CD=1\ 21$ in, $AC=\ 92$ in

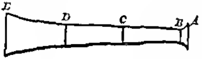


Fig 57

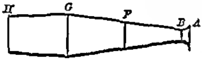


Fig 58

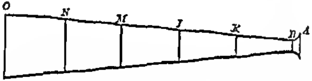


Fig 59

In Figs 57, 58, and 59 AP is a bell mouthed tube with diameter at $B=\frac{3}{4}$ in All the other segments except DE (Fig 57) are conical, and each is 2 in long

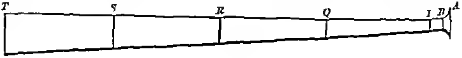


Fig 60

... DE is cylindrical The
... of the cone is least for
...
... 1 ft to 1 5 ft, the
... because the maxim
... tube AS being for
... of 1 19 ft the co

(1) Reference to Figure	(2) Tube	(3) Coefficient for Smallest Diameter	(4) Smallest Diameter	(5) Ratio of Diameter at Discharging End to Smallest Diameter	(6) Taper of Tube, or Rate at which Diameter increases.
			Inches.		
Fig 54	AF	1·40	1·21	2·48	1 in 5·5
„ 55	AF	1·38	1·21	1·24	1 in 14·1
„ 55	$AC+C'F$	1·43	1·21	1·24	1 in 14·1
„ 56	AE	1·57	1·21	1·59	1 in 9·1
Fig 57	AC	1·52	·375	1·58	1 in 9·1
	AD	1·78	·375	2·17	1 in 9·1
	AE	1·57	·375	3·83	1 in 5·6 (mean)
„ 58	AF	1·69	·375	2·33	1 in 4·0
	AG	1·79	·375	3·67	1 in 4·0
	AH	1·79	·375	3·33	1 in 6·6 (mean)
„ 59	AK	1·58	·375	2·0	1 in 5·33
	AL	2·03	·375	3·0	1 in 5·33
	AM	2·07	·375	4·0	1 in 5·33
	AN	2·09	·375	5·0	1 in 5·33
	AO	2·09	·375	6·0	1 in 5·33
Fig 60	AQ	1·48 to 1·60	1·22	1·42	1 in 23·3
	AR	1·98 to 2·16	1·22	2·30	1 in 15·1 (mean)
	AS	2·08 to 2·43	1·22	3·15	1 in 13·7 (mean)
	AT	2·05 to 2·39	1·22	4·0	1 in 13·1 (mean)

In Fig. 54 $EF=3$ in. $CE=9·75$ in

In Fig 55 $EF=1·5$ in $CC'=3·0$ in $C'E=4·1$ in
$CD=C'D$

In Fig 56 $EF=1·92$ in. $CE=6·5$ in

In Fig 57 Diameters at C, D, E are $\frac{19}{32}$ in, $\frac{13}{16}$ in, $1\frac{7}{16}$ in

In Fig 58 Diameters at F, G, H are $\frac{7}{8}$ in, $1\frac{3}{8}$ in, $1\frac{1}{4}$ in.

In Fig 59 Diameters at K, L, M, N, O are $\frac{3}{4}$ in, $1\frac{1}{8}$ in, $1\frac{1}{2}$ in, $1\frac{7}{8}$ in, $2\frac{1}{4}$ in

In Fig 60 Diameters at B, P are 1·22 in, and at Q, R, S, T 1·74 in, 2·81 in, 3·85 in, 4·90 in

Co efficients for Sluices, etc

Kinds of Aperture	Description	Width of Opening	Height of Opening	Co efficient.	Head
Sluice,[1] . .	Shown in Fig 61.	2 0 ft	1 31 ft to 10 ft	61 to 69 (averages)	33 ft to 9 8 ft over upper edge
	As above, but with boards *CF* or *DE* added	Do	Do	64 to 70 (averages)	Do
Do ,	In woodwork 1 77 ft thick at bottom, and 87 ft elsewhere	4 265 ft Do	1 7 ft 39 ft	625 803	6 ft to 14 ft over centre
Iron gates,[2] Bari Doab Canal, India	Working in grooves in the masonry heads of distributaries	4 ft to 10 ft	3 ft to 2 ft	72 to 78 (averages)	25 ft to 4 8 ft
Orifice,[3] .	Shown in Fig 62 1 inch plank placed against a 6 inch space between two 2 inch planks	5 ft 1 0 ft 1 5 ft 2 0 ft 2 5 ft	5 ft ,, ,, ,, ,,	593 607 615 621 626	5 ft over upper edge

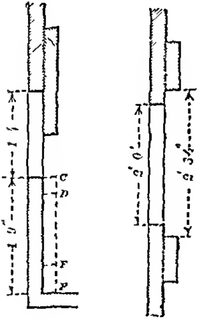


Fig 61

[1] The smaller values of c occurred with the greater height of opening For any given height of opening c varied as the head changed, being generally greatest for a head of about 1 ft

[2] The co efficient includes the *allowance for velocity of approach* which was considerable There was no contraction at the bottom and sides The openings were generally submerged *C* increases as *H* decreases, and it also increases with the size of the opening

[3] *The co efficient varies in a similar manner to that for an orifice in a thin wall*

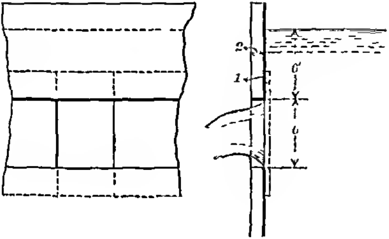


Fig 62

Section IV.—Special Cases

18 Sluices and other Apertures—A sluice is an orifice provided with a gate or shutter. Generally there are adjuncts which complicate the case and render the co-efficient uncertain. When the gate is fully open the case may approximate to that of an orifice in a thin wall. When it is nearly closed the case may resemble that of a prismatic tube. Where accuracy is required the co-efficient must be determined experimentally. It may have any value from 50 to 80, or even outside these limits. The preceding table shows some values. Sometimes when a thick gate is lifted the flow tends to force it down again, especially when it is raised slightly. This is probably due to the formation of a partial vacuum under the gate.

If the sides and lower edge of an orifice are produced externally so as to form a 'shoot' (Fig 63) the co-efficient c may be greatly altered. The air has access to the issuing stream, so that reduction of pressure in the vena contracta cannot take place, as in a cylindrical tube. On the other hand the

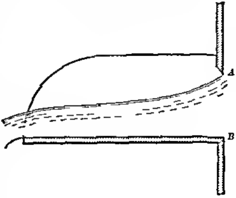


Fig 63

Co efficients for Sluices, etc

Kinds of Aperture	Description	Width of Opening	Height of Opening	Co efficient	Head
Sluice,[1]	Shown in Fig 61	2 0 ft	1 31 ft to 1 0 ft	61 to 69 (averages)	33 ft to 0 8 ft. over upper edge
	As above, but with boards *CF* or *DE* added	Do.	Do	64 to 70 (averages)	Do
Do,	In woodwork 1 77 ft thick at bottom, and 87 ft elsewhere	4 265 ft	1 7 ft	625	6 ft to 14 ft over centre
		Do	39 ft	803	
Iron gates,[2] Bari Doab Canal, India	Working in grooves in the masonry heads of distributaries	4 ft to 10 ft	3 ft to 2 ft	72 to 78 (averages)	25 ft to 4 8 ft
Orifice,[3]	Shown in Fig 62	5 ft	5 ft	593	5 ft. over upper edge
	1 inch plank placed against a 6 inch space between two 2 inch planks	1 0 ft	"	607	
		1 5 ft	"	615	
		2 0 ft	"	621	
		2 5 ft	"	626	

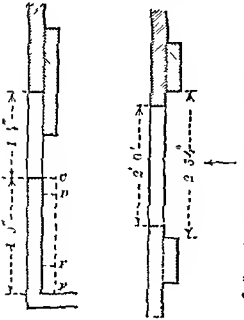


Fig 61

[1] The smaller values of c occurred with the greater height of opening For any given height of opening c varied as the head changed, being generally greatest for a head of about 1 ft

[2] The co efficient includes the allowance for velocity of approach which was considerable There was no contraction at the bottom and sides The openings were generally submerged *C* increases as *H* decreases, and it also increases with the size of the opening

[3] The co efficient varies in a similar manner to that for an orifice in a thin wall

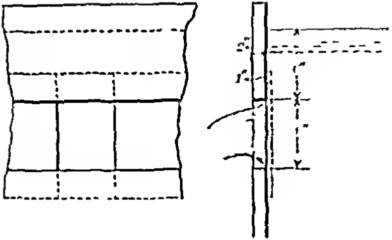


Fig 62

Section IV—Special Cases

18 Sluices and other Apertures—A sluice is an orifice provided with a gate or shutter Generally there are adjuncts which complicate the case and render the co-efficient uncertain When the gate is fully open the case may approximate to that of an orifice in a thin wall When it is nearly closed the case may resemble that of a prismatic tube Where accuracy is required the co-efficient must be determined experimentally It may have any value from .50 to .80, or even outside these limits The preceding table shows some values Sometimes when a thick gate is lifted the flow tends to force it down again, especially when it is raised slightly This is probably due to the formation of a partial vacuum under the gate

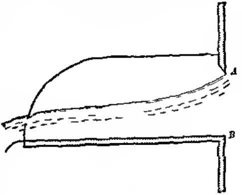


Fig 63

If the sides and lower edge of an orifice are produced externally so as to form a 'shoot' (Fig 63) the co-efficient c may be greatly altered The air has access to the issuing stream, so that reduction of pressure in the vena contracta cannot take place, as in a cylindrical tube On the other hand the

friction of the shoot has to be overcome When the head is more than two or three times the height AB the discharge of the shoot may be nearly the same as that of the simple orifice, but otherwise it is reduced For an orifice 8 inches by 8 inches with H_1 $1\frac{1}{2}$ inches the addition of a horizontal shoot 21 inches long reduced c from ·57 to ·48 With a horizontal shoot 10 feet long the following co efficients have been found,[1] the orifice being ·656 feet wide H_1 and H_2 are the heads over the upper and lower edges of the orifice

$H - H_1$	H_1 in feet						Remarks
	·066	·164	·328	·656	1·64	3·28	
feet ·656	·48	·51	·54	·57	·60	·60	Full contraction
·164	·49	·58	·62	·63	·63	·61	
·656	·53	·55	·57	·59	·61	·61	Lower edge of orifice flush with bottom of reservoir
·164	·59	·61	·63	·65	·65	·65	

19 **Vertical Orifices with small Heads**—Let $ACDL$ (Fig 61) be a bell-mouthed orifice The equations for orifices of different forms are found by integration An orifice is supposed to be divided into an infinite number of horizontal layers The discharge of any layer is $c_v\sqrt{2gH}\cdot l\,dH$ where H is the head over the layer, l its length in the plane of the orifice, and dH its thickness For a rectangular orifice

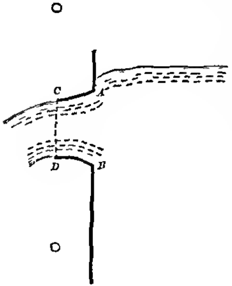


FIG 61

$$Q = c_v L\sqrt{2g}\int_{H_1}^{H_2} H^{\frac{1}{2}}\,dH$$

$$= \tfrac{2}{3}c_v L\sqrt{2g}\,(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}) \qquad (35),$$

where H_1 and H_2 are the heads at C and D respectively The discharge is the difference between the discharges of two weirs

[1] Morin, *Hydraulique*, seconde édition, pp 36 and 37

with crests at C and D respectively, and no contraction For a triangle whose base is upward and horizontal and of length l

$$Q=\tfrac{2}{3}c_v l\sqrt{2g}\left(\tfrac{2}{5}\frac{H_b^{\frac{5}{2}}-H_t^{\frac{5}{2}}}{H_b-H_t}-H_t^{\frac{3}{2}}\right) \qquad (36)$$

For the same triangle with base downwards and horizontal

$$Q=\tfrac{2}{3}c_v l\sqrt{2g}\left(H_b^{\frac{3}{2}}-\tfrac{2}{5}\frac{H_b^{\frac{5}{2}}-H_t^{\frac{5}{2}}}{H_b-H_t}\right) \qquad (37)$$

For a trapezoidal orifice, the lengths of whose upper and lower sides are l_t and l_b respectively, these sides being horizontal, the equation is obtained from equation 35 with 36 or 37 It is

$$Q=\tfrac{2}{3}c_v\sqrt{2g}\left\{l_b H_b^{\frac{3}{2}}-l_t H_t^{\frac{3}{2}}+\tfrac{2}{5}(l_t-l_b)\frac{H_b^{\frac{5}{2}}-H_t^{\frac{5}{2}}}{H_b-H_t}\right\} \qquad (38)$$

For a circle whose radius is R and H the head over its centre

$$Q=\tfrac{2}{3}c_v B\sqrt{2gH}\left(1-\frac{1R^2}{32H^2}-\frac{5}{1024}\frac{R^4}{H^4}-\frac{105}{65{,}536}\frac{R^6}{H^6}-etc\right) \qquad (39)$$

If velocity of approach has to be allowed for nH must be added to each of the heads in equations 35 to 39 Thus equation 35 becomes

$$Q=\tfrac{2}{3}c_v l\sqrt{2g}\{(H_b+nh)^{\frac{3}{2}}-(H_t+nh)^{\frac{3}{2}}\} \qquad (40)$$

In every case the discharge calculated by the above equations is less than that obtained with the same co efficient by equation 9 or 10, p 14, but owing to the much greater simplicity of these last, it is better to use them, and to multiply the result by a second co efficient to correct the error These 'co-efficients of correction,' c_k are given in table x [1] In this table D is the height, measured vertically, between the upper and lower edges of the orifice C and D (Fig 64), and the head in column 2 is that over a point halfway between these edges This, in the case of triangular or semi circular orifices, is not the head over the centre of gravity of the orifice,[2] but this latter head must be used in equation 9 or 10 The correction required is practically negligible when $H=2D$ It is *greatest when $H=50D$, that is when the upper edge of the* orifice is at the surface, which of course it never can be exactly

All the above equations apply to orifices with sharp edges, but they ought to be applied to the vena contracta Not only is D less for CD (Fig 34, p 45) than for AB, but H is greater because of the fall PN which the jet undergoes between AB and CD Thus the ratio in column 2 of table x is always greater for

[1] Smith's *Hydraulics*, chap II

[2] The distance of the centre of gravity of a semicircle from its diameter is 4244 of the radius

CD than for AB. The co-efficients for orifices in thin walls, those above the horizontal lines in the tables (iv to xi and p 54), have however been obtained by applying the above equations to the orifice AB, and for such orifices the co-efficients should be so used, or if equation 9 or 10 is used, c_k should be taken with reference to AB. But for a sluice, cylindrical tube, or other aperture for which some other co-efficient c is to be employed, the correct method is to ascertain c_c and c_v, obtain the approximate dimensions of the jet, and find the fall PN by equation 31 (p 52). This has been done for some square orifices, and the results utilised by adding column 1 to table x. For any entry in this column the corresponding entry in column 2 gives the approximate figure for the jet, and the value of c_k (to be applied to the result found by equation 9 or 10) is that in column 3. For a rectangle whose horizontal side l is less than D, the vena contracta is nearer to the orifice, the fall PN is less, and the contraction of the jet in a vertical direction less, so that the figures in column 1 approach nearer to those in column 2. When l is less than $5D$ column 1 is not needed.

The co-efficients for vertical orifices under small heads are not well determined. The smallness of the margin on the upper side of the orifice tends to produce incomplete contraction there and to increase c, but, on the other hand, there is a fall in the water surface upstream of the orifice, the head is measured above the fall, and this, according to Smith, reduces c. A vortex may also be formed, and possibly it may penetrate the orifice and reduce c. Columns 4 and 7 of the table on page 54 do not agree, though in both cases the head was measured back from the orifice. Smith's co-efficients are to be preferred.

With an orifice in a horizontal plane under a small head the proportion of water approaching axially is reduced and the contraction is probably increased, except with bell-mouths. The co-efficients for such cases having nearly all been obtained for orifices in vertical planes, are not likely to apply correctly to others, even if the head is measured to the vena contracta.

The matter in this article refers to cases where H is small compared to the orifice. If, in addition, H is actually small, the difficulties attending such cases (chap II art 7) are added.

EXAMPLES

Example 1.—Water enters the condenser of a steam engine at the sea level from a reservoir whose water surface is 10 feet above the injection orifice. The pressure in the condenser is 3 lbs per

square inch Find the theoretical velocity of flow into the condenser

The atmospheric pressure in the reservoir is 14 7 lbs per square inch The resultant pressure is thus 11 7 lbs per square inch or 1685 lbs per square foot This is equivalent to a head of $\frac{1685}{62\ 4}=27$ feet The total effective head is therefore 37 feet From table i the velocity is 48 7 feet.

Example 2—Find the discharge from a circular bell mouthed tube, 1 foot in diameter, situated in the middle of the end of a horizontal trough of rectangular section, 2 feet wide and 2 feet deep

The head is 1 foot From the table in article 14 c, is probably 96 From table x the co efficient of correction for small heads is 992 A is 4 square feet and a is 7854 square feet $\frac{A}{a}=\frac{4}{7854}=5\ 01$ From table iii the co-efficient of correction for velocity of approach is 1 02 From table i $\sqrt{2gH}=8\ 02$ Then $Q=96\times 8\ 02\times 785\times 992\times 1\ 02=6\ 12$ cubic feet per second

Example 3—A culvert 3 feet long, consisting of a semicircular arch of 1 foot radius resting on a level floor, has to pass a discharge of 9 feet per second There is a free fall downstream What will be the water level upstream?

From table ix c may be taken to be 80 Also $a=2\times 785=1\ 57$ square feet

To obtain an approximate solution

$$Q=9=80\sqrt{2gH}\times 1\ 57 \qquad \sqrt{2gH}=\frac{9}{80\times 1\ 57}=7\ 17$$

From table i $H=80$, or the water will be 80 foot above the centre of gravity of the aperture or 22 foot above the crown of the arch

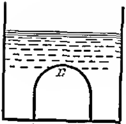
The contraction, supposed to be complete elsewhere, is nearly absent at the crown, and may be taken to be suppressed on one fourth of the perimeter, thus (table ii) making

$$c=80\times 1\ 04=832$$

In table x $D=1\ 0$ foot, and the head over the centre of the orifice is $22+50=72$ foot or $72D$ This corresponds to 8D for the vena contracta, and the figure in column 8, differing no doubt hardly at all from column 4, is 980

The above two corrections are 4 per cent plus and 1 per cent minus, so that Q is really 3 per cent more than assumed To make it right deduct 6 per cent from H, which will then be $80\times 94=752$ foot, that is, the water is 18 foot above the crown.

Example 4—For the culvert shown in the annexed diagram (2 feet wide and 5 feet long), let there be an open approach channel 4 feet wide, with vertical walls and floor level with that of the culvert Find the discharge when the upstream head is 1 foot above the crown of the arch, and the downstream head 6 inches above it



In this case there is incomplete contraction on all sides, and also velocity of approach From example 3, $a=3\ 57$ square feet, $A=12\ 0$ square feet, $P=4\ 0+3\ 14=7\ 14$ feet, $S=2\ 0$ feet If the contraction were complete on AEB, c_p would be (art 3) about $80\times(1+\ 152\times\frac{5}{7})=\ 80\times1\ 043=\ 834$ The average margin on AEB is about 1 30 feet Therefore $\frac{G}{d}=\frac{1\ 30}{2}=\ 65$, and $\frac{S'}{P}=\frac{5\ 14}{7\ 14}=\ 75$ From table II $\frac{c_i}{c}=1\ 035$ about Therefore

$$c_i=\ 834\times1\ 035=\ 863$$

The head is 5 foot, and as the orifice is wholly submerged no correction for small head is needed From table I $\sqrt{2gH}$ is 5 67 $Q=\ 863\times5\ 67\times3\ 57=17\ 47$ cubic feet per second

To allow for velocity of approach by the usual method,

$$v=\frac{17\ 47}{12}=1\ 46 \text{ feet per second} \quad \text{Let } n=1\ 0$$

From table I $h=\ 033$, $H+h=\ 533$ From table I $V=5\ 87$ Then $Q=\ 863\times5\ 87\times3\ 57=18\ 08$ cubic feet per second

To allow for velocity of approach by a co efficient of correction, for the contracted section c_v is (art 12) about 1 30, and $c_v=\frac{863}{1\ 30}=\ 664$ *Therefore* $a=3\ 57\times\ 66=2\ 36$ square feet, and $\frac{A}{a}=\frac{12\ 0}{2\ 36}=5\ 09$ From table III, noting that c_v is about 1 30 instead of 97, and that the figures in column 3 are to be increased, c_a is about 1 03, that is, 3 per cent must be added to 17 47, *making* 17 99 *cubic feet per second*

Note—Further examples may be obtained by taking cases analogous to some of those in examples of chap II

TABLE I—HEADS AND THEORETICAL VELOCITIES (Art 1)

For a head greater than 10 feet divide the head by 100 and take ten times the corresponding velocity Thus for a head of

120 feet the velocity is 87·9, or ten times the velocity given for a head of 1·2 feet. For a velocity over 25 divide it by 10 and multiply the corresponding head by 100. The same methods can be adopted to facilitate interpolations. Thus for $H = ·032$ look out 3·2.

In the first fifteen entries the heads correspond to certain definite velocities. These entries may be useful in cases of velocity of approach. After that the velocities correspond to definite heads.

H	V	H	V	H	V	H	V	H	V	H	V
·0022	·38	·13	2·89	·45	5·38	·84	7·35	2·3	12·2	6·2	20·0
·0025	·40	·135	2·95	·46	5·44	·85	7·40	2·4	12·4	6·3	20·1
·0027	·42	·14	3·00	·47	5·50	·86	7·44	2·5	12·7	6·4	20·3
·0030	·44	·145	3·05	·48	5·56	·87	7·48	2·6	12·9	6·5	20·5
·0033	·46	·15	3·11	·49	5·62	·88	7·53	2·7	13·2	6·6	20·6
·0036	·48	·155	3·16	·50	5·67	·89	7·57	2·8	13·4	6·7	20·7
·0039	·50	·16	3·21	·51	5·73	·90	7·61	2·9	13·7	6·8	20·9
·0042	·52	·165	3·26	·52	5·79	·91	7·65	3	13·9	6·9	21·0
·0045	·54	·17	3·31	·53	5·85	·92	7·70	3·1	14·1	7	21·2
·0049	·56	·175	3·36	·54	5·90	·93	7·74	3·2	14·3	7·1	21·3
·0052	·58	·18	3·40	·55	5·95	·94	7·78	3·3	14·5	7·2	21·5
·0056	·60	·185	3·45	·56	6·00	·95	7·82	3·4	14·8	7·3	21·6
·0066	·65	·19	3·50	·57	6·06	·96	7·86	3·5	15·0	7·4	21·8
·0076	·70	·195	3·55	·58	6·11	·97	7·90	3·6	15·2	7·5	22·0
·0087	·75	·20	3·59	·59	6·17	·98	7·94	3·7	15·4	7·6	22·1
·01	·80	·21	3·68	·60	6·22	·99	7·98	3·8	15·6	7·7	22·2
·015	·98	·22	3·76	·61	6·28	1	8·02	3·9	15·8	7·8	22·4
·02	1·13	·23	3·85	·62	6·32	1·05	8·22	4	16·0	7·9	22·5
·025	1·27	·24	3·93	·63	6·37	1·1	8·41	4·1	16·2	8	22·7
·03	1·39	·25	4·01	·64	6·42	1·15	8·60	4·2	16·4	8·1	22·8
·035	1·50	·26	4·09	·65	6·47	1·2	8·70	4·3	16·6	8·2	23·0
·04	1·60	·27	4·17	·66	6·52	1·25	8·97	4·4	16·8	8·3	23·1
·045	1·70	·28	4·25	·67	6·57	1·3	9·15	4·5	17·0	8·4	23·2
·05	1·79	·29	4·32	·68	6·61	1·35	9·32	4·6	17·2	8·5	23·4
·055	1·88	·30	4·39	·69	6·66	1·4	9·49	4·7	17·4	8·6	23·5
·06	1·97	·31	4·47	·70	6·71	1·45	9·66	4·8	17·6	8·7	23·6
·065	2·04	·32	4·54	·71	6·76	1·5	9·83	4·9	17·7	8·8	23·8
·07	2·12	·33	4·61	·72	6·81	1·55	9·99	5	17·9	8·9	23·9
·075	2·20	·34	4·68	·73	6·86	1·6	10·2	5·1	18·1	9	24·1
·08	2·27	·35	4·75	·74	6·91	1·65	10·3	5·2	18·3	9·1	24·2
·085	2·34	·36	4·81	·75	6·95	1·7	10·5	5·3	18·5	9·2	24·3
·09	2·41	·37	4·87	·76	6·99	1·75	10·6	5·4	18·7	9·3	24·4
·095	2·47	·38	4·94	·77	7·04	1·8	10·8	5·5	18·8	9·4	24·6
·10	2·54	·39	5·01	·78	7·09	1·85	10·9	5·6	19·0	9·5	24·7
·105	2·60	·40	5·07	·79	7·13	1·9	11·1	5·7	19·2	9·6	24·8
·11	2·66	·41	5·14	·80	7·18	1·95	11·2	5·8	19·3	9·7	24·9
·115	2·72	·42	5·20	·81	7·22	2	11·3	5·9	19·5	9·8	25·0
·12	2·78	·43	5·26	·82	7·26	2·1	11·7	6	19·6	9·9	25·2
·125	2·84	·44	5·32	·83	7·31	2·2	11·9	6·1	19·8	10	25·4

TABLE II.—IMPERFECT AND PARTIAL CONTRACTION FOR RECTANGULAR ORIFICES WITH SHARP EDGES (Art 3)

$\frac{s}{l}$	Values of $\frac{f}{l}$: 3	2 67	2	1	5	0	Remarks
	Approximate Values of $\frac{c}{c}$						
25	1	1 000	1 002	1 006	1 015	1 04*	* *These are cases* of partial contraction, see equation 20, page 46
50	1	1 001	1 003	1 013	1 030	1 08*	† These are quite doubtful
75	1	1 001	1 004	1 019	1 045	1 12*	‡ When the contraction is suppressed on all sides c_i is 1 0 that is, it is increased by 61 per cent above its mean value (62)
875	1	1 001	1 005	1 04†	1 10†	1 40†	
1	1	1 002	1 006	1 05†	1 20†	1 61‡	

TABLE III.—COEFFICIENTS OF CORRECTION FOR VELOCITY OF APPROACH (Art 5)

($c_i = 97$ $n = 1 0$)

(1)	(2)	(3)	(4)	(5)	(6)
$\frac{A}{a}$	$\frac{a^2}{A^2}$	$c_i^2 n \frac{a^2}{A^2}$	$1 - c_i^2 n \frac{a^2}{A^2}$	$\sqrt{1 - c_i^2 n \frac{a^2}{A^2}}$	$\frac{1}{\sqrt{1 - c_i^2 n \frac{a^2}{A^2}}}$ or C_v
1 33	5625	529	471	587	1 456
1 5	4444	418	582	763	1 311
2	2500	23[illegible]	765	875	1 143
2 5	1596	150	850	922	1·072
3	1111	104	896	947	1 0[illegible]6
5	0400	038	962	981	1·010
10	·0100	010	990	995	1·005
15	·0044	004	996	·997	1·003
20	·0025	·0024	9976	·999	1·001

Table IV — Co-efficients of Discharge for Circular Orifices in Thin Walls (Art 8)

Head	Diameter of Orifice in Feet									
	·02	·03	·04	·05	·07	·1	·15	·2	·6	1
Feet.										
·3(¹)				637	628	621	608			
·4			637	631	624	618	606			
·5		643	633	627	621	615	605	600	592	
·6	655	640	630	624	618	613	605	601	593	
·8	648	634	626	620	616	610	603	601	594	591
1	644	631	623	617	612	608	603	600	595	591
1·5	637	624	618	612	608	605	601	600	596	593
2	632	621	614	610	607	604	600	599	597	595
2·5	629	619	612	608	605	603	600	599	598	596
3	627	617	611	606	604	603	600	599	598	597
4	623	614	609	605	603	602	599	599	597	596
6	618	611	607	604	602	600	599	598	597	596
8	614	608	605	603	601	600	598	598	596	596
10	611	606	603	601	699	598	597	597	596	595
20	601	600	599	598	597	596	596	596	596	594
50 (¹)	596	596	595	595	594	594	594	594	594	593
100 (¹)	593	593	592	592	592	592	592	592	592	592

Table V — Co-efficients of Discharge for Square Orifices in Thin Walls (Art 8)

Head	Side of Orifice in Feet.									
	·02	·03	·04	·05	·07	·1	·15	·2	·6	1
Feet.										
·3(¹)				642	632	624	612			
·4			643	637	628	621	611			
·5		648	639	633	625	619	610	605	597	
·6	660	645	636	630	623	617	610	605	598	
·8	652	639	631	625	620	615	608	605	600	596
1	648	636	628	622	618	613	608	605	60	598
1·5	641	629	622	617	614	610	606	605	602	601
2	637	626	619	615	612	608	606	605	604	602
2·5	634	624	617	613	610	607	606	605	604	602
3	632	622	616	612	609	607	606	605	604	603
4	628	619	614	610	608	606	605	605	603	602
6	623	616	612	609	607	605	605	604	603	602
8	619	613	610	608	606	605	604	604	603	602
10	616	011	608	606	605	604	603	603	602	601
20	606	605	604	603	602	602	602	602	601	600
50 (¹)	602	601	601	601	601	600	600	600	599	599
100 (¹)	599	598	598	598	598	598	598	598	598	598

For circular and square orifices 2 feet in diameter under heads of 2 to 10 feet co-efficients of about ·60 have been found, and for a horizontal circular orifice ·033 foot in diameter under a head of 25 feet a co-efficient of ·25 foot

TABLE VI —COEFFICIENTS OF DISCHARGE FOR RECTANGULAR ORIFICES, ONE FOOT WIDE, IN THIN WALLS (Art 8)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Head	Height of Orifice in Feet.							
	125	25	50	5	1	1 5	2	4
Feet								
2	634							
3	634	632						
4	6 13	632	621					
5	633	632	619	615				
6	633	632	619	613	610			
8	633	633	618	612	606	630		
1	632	632	618	612	605	624		
1·25	631	632	618	611	604	624	632	
1 5	630	631	618	611	604	610	627	
2	629	630	617	610	005	617	628	
2 5	628	628	616	610	605	615	627	645
3	627	627	613	610	(05	613	619	637
4	624	624	614	609	603	611	616	630
6	615	615	609	604	602	606	610	618
8	609	607	603	602	601	602	604	610
10	600	603	601	601	601	601	602	604
20	607	604	602	601	601	601	602	605
30	609	604	603	602	601	602	603	605
40	611	606	604	603	602	603	605	607
60	614	607	605	604	602	603	606	609

TABLE VII —COEFFICIENTS OF DISCHARGE FOR SMALL ORIFICES (area 196 square inch) IN THIN WALLS (Art 8)

Head	Equilateral triangle base upward	Square* with sides vertical.	Circular	Rectangle with long side horizontal		Remarks
				4 to 1 †	16 to 1 ‡	
Feet.						* With diagonal verti
1	636	627	620	643	664	cal c is about 0014 greater
2	628	620	613	636	651	† With long side verti
4	623	616	608	629	642	cal c is about 0014 less
6	620	614	607	627	637	‡ With long side verti
10	618	612	605	624	633	cal c is about ·0005 less
14	618	610	604	622	630	for heads up to 10 feet
20	616	609	603	621	629	and about ·0005 more for
						the greater heads

TABLE VIII.—COEFFICIENTS OF DISCHARGE FOR SUBMERGED ORIFICES IN THIN WALLS. (ART. 10.)

Head	Size of Orifice in Feet.				
	Circle [illegible] ft.	Square [illegible]	Circle [illegible] ft.	Square [illegible] ft.	Rectangle [illegible] ft. × [illegible] ft.
Feet. .5	·616	·620	·602	·609	·622
1	·610	·615	·602	·606	·622
1.5	·6[illegible]7	·612	·601	·605	·621
2	·604	·6[illegible]2	·601	·604	·620
2.5	·6[illegible]2	·6[illegible]	·599	·604	·619
3	·6[illegible]2	·6[illegible]	·599	·604	·618
4	·6[illegible]1	·6[illegible]	·599	·605	...

TABLE IX.—COEFFICIENTS OF DISCHARGE FOR CYLINDRICAL TUBES. (ART. 12.)

Head	Diameter of Tube in Inches.			
	[illegible]	[illegible]	1	3
Feet. .5	84	83	82	
2	83	82	81	80
22	. .		80	80

TABLE X —COEFFICIENTS OF CORRECTION FOR VERTICAL ORIFICES WITH SMALL HEADS (Art 19)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Head over centre of square orifice with sharp edges	Head over centre of bell mouthed orifice or of vena contracta for sharp edged orifice	Rect angle	Circle or semicircle with diameter vertical	Triangle with base upward	Triangle with base downward	Semicircle with diameter upward	Semicircle with diameter downward	Remarks
	50*D*	943	960	924	979	937	965	
	52*D*	950	965					The coefficients have not been worked out in detail for triangles and semicircles, but can be easily estimated from the figures given in the first and tenth lines When the head is greater than *D* the coefficients for orifices of all shapes are nearly equal
	55*D*	957	970					
	60*D*	966	975					
52*D*	70*D*	976	982					
64*D*	80*D*	982	987					
78*D*	90*D*	986	990					
92*D*	1 0*D*	989	992					
1 13*D*	1 2*D*	992	994					
1 44*D*	1 5*D*	995	997	996	998	996	997	
	2 0*D*	997	998					
	2 5*D*	998	999					
	3 0*D*	999	999					
	4 0*D*	999	1 000					

CHAPTER IV

WEIRS

[For preliminary information see chapter II articles 4, 6, 7, 14, and 15]

SECTION I—WEIRS IN GENERAL

1 General Information —The following statement shows a few typical kinds of weirs, and gives some idea as regards the co efficients Further co efficients will be given in subsequent articles, and from them the values for many cases occurring in practice can be inferred, but the varieties of cross section are innumerable, the co efficients vary greatly, and generally can only be found accurately by actual observation When the length, l, of a weir is great relatively to H, it makes little difference whether there are end contractions or not

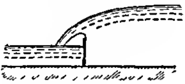
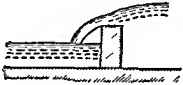
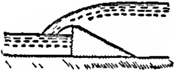
To ensure complete contraction iron filed sharp should be used for the upstream edges with small heads For heads of over a foot planks or masonry may be used

Since the inclusive co efficient C increases with H, it follows that when there is velocity of approach Q increases faster than $H^{\frac{3}{2}}$ If H is doubled Q is about trebled To double the discharge H must be multiplied by 1 5 If a given volume of water passes in succession over two similar weirs, one of which is three times as long as the other, the head on it will be half that on the other If a volume of water, passing in succession over two weirs, alters, the heads on both will alter in nearly the same ratio These rules are only approximate, and when there is no velocity of approach they are somewhat modified To facilitate calculations the values of $H^{\frac{3}{2}}$ corresponding to different values of H are given in table XI

Smith states that with low heads such as 2 foot the discharge may be affected by a change in the temperature of the water of 30° Fahr If the water is disturbed by waves or eddies the discharge is probably reduced, unless 'baffles' are used to calm it

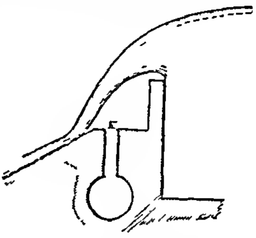
In the sheet of water passing the edge of a weir in a thin wall

Various Kinds of Weirs and their Co-efficients.

Type of Weir	Dimensions of Weirs for which Co-efficients are quoted: Height	Top Width.	Upstream Slope.	Down stream Slope.	Co-efficient C for Head of 1 foot.	Manner in which Co-efficient varies as Head Increases
	Feet	Feet				
 Fig. 65. Thin Wall.	1·64				·67	Increases slowly
 Fig. 66. Flat top, vertical face and back	2·46	1·31	vertical	vertical	·54	Increases rapidly
 Fig. 67. Steep back and sloping face	1·64	·33	2 to 1	vertical	·75	Increases
 Fig. 68. Steep face and sloping back.	1·64	·33	vertical	5 to 1	·61	Increases
 Fig. 69. Rounded.	1·64				·65	Increases

These weirs are some of the types used by Bazin in his experiments. There were no end contractions. The co-efficient C includes the allowance for velocity of approach.

the velocity is greatest at the lower side, but with a broad topped weir the friction on the top reduces the velocities nearest the weir In every case the initial horizontal velocity of the whole sheet may be taken to be $\frac{2}{3}\sqrt{2gH}$, and the path of the sheet calculated as for orifices (chap iii art 7) Fig 70 shows a separating weir as used for water supplies of towns After heavy rain the water is discoloured and H is great, so that the sheet falls as shown and the water is conveyed to a waste channel At other times the water falls into the opening K and is conveyed to the service reservoirs The velocity at the ends of a weir is generally less than elsewhere, and it increases up to a point distant about $3H$ from the ends The pressure in the water passing over the crest of a weir is less than that due to the head



Fa 0

The following statement shows the chief experiments on weirs in thin walls —

Observer	No of Observations made	Length of Weir	Head		Height of Weir	State of Contraction	Distance of Measuring Section from Crest of Weir
			From	To			
		Feet	Feet.	Feet.	Feet		Feet
Francis	46	10	6	1 6	4 6	Com	6 0
,,	19	10	6	1 0	2 0	plete	6 0
	6	4	7	1 0	4 6 & 2 0	or	6 0
Smith	12	2 6	6	1 7	3 8	nearly	7 6
Lesbros	21	1 77	1	6	1 8	com	11 5
Poncelet & Lesbros	6	66	08	7	1 8	plete	11 5
Fteley & Stearns	54	2 3 to 5	15	94	3 6	Vari	6 0
Lesbros	34	66	06	7	1 8	able	11 5
Francis	1"	10	7	1 0	4 6		6 0
Fteley & Stearns	10	19	5	1 6	6 6	End	6 0
,,	30	5	07	8	3 2	con	6 0
Lesbros	14	66	06	8	1 8	trac	11 5
Bazin	295	6 56	·23	1 0	3 7 to 8	tions	16 4
	38	3 28	23	1 3	3 3	absent	16 4
	48	1 64	23	1 8	3 3		16 4

Bazin's observations are the most recent and extensive They included observations of the form of the upper and under surfaces of the falling sheet and of the air pressure beneath it Bazin states that under some circumstances the discharge of a weir can be ascertained better by observing this pressure than by observing the head (Cf art 8)

2. Formulæ —The ordinary weir formula (equation 11, p 15) and the other formulæ deduced from it are defective in form It is often said that the head ND (Fig 71) ought to be taken into account, the discharge of the weir being considered to be that of an orifice whose bottom edge is C and top edge D, but the formulæ would be much more complicated, and the height ND is not well known Any shortcomings in the formulæ are made good by the values given to the co efficients Moreover there is no special reason why the section NC should be selected for measurement From C to F the under side of the sheet rises if the weir has its upper edge sharp The heads should probably be measured, as with an orifice in a thin wall, to F and G at the contracted section The flow over a weir with a wide top is still more complex The case is really one of variable flow in a short open channel

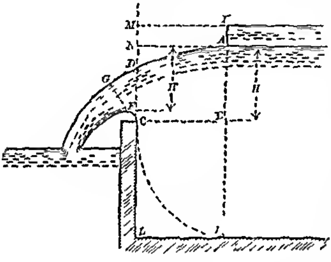


Fig 71

In all weir formulæ m can be written for $\frac{2}{3}c$, and this plan is adopted by Bazin, but c is the true co efficient expressing the relation between the actual and the theoretical discharge, and, following the usual practice, c will be used both in formulæ and in tables Since $\frac{2}{3}\sqrt{2g}=5{\cdot}35$ this figure can be used in calculations instead of 8·02, and multiplication by $\frac{2}{3}$ is thus unnecessary The values of $\frac{2}{3}c\sqrt{2g}$ corresponding to different values of c are given in table xii and denoted by q_w being the discharges per foot run over a weir with $H=1$ foot

3 Incomplete Contraction—From a comparison of the co-efficients obtained for various weirs in thin walls, Smith arrives at the formula

$$c_p = c\left(1 + 16\frac{S}{l}\right)$$

where c_p and c are the co-efficients for two equal weirs, one with partial and one with full contraction P is the complete perimeter of the weir, that is $l+2H$, S the length of the perimeter over which the contraction is suppressed This formula applies for heads ranging from 3 foot to 1 0 foot, it is not exact, but may be used for finding co-efficients not otherwise known

When the contraction is imperfect,[1] whether or not the margin is sufficient to give a negligible velocity of approach, the formula arrived at by Smith is

$$c_i = c\left(1 + x\frac{S}{l}\right)$$

where c_i is the co-efficient for the weir with imperfect contraction, S the length of its perimeter on which the contraction is imperfect, and x is as follows, d being the least dimension of the weir and G the width of the clear margin

$\frac{G}{d}=$	3	2 67	2	1	5	0
$x=$	0	0016	005	025	06	16

When the contraction is imperfect over the whole perimeter $S=P$, and when

$\frac{G}{d}=$	3	2 67	2	1	5	0
the increase in c per cent $=$	0	16	50	2 5	6	16

But when S is a very large fraction of P, or when $S=l$ and $\frac{G}{d}$ is very small—that is, when there is not much contraction left except at the surface—the rules become of doubtful application

4 Flow of Approach—Bazin observed some surface curves for weirs 3 72 feet and 1 15 feet high, and for each weir with several heads ranging from 5 feet to 1 5 feet He finds y (Fig 71) to be in every case about $3H$, but the upper portions of the curves are so flat, especially for the lower heads, that it is impossible to say exactly where they begin Observations made by Fteley and Stearns, with H nearly constant and different values of G, give results somewhat similar to Bazin's, but when G is less than H, y is

[1] For definitions of partial and imperfect see chap III art 3

about 2 5G The above indicates the proper distance from the weir to the measuring section In weirs with end contractions G, the distance of the end of the weir from the side of the channel must be used instead of G if it exceeds G In a weir with a long sloping face Smith found y to be 40 feet with $H=7$ 24 feet

The fall ND or F for weirs in thin walls is generally between $\frac{H}{10}$ and $\frac{H}{4}$ It is much greater with broad topped weirs In the above experiments with weirs in thin walls $\frac{F}{H}$ was found to be as follows —

$G=$	3 56	1 7	5	3 72	1 15	feet
$H=$	614	606	564	5 to 1 5	5 to 1 5	„
$\frac{F}{H}=$	148	145	114	149	143	„
	Fteley and Stearns			Bazin		

Some other values are

$H=$ 68	37	·20	08 feet		Poncelet and Lesbros, weirs in thin walls, full contraction, length 66 foot
$\frac{F}{H}=$ 08	11	15	25 „		

And for flat topped weirs

	$H=$ 5	1	5	1	5	1 feet
	$\frac{F}{H}=$ 27	·28	29	40	64	67 „
Top width	5 inch		2 inches		3 inches	

According to Smith F is somewhat greater in weirs with no end contractions than in others, and increases slightly with l

Fteley and Stearns found that just upstream of a weir the pressure, at least near the bottom, is greater than at the same level further upstream Generally the difference is nearly as h or $\frac{v^2}{2g}$, and it also increases as G decreases It never exceeded the amount due to a head of 03 foot, and was generally much less

5 **Velocity of Approach** —The ordinary formulæ for weirs with velocity of approach are

$$\left.\begin{aligned} Q&=\tfrac{2}{3}cl\sqrt{2g}\,(H+nh)^{\frac{3}{2}} \\ &=ml\sqrt{2g}\,(H+nh)^{\frac{3}{2}} \end{aligned}\right\} \quad \ldots \quad (41)$$

By using a variable co-efficient of correction c_a we obtain the inclusive co-efficients $C=cc_a$ and $M=mc_a$

The formulæ with inclusive co-efficients are

$$\left.\begin{aligned} Q&=\tfrac{2}{3}Cl\sqrt{2g}\,H^{\frac{3}{2}} \\ &=Ml\sqrt{2g}\,H^{\frac{3}{2}} \end{aligned}\right\} \qquad (42)$$

For weirs in thin walls with complete contraction equation 42 is not ordinarily suitable, because while the values of c are known and tabulated those of C are not known, and if calculated for many different values of v would fill a formidable set of tables. But for other kinds of weirs C is often known as well as or better than c. In these cases, and also in cases where Q is to be measured for some particular weir, and the co-efficients ascertained and recorded, equation 42 is eminently suitable.

Where c is not known the use of c_a renders the adoption of the indirect or tentative solution unnecessary in certain cases, and so saves trouble (see examples 1 and 5). It is not convenient to give a formula, as in the case of orifices (equation 22, p. 48), for calculating c_a, because equation 11 gives Q and not v. In order to find v it would be necessary to separate c into c_a and c_v, and these quantities are not properly known. Values of c_a have, however, been found by working out various cases, and are given in table XIII for two values of c. Others can be interpolated if required. The excess of c_a above 1.0 is nearly as c^2, and for a given value of c nearly as n. The co-efficient c_a may be used either for solving ordinary problems or for obtaining values of C from c or M from m.

The inverse process of finding c from C or m from M is as follows:—

$$\text{Since } Q = vA,$$

Therefore from equation 42 $$\frac{v^2}{2gH} = \frac{M^2 l^2 H^2}{A^2} = M^2 \frac{a^2}{A^2} \qquad (42\text{A})$$

$$\text{But } Q = ml\sqrt{2g}\left(H + n\frac{v^2}{2g}\right)^{\frac{3}{2}}$$

$$= ml\sqrt{2g}H^{\frac{3}{2}}\left(1 + n\frac{v^2}{2gH}\right)^{\frac{3}{2}}$$

Since the last term in the brackets is small compared to the first term, the expression in brackets is nearly equal to $1 + \frac{3}{2}n\frac{v^2}{2gH}$. Adopting this value and substituting from equation 42A

$$Q = ml\sqrt{2g}H^{\frac{3}{2}}\left(1 + \frac{3}{2}nM^2\frac{a^2}{A^2}\right) \qquad (43)$$

From equations 42 and 43

$$m = \frac{M}{1 + \frac{3}{2}nM^2\frac{a^2}{A^2}} \qquad (44)$$

It is of course impossible to observe either m or n directly. The

observations give M directly, and either m or n can be found by assuming a value for the other. Generally m is assumed or deduced from its values for a similar weir with no velocity of approach, and n is then calculated. When the length of a weir is the same as the width of the channel of approach and G is the height of the weir equation 44 becomes

$$m=\frac{M}{1+\frac{3}{2}nM^2\frac{H}{(G+H)}} \qquad (45),$$

and in this form is given by Bazin.

On the assumption that the effect of the energy due to the velocity of approach is the same as that of raising the water level by a height AK (Fig 71) equal to $\frac{v^2}{2g}$, the discharge is the same as that through an orifice with heads KA and KE, and the old form of equation was

$$Q=\tfrac{2}{3}cl\sqrt{2g}\left\{(H+h)^{\frac{3}{2}}-(h)^{\frac{3}{2}}\right\},$$

which is similar to equation 35, p 70. This equation cannot be of the true theoretical form, chiefly because the original weir formula (equation 11 p 15) is not so. It would, however, be right to use it as the best attempt at a theoretical formula, if there were any advantage in doing so. But the last term $h^{\frac{3}{2}}$ is generally small and often minute, while the formula is more complicated than equation 12. The method of allowance for v is largely empirical, and it is better to use the more simple formula 12. With this formula n might be expected to be somewhat less than unity.

From article 7, chapter II it is clear that for weirs with velocity of approach the contraction may be either perfect or imperfect. When it is imperfect the increase of discharge is due partly to the energy of the water represented by $\frac{v^2}{2g}$ and partly to reduced contraction due to smallness of the margin. The value of n from both causes combined has been found to be, for weirs in thin walls, from 1 0 to 2 5. Smith rightly separates the two causes, and, discussing various experiments concludes that n should be 1 4 for weirs with full contraction, and 1 33 for weirs with no end contractions. The effect of reduced contraction, if any, was estimated separately, but the allowance made in the cases of weirs with no end contractions was not quite sufficient according to the rules given in article 3 above, so that n was a little overestimated and Smith himself suggests that this may be so. Since Smith wrote, the results of Bazin's experiments on weirs with no end contractions have appeared. Owing to their general regularity and extent they are entitled to great weight. In analysing them on

Smith's principle it is found that n varies from .86 to 1.37, and averages about 1.1. For moderate velocities of approach Q depends only a little on n (see table XIII.), and it is not worth while to give here the detailed analysis.[1] Bazin himself gives 1.54 as the mean value of n, but this includes the effect of reduced contraction. Both sets of experiments, namely Bazin's and those discussed by Smith, include high velocities of approach, the ratio $\frac{A}{a}$ being sometimes only 1.6. For weirs with full contraction the experiments discussed by Smith are not numerous, and his resulting figure 1.4 somewhat doubtful. It seems high in comparison with the others, and may be put at 1.33.

The variations in n, and especially its exceeding the value 1·0 are not easy to explain. A weir is usually in the centre of a channel, and the average deflection of the various portions of the approaching stream is then a minimum, especially if its greatest velocity is also in the centre, so that a large proportion of the water flows straight. In a weir so placed n will be a maximum, but this is no reason for its being greater than unity. The whole of the water, and not only the quickest water, has to pass over the weir. At the approach section the velocity distribution (chap. II. art. 21) is normal. The total energies of the various portions of the stream may (chap. II. art. 10) exceed the energy due to v, but the difference is probably only a few per cent, and nothing like 33 or even 20. Moreover, some little energy must be lost in eddies between the approach section and the weir. Thus in no case will the available energy appreciably exceed that due to $\frac{v^2}{2g}$. A high velocity of approach does not of itself reduce contraction. The high velocity occurs in the portion EB (Fig. 71) as well as in AE. With an orifice in the side of a reservoir a high velocity does not cause reduced contraction, but rather the contrary. The surface curves for weirs do not indicate any reduced surface contraction when v is high. Reduction of the clear margin is allowed for separately, and there are high values of n for cases in which the clear margin is ample.

It is probable that the deviations of n from unity are chiefly due to the incorrect form of the equation used. If a curved crest FC is added, the flow will not be appreciably affected, but the head will now be H' instead of H. The co-efficients of the two weirs must be such that $cH^{\frac{3}{2}}=c'H'^{\frac{3}{2}}$. Suppose A now reduced so that v becomes considerable, then $c(H+nh)^{\frac{3}{2}}$ must equal $c'(H'+n'h)^{\frac{3}{2}}$, and this occurs when $n'=n\frac{H'}{H}$. If c is .60 and c' is .80 (values likely to occur in practice), $\frac{H}{H'}=\frac{n}{n'}=1.2$. Thus it

[1] It will be found in Appendix A.

can be seen how imperfections in the formula may cause n to change, and also that for a weir with a sharp edge n is greater than for a rounded weir

The following values for n seem suitable for weirs situated in the centre of the stream —

	Weirs with end contractions	Weirs without end contractions
Weir with sharp edge,	1 33	1 2
Rounded weir,	1 1	1 0

For other kinds of weirs the value can be estimated For a weir *not in the centre a reduction can be made* When the edges are sharp, and the margin insufficient for complete contraction, an additional allowance for this must be made by the rules of article 3

Section II — Weirs in Thin Walls

6 **Co efficients of Discharge** — The chief experiments on weirs in thin walls, except Bazin s, have been analysed by Smith, who has prepared tables of the values of c at which he arrives, and his results somewhat condensed are shown in tables XIV to XVI, but he notes that when H is less than 2 foot the figures are not reliable The first part of the following statement gives an abstract of Smith s results (except for very short weirs), disregarding decimals of 001 or 002 The figures marked * Smith considered *doubtful*, owing to the absence of observations for such cases For the others he gives the probable error as only 3 per cent It is of course known that end contractions reduce the discharge, and that their effect increases with H and decreases with l Smith in his analysis considers all the experiments (except Bazin s) mentioned in article 1—those with and those without end contractions and those having various degrees of contraction—together, and to a certain extent infers one set of values from the other

Bazin's extensive observations, already to some extent discussed in article 5, give results differing somewhat from Smith s They are shown in the lower part of *the following statement* Smith s co-efficients for any weir without end contractions attain a mini

mum as H increases and then increase, but Bazin's decrease as long as H increases Smith's co-efficients increase as l decreases, but Bazin's are constant. The discrepancies are not very large, and they occur chiefly for small heads, but they are important because of the different laws which they indicate, and because of

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Kind of Weir	Head	Length of Weir in Feet							Remarks
		1·64	4	3·3	5	6·6	10	19	
Full Contraction	10		65	65		65	655	655	Smith's Co efficients
	25		·62	625		625	635	63	
	50		605	61		615	615	615	
	1·0		59	595		605	61	61	
	1 4		58	59		60	60	61	
	1 7					59	60	605	
No End Contractions	10				66	66	66	655	Smith's Co-efficients
	·25		64*	64*	635	635	63	63	
	50		635*	635*	625	625	62	62	
	1·0		65*	64*	635	63	625	62	
	1 4			645*	64	635	63	62	
	1 7				645	64	63	625	
No End Contractions	10								Bazin's Co-efficients
	25	66		66		66			
	50	64		64		64			
	1 0	63		63		63			
	1 4	63		63					
	1 7	625							

the high standard of accuracy obtainable with weirs in thin walls The methods used for observing the head are described in chapter viii article 6 The measuring sections of Francis and Fteley and Stearns could not have come within the surface curve, but it seems possible that the erratic pressures (art 4) may have had some effect Bazin's measuring section was far enough away to avoid this, and yet not far enough for the surface fall to cause over estimation of H Bazin's arrangements for starting and stopping the flow, and for measuring the volume discharged, were not quite so good as those of the American observers, and his individual experiments show more fluctuation among themselves, but the

can be seen how imperfections in the formula may cause n to change, and also that for a weir with a sharp edge n is greater than for a rounded weir

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(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
		Length of Weir in Feet							
Kind of Weir	Head.	1·64	2	3·3	5	6·6	10	19	Remarks.
Full Contraction	·10		·65	·65		·65	·635	·635	Smith's Co-efficients
	·25		·62	·625		·625	·625	·63	
	·50		·605	·61		·615	·615	·615	
	1·0		·59	·595		·605	·61	·61	
	1·4		·58	·59		·60	·60	·61	
	1·7					·60	·60	·605	
No End Contractions	·10				·66	·66	·66	·655	Smith's Co-efficients
	·25		·64*	·64*	·635	·635	·63	·63	
	·50		·635*	·635*	·625	·625	·62	·62	
	1·0		·63*	·64*	·635	·63	·625	·62	
	1·4			·645*	·64	·635	·63	·62	
	1·7				·645	·64	·63	·625	
No End Contractions	·10								Bazin's Co-efficients
	·25	·66		·66		·66			
	·50	·64		·64		·64			
	1·0	·63		·63		·63			
	1·4	·63		·63					
	1·7	·625							

the high standard of accuracy obtainable with weirs in thin walls The methods used for observing the head are described in chapter VIII article 6 The measuring sections of Francis and Fteley and Stearns could not have come within the surface-curve, but it seems possible that the erratic pressures (art 4) may have had some effect Bazin's measuring section was far enough away to avoid this, and yet not far enough for the surface fall to cause over estimation of H Bazin's arrangements for starting and stopping the flow, and for measuring the volume discharged, were not quite so good as those of the American observers, and his individual experiments show more fluctuation among themselves, but the

number of his observations was far larger, and his figures when averaged are remarkably consistent

On the whole it seems that Bazin's results are probably more accurate than the others, and the use of his co efficients for weirs 1 5 to 7 feet long, and without end contractions, is recommended For longer weirs there is nothing to shake Smith's figures Decrease of c with increase of l may seem improbable, but it is quite possible It may, for instance, be due to the surface at the measuring section being higher at the sides than elsewhere The head is measured at the side, and this would make H seem to be greater than its real average value from side to side of the stream, especially for long weirs

The detailed values of Bazin's co efficients given in table xvi are, owing to Bazin's values of n not being accepted (art 5), slightly higher for the greater heads than the values arrived at by Bazin himself They accordingly differ less from Smith's figures Bazin calculated c, or rather m, for heads ranging from 16 to 1 97 feet, but his actual observations were within the range shown in table xvi Bazin also gives a complete table of the values of M, and from it table xviii giving values of C has been framed

It has been found that when there are no end contractions the sheet of water after passing the crest of a weir tends to expand laterally, except when H is less than 20 feet, and the side-walls have usually been prolonged downstream of the crest, openings for free access of air beneath the sheet being left If the sides are not so prolonged c will be increased about 25 per cent when $H=\frac{l}{10}$, and more or less as H is more or less It also appears that in such weirs moderate roughness of the sides of the channel has no appreciable effect on the discharge

For small weirs of triangular section in thin walls, with the apex a right angle, c has been found to be 617

7 **Laws of Variation of Co efficients** —The following laws, governing the variation of the co efficient for complete contraction, are apparent —

(1) When the length of the weir is 246 feet or more, c is a minimum when l is equal to H or thereabouts, that is, when the section of the stream is nearly square, and it increases as the section deviates from a square The deviations are all in the direction of increase in $\frac{l}{H}$

(2) When l is less than 2 46 feet deviation from the square section is due to decrease in $\frac{l}{H}$ The co efficient c does not increase with the deviation, and for the smallest weirs it decreases

(3) For sections of the same shape c is less as the section is greater

(4) The value of c is less or greater than for an orifice of the same size and shape according as the length of the weir is greater or less than 2 46 feet

Laws (1) and (3) are the same as for orifices and are due to the same causes As to law (4), having regard to the remarks made above (Art 5) concerning the defective form of the weir formula, it is clear that the two sets of co efficients could not be expected to agree The reasons for law (2) are not known It is not certain that it applies to any except very small weirs

8 **Flow when Air is excluded** —With four weirs in thin walls, of heights 2 46 feet, 1 64 feet, 1 15 feet, and 79 foot, further observations were made by Bazin, the access of air beneath the falling sheet being prevented by the closure of the openings which had been left for that purpose The following statement shows the results noticed The pressures under the sheets were observed, and the discharge was found to increase as the pressure decreased

An interesting point for consideration is the conditions under which the different forms are assumed This is stated by Bazin, and is shown in the above statement With weirs not exactly similar to those of Bazin, it may be difficult to say when the various changes will occur, but it will at least be possible to foresee them and to take some account of them when they do occur The occurrence of the form called 'drowned underneath' will obviously be affected by the condition of flow in the down stream reach One lesson to be learnt is, that if complications are to be avoided and discharges accurately inferred the free access of air under the sheet is essential

9 **Remarks** —

For a given weir in a thin wall c decreases as H increases the effect of the end contractions increasing with H It has been stated that if the sides are given a slope of $\frac{1}{4}$ to 1, c is constant for all heads the sloping sides having the effect of lengthening the weir as H increases This has however, been proved to be true only for some weirs whose length of crest did not exceed 1 foot In order that for a given weir c may be constant for all heads the side is more likely to be curved than straight, and it is unlikely that its general slope will be the same for weirs of different length

Reference to Fig	Name given to Case by Bazin	Description of Case	Conditions under which it occurs	Effect on the Co efficient of Discharge, C
Fig 72	Adherent sheet	Sheet in contact with weir and no air under it, or it may spring clear from the iron plate, enclose a small volume of air, and then adhere to the plank, or it may adhere to the top and bevelled edge and then spring clear, enclosing air as in the case following	Under small heads	C may possibly exceed that for a free sheet by 33 per cent
Fig 73	Depressed sheet	Air partly exhausted by the water and at less than atmo [illegible]	When case 1 does not occur, or when it occurs and H is [illegible]	C is higher than for free sheet, generally only slightly, but it may be 10 per cent higher when [illegible] point of as [illegible] 'drowned
	Sheet drowned underneath	Water under sheet rises to level of crest and all air is expelled	When H is further increased so that H is not less than about $4G$	
Fig 74		(a) If *are at a distance*	When the fall H_1+H_2 is greater than about $\frac{2}{3}G$	Value of $\frac{H_1}{l}$ / Value of $\frac{C'}{C}$: 05 — 1 22 40 — 1 19 50 — 1 13 60 — 1 09 80 — 1 04 1 00 — 1 005 1 20 — 98 1 40 — 96 1 60 — 95 C is the co efficient for a free sheet and C' for the case in question
Fig 75		(b) If *are covering foot of sheet*	When the fall H_1+H_2 is not greater than about $\frac{2}{3}G$ For a given head H_1 the greatest value of H_2 is $\frac{2}{3}G-H_1$	The level of the tail water affects the discharge, and approximately $\frac{C''}{C'}=\left(1{\cdot}05+15\frac{H_2}{H_1}\right)$ (46) See also article 13

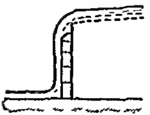

Fig. 72

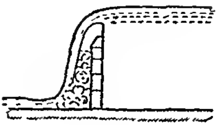

Fig. 73

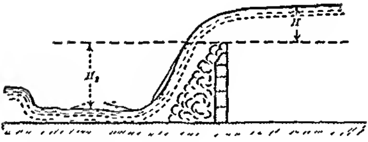

Fig. 74

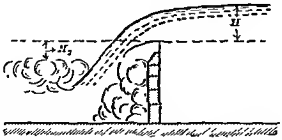

Fig. 75

Francis found that end contraction might be allowed for by considering the length of the weir to be reduced by $20H$, that is, by substituting $(l-2H)$ for l in equation 11, page 15 He found that with the formula thus modified, the co efficient, provided l is not less than $3H$ or $4H$, is nearly constant, its value being 620 to 624, and averaging 623 for heads ranging from 5 to 19 inches Results obtained by this formula are liable to differ by 1 or 2 per cent from those of the ordinary formula with Smith's co efficients Francis' formula is specially mentioned here because it is well known

Bazin, taking mean values of M and n, puts equation 45 (p 88) in the form

$$m = \frac{M}{1+55\left(\frac{H}{G+H}\right)^2} \qquad (47)$$

But he admits that simplicity is obtained at some expense of accuracy His value of n, as shown above (art 5), is not correct, and this formula should not be used as a general one For Bazin's own experiments it is fairly accurate, but there is no use for it since his co efficients c and C are tabulated Bazin also states that m may be taken as $405+\frac{003}{H}$ (H being in metres) and this agrees closely with his values of ~~n~~ m, but as these have now been slightly altered the equation does not agree with them

Section III — Other Weirs

10 **Weirs with flat top and vertical face and back.**—Generally the water at B (Fig 76) holds back that upstream of it, and the discharge is less than for a weir in a thin wall under the same head It is a sort of drowned weir, B being the tail water level At A there is eddying water When H is about 1 6W to 2W—W being the top width—the sheet springs clear from the top, and the case becomes a weir in a thin wall But if the sheet nearly

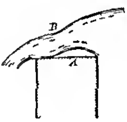

Fig 76

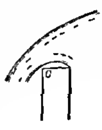

Fig 77

touches at C (Fig 77) the water gradually abstracts the air, and the sheet is pressed down, touches at C, and Q is slightly greater than for a weir in a thin wall Table xvii (prepared by Fteley and Stearns) shows the corrections to be applied to c, the co efficient for weirs in thin walls, in order to give c_w the co efficient for weirs with flat top and vertical face and back The corrections apply

$$\frac{C}{C'} = \cdot 70 + \cdot 185 \frac{H}{H'} \qquad (48)$$

The results given by this formula agree with the observed results generally within about 2 per cent., but for the widths of 0·56 feet, 2·62 feet, and 1·31 feet the error may be 3 or 4 per cent They also agree with Fteley and Stearn's results within 1 or 2 per cent When H was increased to about $2H'$ the sheet sprang clear, but if H was gradually lowered the sheet remained clear till H was about $1\cdot 6H'$ Between these limits it was unstable When the sheet springs clear the above formula of course is not needed The thick lines in the table mark off the cases when H was less than $2H'$ While H varies from $\frac{3H'}{2}$ to $2H'$, the ratio $\frac{C}{C_w}$ may change from ·98 to 1·07 if the sheet remains attached to the crest.

When air was excluded depressed and drowned sheets occurred under somewhat similar conditions to those with weirs in thin walls Remarks regarding them are given in table xix Their occurrence sometimes preceded and sometimes succeeded that of detachment of the sheet from the back or top of the weir, and rendered the conditions very complicated

11 **Weirs with sloping face or back.**—Bazin's chief results for weirs of this class are given in tables xxi and xxii, and the

[1] *Transactions of the American Society of Civil Engineers*, vol xliv

Cornell results are included Table xvi contains the cases where the back of the weir was steep, so that the sheet generally sprang clear of it Apparently no air openings were left, and the adherent depressed and drowned sheets often occurred Table xxii shows the cases where the back slopes gradually In these last the stream flowing down the back is in uniform flow in an open channel Weirs of this kind with back slopes about 10 to 1 are used on some large canals in India and termed 'Rapids,' the profile of the water surface being as sketched in Fig 68, page 82 The flow at the crest is virtually that of a drowned weir At the foot there is a standing wave (chap vii art 11)

In weirs of these classes there are several variable elements Pairs of cases in the tables can be compared in which only one element varied, so that its effect can be traced By studying these cases and the tables generally it will be seen that C generally increases as the height of the weir decreases, as the top width of the weir decreases (but not so much for the greater heads), as the upstream slope is flattened, and as the downstream slope is made steeper For a weir with slopes of face and back both 3 to 1 and sharp top, C has been found to be 51 and 43 for heads of 2 5 feet and 10 feet respectively

12 Miscellaneous Weirs —For a weir made of plank with a rounded crest of radius h the discharge with head H' is about the same as for a weir in a thin wall with a head H The following table is given by Smith[1] —

H	Values of h		
	25 in	·0 in	·4 in
	Values of $H' - H$		
116	006	004	003
166	014	013	015
217		021	018
284	011	029	028
351	015	028	·039
41	014	028	044
49	015	·030	052

The chief results of the Bazin and Cornell observations on rounded weirs are given in table xx

[1] *Hydraulics* chap v

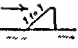

FIG. 78

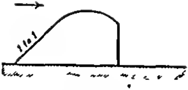

FIG. 78A

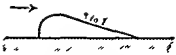

FIG. 78B

For a weir formed entirely by lateral contraction of the channel, and having a crest length of 2 feet to 6 feet (Fig 82, p 107), c is ·65 to ·73 and C is ·70 to ·78, being greater for the larger sizes

For a fall (Fig 79) in which there is neither a raised weir nor a lateral contraction there is no local reduction of the approaching stream due to eddies or walls, and therefore no local surface fall of the kind ordinarily occurring The surface curve is due to draw (chap II art. 11) If the slope AB is not very steep the curve extends for a great distance

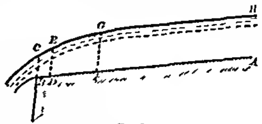

FIG. 79

If V is the velocity at DE near to BC, then V is both the velocity of approach and the velocity in the weir formula, so that

$$V^2 = \tfrac{4}{9}c^2 2g\left(H + n\frac{V^2}{2g}\right)$$

or
$$V^2 = \tfrac{4}{9}c^2 2gH + \tfrac{4}{9}nc^2 V^2$$
$$V^2(1 - \tfrac{4}{9}nc^2) = \tfrac{4}{9}c^2 2gH.$$

If $c = ·79$ and $c^2 = ·63$ and $n = 1·0$,

$$V^2 = \frac{·28}{1 - ·28}c^2 2gH$$
$$V = ·62c\sqrt{2gH}$$

If the channel AB be supposed to be very smooth or steep the water surface HG will be parallel to the bed, but there will always be a short length GC in which draw will occur Falls of this kind occur at the ends of wooden troughs and shoots They were used on one of the older of the great Indian canals, but the high velocity due to the draw caused such scour and damage that raised weirs had to be added

SECTION IV —SUBMERGED WEIRS

13 **Weirs in Thin Walls**—The following statement shows the chief experiments which have been made

Observer	Length of Weir	Upstream Head H_1		Downstream Head H_2		Height of Weir
		From	To	From	To	
	Feet.	Feet.	Feet.	Feet	Feet.	Feet
Francis,	11	1 0	2 3	·24	1 1	5 8
Fteley and Stearns,	5	·33	·81	·07	·80	3 2
Bazin,	6 56	·19	1 49	·79	1 26	·8 to 2 5

The weirs were all without end contractions The level of the tail water was measured at M (Fig 80), which is theoretically wrong, the surging of this water renders exact measurements difficult The co efficients for submerged weirs are not, in most cases, well known, and exact results cannot be expected from them

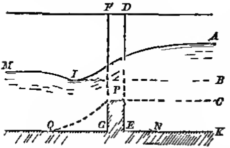

FIG 80

Let q_1 be the discharge through AB and q_2 through LC Then

' H + H₂ be heights of A + M above P
'' H A - - M

$$q_1 = \tfrac{2}{3} c_1 l \sqrt{2gH}\ \ H \qquad (49)$$

$$q = c_2 l \sqrt{2gH}\ \ H_2 \qquad (50)$$

If c has the same value for both portions,

$$q = \tfrac{2}{3} c l \sqrt{2gH}\left(H + \frac{3H_2}{2}\right) \qquad (51),$$

$$\left.\begin{aligned} &\text{or } q = cl\sqrt{2gH}\left(H_2 + \frac{2H}{3}\right) \\ &\text{or } q = cl\sqrt{2gH}\left(H_1 - \frac{H}{3}\right)\end{aligned}\right\} \qquad (52)$$

The last two formulæ are those for an orifice having a height equal to the downstream head plus two thirds of the fall If there is velocity of approach $H + nh$ must be put for H and $H_1 + nh$ for H_1, but H_2 is left unaltered

Francis makes $c = 921 c_1$, that is, he multiplies H_2 in equation 52 by 921 Smith, discussing the experiments of Francis and Fteley and Stearns, and reviewing a previous discussion by Herschel, substitutes 915 for 921 and recommends the formula—

$$Q \quad q = c_1 l \sqrt{2g(H + nh)}\left(915\ H_2 + \frac{2(H + nh)}{3}\right) \qquad (53)$$

This formula is for weirs in thin walls without end contractions c_1 is the co efficient taken from table xvi for the equivalent weir with a free fall (that is, the weir with a free fall giving the same discharge) and n is 1 33 The formula may be applied to weirs with end contractions and the same co efficients used if $l - 2H_1$ be substituted for l

If Q is the discharge for a free weir, and if H_1 remains constant while the tail water is raised by some cause operating in the

downstream reach, Q decreases very slowly till H_2 is about $\frac{H_1}{2}$ The discharge through AB is the same as before, while the velocity in FC is altered in the ratio $\sqrt{\frac{H+\frac{1}{2}H_2}{H}}$ The relative discharges are as follows, c being constant and velocity of approach being supposed to be negligible —

$\frac{H_2}{H_1}=$	00	25	33	50	66	75
or $\frac{H_2}{H}=$	·00	33	50	1 0	2 0	3 0,
$\frac{q}{Q}$ (equation 52) $=$	100	97 4	95 3	88	77	69
$\frac{q}{Q}$ (equation 53) $=$	100	94 5	93	84	71	61

Practically, this law is somewhat modified Let it be supposed that for the free weir there is ample access of air As the tail water rises above the crest the air is shut out The under side of the sheet springs up to a somewhat higher level than the crest, but the surging of the tail water shuts out the air almost at once The sheet of water is pressed down, and the discharge instead of decreasing increases a little Practically it remains nearly constant during a certain rise of the tail water and then decreases If the air passages become obstructed just before the tail water rises to the crest level, Q will begin to increase then, but this does not necessarily occur Neither equation 52 nor 53 takes account of the increase in discharge when the tail water rises above the crest If the air was shut out from the commencement, Q begins to decrease as soon as the tail water begins to rise See equation 46, page 94

Bazin uses the simple weir formula $q=C_d\sqrt{2gH_1}$ (where C_d is the inclusive co-efficient for the drowned weir and H_1 the upstream head) and finds the ratio $\frac{C_d}{C}$, C being the inclusive co-efficient for the 'standard weir,' 3 72 feet high with a free fall and with the same head H_1 His results are as follows —

$\frac{H_2}{G}$ or Ratio of Down stream Head to Height of Weir	$\frac{H}{G}$ or Ratio of Fall in Water to Height of Weir												
	05	10	15	20	·25	30	35	40	45	50	60	0	F*
	Ratio $\frac{c_d}{c}$												
0	1 05	1 05	1 05	1 05	1 05	1 05	1 05	1 05	1 05	1 05	1 05	1 05	1 06
05	84	93	96	98	1 00	1 01	1 01	1 02	1 02	1 03	1 03	1 04	1 05
10	74	85	90	94	96	97	98	99	1 00	1 01	1 02	1 02	1 04
15	68	80	86	90	92	94	96	97	98	99	1 00	1 01	1 03
20	64	76	82	87	90	92	94	95	96	98	99	1 00	1 02
30	58	70	77	82	86	88	90	92	94	95	98	99	1 00
40	54	66	74	79	82	85	88	90	92	93	96	98	99
60	50	61	69	74	78	81	84	87	89	90	93	96	97
80	47	58	66	71	75	79	82	84	87	89	92	94	95
1 00	45	57	64	69	74	77	80	83	85	87	91	94	94
1 20	44	66	63	68	72	76	79	82	84	87	90	93	93
1 50	43	54	61	67	71	75	78	81	84	86	89	92	92

* This column shows $\frac{C_d}{C}$ when the tail water is below the crest and the standing wave is at a distance (art 8).

Actually the ratio $\frac{C_d}{C}$ $\frac{C_a}{c}$ is somewhat different with the weirs of different heights for the same values of $\frac{H_2}{G}$ and $\frac{H}{G}$, but the error in the figure given is usually only 1 or 2 per cent, except for very small values of $\frac{H_1}{G}$ and $\frac{H}{G}$, and in these cases the ratio is always uncertain The values 1 05 in the first line of the table agree with the figure obtained by equation 46 (p 94), when $H_2=0$ If, for any given weir, G is supposed to be 1 0, the above figures show $\frac{q}{Q}$ for various values of H and H_2 In this case, for a given value of $\frac{H}{H_2}$, the figures are high when H is high This is due to velocity of approach, the standard weir having been high

Bazin's figures may be compared with those given on page 101 Take for instance the cases where $H_2=2H$

$$\left.\begin{array}{llllll} \frac{H}{G}= & 70 & 50 & 30 & 10 & 05 \\ \frac{H_2}{G}= & 1\ 40 & 1\ 00 & 60 & 20 & 10 \\ \frac{C_d}{C}= & 93 & 87 & 81 & 76 & 74 \end{array}\right\}$$

The figures on p 101 are 77 and 71

Again for the case where $H_1 = \frac{H}{3}$

$\frac{H}{G} =$	70	45	15	The figures on p 101 are 97 4 and 94 5
$\frac{H_1}{G} =$	23	15	05	
$\frac{C_e}{C} =$	1 00	98	96	

In the above case, where $\frac{H}{G} = 70$, $\frac{H_1}{G} = 2$ 10 and $\frac{a}{A}$ or $\frac{H_1}{G+H_1} = \frac{2\ 1}{3\ 1}$ The excessive velocity of approach accounts for the high value of $\frac{C_e}{C}$

Bazin found that when H is reduced to about 16G or ·21G, the sheet, instead of plunging beneath the surface (Fig 76), suddenly assumes the form shown in Fig 80 (which he terms the 'undulating' form, there being generally waves near M) but this does not affect the co efficient If H is now gradually increased, the undulating form remains till H is about ·28G or ·27G, but is unstable or liable at any moment to revert to the other form 29 G

14 Other Weirs —The results of Bazin s observations on weirs of other kinds are shown in the following table Instead of giving the co-efficient ratios Bazin gives the equivalent heads The conditions of flow are complicated in such cases, and formulæ can probably apply only with the co-efficient varying to a great extent The height H_2, to which the tail water can rise before it begins to affect the discharge, varies greatly for different weirs For a weir in a thin wall it is very small, and it is largest for weirs with flat tops For the weir No 5 in the table H_2 was $\frac{5}{6}H_1$ For weirs with a sharp top it was *minus*, *zero*, and plus for downstream slopes of 1 to 1, 3 to 1, and 5 to 1 respectively, the flat downstream slope in the last case having the same effect as a large top width For weirs with flat tops 66 foot wide, back slopes varying from 2 to 1 to 5 to 1, H_2 is nearly $\frac{H_1}{2}$, but when the top was 1 32 feet wide H_2 was $\frac{2H_1}{3}$

Reference Number	Dimensions of the Weirs: Down stream Slope	Top width in metres	Up stream Slope	Height in metres	Downstream Head H on the Weirs	Discharges per foot run of Standard Weir in Thin Wall cubic metres per second: 061 / Heads H on Standard Weir metres: 10	110 / 15	169 / 20	310 / 30	480 / 40	Remarks
						Corresponding Heads H_1 on the other Weirs metres					
					Weirs with sloping face or back						
1	1 to 1	0 0	Vertical	75	E*		14	18	27	36	$H_1 < H$ for the greater discharges and when H_2 is small
					– 06		14	19	27	36	
					06		16	20	29	38	
					12		18	22	31	40	
					24				35	43	
2	1 to 1	2	½ to 1	75	E*		16	21	29	37	
					12		17	21	29		
					24			27	32	39	
3	5 to 1	0 0	Vertical	75	E*		16	21	31	42	$H_1 > H$
					12		17	21			
					24			27	33		
					36				40	45	
4	5 to 1	2	½ to 1	75	E*		17	22	31	41	
					12		17	22			
					24				33	41	
					36					42	
					Weirs with flat top and vertical face and back						
5		2 0		75	12	14	18				
					24			27	35		
					36				40		
6		2		75	E*	12	17	21	29	38	For small discharges $H_1 > H$ For greater discharges $H_1 < H$ when H_2 is small and $H_1 > H$ when H_2 is larger
					12	14	18	22	31	39	
					24			28	34	41	
					36				41		
7		2		35	E*	12	17	21	29	37	
					12	13	17	22	30	39	
					24						
					36				41		
8		1		75	E*	11	15	19	27	35	
					12	14	18	22	31	40	
					24				35	43	
9		1		35	E*	11	15	19	27	37	
					12	14	17	21	28		
					24			33	36	40	
					36					44	

* Tail water below crest and wave at a distance

Hughes, adopting equation 51 with $n=1$, has worked out[1] the values of c for weirs Nos 5 and 6 on the above list, and the results condensed are as follows —

Discharge in cubic metres per second	Weir No. 5			Weir No 6		
	H_1 metres.	H_2 metres.	c	H_1 metres	H_2 metres	c
·061	122	·031	50	119	·000	50
	122	091	70	123	·090	67
	161	150	87	135	120	85
				163	150	74
169	·236	151	61	216	·000	56
	·247	·211	81	·219	·060	56
	·293	·271	84	·220	120	63
				·233	180	74
				277	240	72
310	353	·242	63	301	000	61
	360	303	80	307	120	67
	396	361	88	319	210	71
				413	360	72
392	409	300	68			
	418	360	83			
	439	389	84			
450				382	000	65
				384	060	63
				406	·240	70
				442	300	68

It will be seen that c increases rapidly with H_2, and apparently attains a maximum and then decreases

The effect of a submerged weir varies greatly according to the state of the discharge With low water it may act as a free weir, and have great effect, for however small the discharge may be, the upstream water surface must be higher than the top of the weir With larger discharges the heading up is less, and with a great depth of water the weir may be almost imperceptible

15 Contracted Channels—These are (chap ii arts 6 and 19) analogous to submerged weirs The co efficients are very roughly known When an open stream issues from a reservoir, or from a

[1] Madras Government Paper on Bazin s New Experiments on Flow over Weirs

larger channel, or passes between contracted banks, or bridge abutments, or piers, c may have any value from 50 to 95, being smallest when the angles of the apertures are sharp and square (especially if there is a decrease in section both vertically and laterally), greater if the angles are chamfered or curved, and greatest when there are bell-mouths The co efficients are also greater for large than for small openings [1]

When a bridge or other obstruction in a stream has a waterway less than that of the stream the real obstruction is generally much less than it seems to be It is to be measured, not by the difference between the waterway at the obstruction and that upstream of it, but by the difference in the upstream and down stream water levels This is very often inconsiderable A fall of 1 foot gives a theoretical velocity of 8 feet per second, and 25 foot gives 4 feet per second Bridges are sometimes un necessarily altered or rebuilt owing to 'obstruction' which is nearly harmless Heading up is most likely to be considerable with high discharges, because the mean width of the channel is then increased, while perhaps that of the contracted place is not Thus the effect varies in just the opposite manner to that of a submerged weir.

The real objection to a contraction is very often the expansion which succeeds it and the eddies and scour which occur (chap ii arts 17 and 23, and chap vii art 1)

SECTION V—SPECIAL CASES

16 **Weirs with Sloping or Stepped Side-walls**—For a weir of triangular section the formula is obtained by putting $H_1=0$ and $l_1=l$ in equation 36 (p 71) Thus—

$$Q=\tfrac{4}{15}c\sqrt{2gl}H^{\frac{3}{2}} \qquad (54)$$

Since l increases as H, in any triangular weir in which c does not vary greatly, Q is nearly as $H^{\frac{5}{2}}$, that is, it varies much more rapidly than with an ordinary weir If two weirs, one triangular and one rectangular, are so designed (Fig 81) as to hold up the water of a stream to a given level with ordinary supplies, the triangular weir will allow floods to pass with a smaller head This applies to any weir with sloping sides The triangular form

[1] The co efficients for narrow openings are, roughly, for square piers, 6; obtuse angled, 7, curved and acute, 8 For wider openings add 1

is suitable for small drains. By making the sides of a weir at any given level DF (Fig. 81) horizontal, and extending them outwards, the rise of the water above DF can be limited.

FIG. 81

The formula for the discharge of a trapezoidal weir (Fig. 82) is

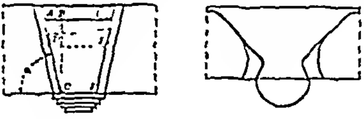

FIG. 82

obtained by putting $H_1=0$ in equation 35 (p. 71). Thus—

$$Q=\frac{2}{3}c\sqrt{2g}H^{\frac{3}{2}}\{l_b+\tfrac{2}{5}(l_t-l_b)\} \quad . \quad . \qquad (55)$$

The quantity in the outer brackets is the crest length of the equivalent ordinary weir. This length is less than $\frac{l_t+l_b}{2}$ because the velocity of the water at the bottom of the section is greater than at the top. If there is velocity of approach $(H+nh)$ must be put for H in equation 55, or else C' put for c. If r is the ratio of the side slopes, that is, the ratio of AB to BC, then $\frac{AB}{BC}=r=\cot\alpha$, $AB=rH=H\cot\alpha$, and $l_t-l_b=2rH=2H\cot\alpha$. Thus equation 55 may be written—

$$Q=\frac{2}{3}C\sqrt{2g}H^{\frac{3}{2}}(l_b+\tfrac{4}{5}rH) \qquad (56)$$

17. Canal Notches.—A common problem on irrigation canals is to design a weir so that the water levels, CD, EF, etc. (Fig. 83), upstream of it, corresponding to different discharges in the channel of approach, shall be the same as they would have been if the weir had not existed and the channel had continued uniform and uninterrupted. If the cross section of the channel of approach is trapezoidal, the form of the aperture will be approximately

trapezoidal, and its crest will be at the bed level of the canal Such a weir is termed a notch It is usually, for convenience in construction, built exactly trapezoidal and of the form shown in Fig 82, the lip being added to cause the falling water to spread out and exert less effect on the downstream floor In a large channel two or more notches are built side by side instead of one very large notch The co efficients, so far as known, are given in art 12 If C is the same for all heads the true theoretical form of the notch is curved, the angles at C, F (Fig 82) being rounded The slope of the sides is great for small depths because the co efficient for flow in channels increases rapidly for small depths, but if C increases fast with the head at small depths, as is highly probable, judging from other weir co efficients, the form is more nearly a trapezoid To design the notch, find Q and q, the discharges (or the fractions of the discharges if there are to be several openings) of the channel for two depths D and d Then from equation 56

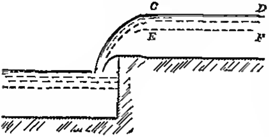

Fig 82

$$l_b + 8rd = \frac{q}{\frac{2}{3}C_1\sqrt{2g}\,d^{\frac{3}{2}}} \qquad (57)$$

$$l_b + 8rD = \frac{Q}{\frac{2}{3}C_2\sqrt{2g}\,D^{\frac{3}{2}}} \qquad (58)$$

Therefore $$8r(D-d) = \frac{Q}{\frac{2}{3}C_2\sqrt{2g}\,D^{\frac{3}{2}}} - \frac{q}{\frac{2}{3}C_1\sqrt{2g}\,d^{\frac{3}{2}}}$$

Or $$r = \frac{\frac{2}{3}\sqrt{2g}(C_1Qd^{\frac{3}{2}} - C_2qD^{\frac{3}{2}})}{8\times\frac{4}{9}\times 2g(D-d)C_1C_2d^{\frac{3}{2}}D^{\frac{3}{2}}}$$

$$= \frac{C_1Qd^{\frac{3}{2}} - C_2qD^{\frac{3}{2}}}{42.8(D-d)C_1C_2d^{\frac{3}{2}}D^{\frac{3}{2}}} \qquad (59)$$

The depths d and D can be so selected as to make the notch specially accurate for any given range of depth In irrigation canals (and still more in their distributaries) there is a certain minimum depth, d_I below which the channel is not run In such a case it does not matter if the notch is inaccurate for depths less than d_I To make its accuracy a maximum for depths between d_I and any greater depth, D_1, the range of depth should be divided

into four parts and the depths d and D taken at the quarter points Thus if

$$D_1 - d_1 = l$$

$$d = d_1 + \frac{l}{4}$$

$$D = d_1 + \frac{3l}{4}$$

If general accuracy is required over a range of depth from zero to D_1, then $d = \frac{D_1}{4}$ and $D = \frac{3D_1}{4}$ The formulæ are, however, most simple when $D = 2l$ In this case equation 59 becomes—

$$r = \frac{C_2 Q d^{\frac{3}{2}} - 2.828 C_1 q d^{\frac{3}{2}}}{4.28 C_1 C_2 d^{\frac{3}{2}} \times 2.828 d^{\frac{3}{2}}}$$

$$= \frac{C_2 Q - 2.828 C_1 q}{12.10 C_1 C_2 d^{\frac{3}{2}}} \quad . \quad (60)$$

Substituting this value of r in 57

$$l_1 = \frac{q}{\frac{2}{3} C_1 \sqrt{2g} d^{\frac{3}{2}}} - \frac{8(C_2 Q - 2.828 C_1 q)}{12.10 C_1 C_2 d^{\frac{3}{2}}}$$

$$= \frac{2.262 C_2 q - 8 C_1 Q + 2.262 C_1 q}{12.10 C_1 C_2 d^{\frac{3}{2}}}$$

$$= \frac{2.262 C_2 q - 4 C_1 Q}{6.05 C_1 C_2 d^{\frac{3}{2}}} \quad . \quad (61)$$

If C_1 and C_2 are each assumed to be equal to C,

$$r = \frac{Q - 2.828 q}{12.10 C d^{\frac{3}{2}}} \qquad (62)$$

And

$$l_1 = \frac{2.262 q - 4Q}{6.05 C d^{\frac{3}{2}}} \qquad (63)$$

If it is desired to build a notch to the true form, that is not strictly trapezoidal, the lower part corresponding to a small depth in the channel may first be designed trapezoidal and the upper parts designed in instalments, working upwards

In deciding in which direction a notch is to deviate from the true form, and for what water levels accuracy is to be aimed at, regard must be had to the special circumstances of the case If scour of the canal bed is feared or if there is difficulty, with low supplies, in getting enough water into the distributaries, the notch can be designed narrow

If a notch is drowned its true form is modified In Fig 82 let

DE be the upstream water level when the tail water is just level with the crest CF. The portion $CDEF$ of the notch obviously need not be altered. As the tail water rises above CF the discharge through the notch becomes gradually less than it would be for a free notch with the same upstream water level, and the upper part of the notch must be widened as shown by the dotted lines. In this case also a trapezoid can be drawn so as to closely agree with the true form. As before, the trapezoid can be designed so as to give nearly exact discharges for any particular range of depths, or the notch can be designed to the true form as above explained. The formulæ for a drowned notch are as follows. For an upstream depth d let q_1 be the discharge through $ADEG$ and q_2 through $DCFE$.

$$\begin{aligned} q &= q_1 + q_2 \\ &= \tfrac{2}{3}C_1\sqrt{2g}(d-h)^{\frac{3}{2}}[l_b + 2rh + \tfrac{8}{15}r(d-h)] \\ &\quad + C_2\sqrt{2g(d-h)}(l_b + rh)h \qquad (64) \end{aligned}$$

For a greater discharge let D and H be the heights of AG and DE above CF. Then

$$\begin{aligned} Q &= \tfrac{2}{3}C_1\sqrt{2g}(D-H)^{\frac{3}{2}}[l_b + 2rH + \tfrac{8}{15}r(D-H)] \\ &\quad + C_2\sqrt{2g(D-H)}(l_b + rH)H \qquad (65) \end{aligned}$$

If the upstream and downstream channels are similar in all respects $d-h=D-H$ and $D-d=H-h$. Let $D=2d$. Then $d=D-d=H-h$ and $H=d+h$. Therefore

$$\begin{aligned} Q &= \tfrac{2}{3}C_1\sqrt{2g}(d-h)^{\frac{3}{2}}[l_b + 2rH + \tfrac{8}{15}r(d-h)] \\ &\quad + C_2\sqrt{2g(d-h)}(l_b + rH)H \qquad (66) \end{aligned}$$

Subtracting 64 from 66 and putting $C_1=C_2=C$ and $C'_1=C'_2=C'$,

$$\begin{aligned} Q-q &= \tfrac{2}{3}C\sqrt{2g}(d-h)^{\frac{3}{2}}[2rd] \\ &\quad + C'\sqrt{2g(d-h)}[l_b(H-h) + r(H^2-h^2)] \qquad (67) \end{aligned}$$

from which r can be found, and l_b can then be found from 65, Q and Q_1 being selected at such depths as to make the trapezoid most accurate at the points desired. If D is not taken as $2d$, or if C_1 and C_2 differ, the equation will be complicated, and it may be easiest to adopt the instalment process and design the notch to the true curve, afterwards straightening it if necessary.

18. **Oblique and Inclined Weirs.**—If a weir is built obliquely across a stream the discharge is that due to the full length of the weir, provided the section of the stream passing over the weir is small compared to that of the stream at the approach section. In this case the water approaches the weir nearly at right angles

But when the stream is in flood, or when, under any circumstances, the two sections become more equalised, the water passing over the weir travels more nearly parallel to the axis of the stream, and the discharge over the weir tends to become equal to that over a weir found by projecting the oblique weir on a plane perpendicular to the axis of the stream By making a weir extend from bank to bank of a stream with its alignment very oblique, so that its length projected on the axis of the stream is considerable, it can be made to offer less obstruction to floods, but its length is also increased so that it does not hold up low supplies so well The problem of constructing a weir so that it will hold up low supplies and yet not form a serious obstruction to floods cannot usually be solved by means of oblique weirs The only solution is to have a 'movable weir,' that is, gates or shutters which can be placed across the stream at times of low supply and removed or placed parallel to the stream in floods or high supplies

When the plane of a weir in a thin wall, instead of being vertical, is inclined, the co efficients can be obtained by multiplying that for a vertical weir by a co efficient of correction c_k, whose value was found by Bazin to be as follows —

Inclination of plane of weir—

	Upstream				Downstream				
	1 to 1,	⅔ to 1,	⅓ to 1,	vertical,	⅓ to 1,	⅔ to 1,	1 to 1	2 to 1,	4 to 1
Average value of c_k—	93	94	96	1 0	1·04	1 07	1 10	1 12	1 09

The heights of the weirs when vertical were 3 72 feet, 1 64 feet, and 1 15 feet The co efficient is a maximum when the weir is inclined downstream at 2 to 1, that is, when the height of the crest above the bed is half the distance of the crest downstream from the base of the weir The weirs were without end contractions, and the head ranged in each case from about 33 feet to 1 48 feet

Examples

Example 1 —A weir in a thin wall is 25 feet long and 3 feet high, and H is 1 foot The channel of approach is 30 feet wide Find Q

The crest contraction is complete, and the end contraction so nearly complete that no allowance need be made for it From table xiv c is probably 612 From table xii $q_r = 3\ 275$ Then $Q = 25 \times 3\ 275 = 81\ 88$ cubic feet per second

To allow for v by the usual method, $A = 30 \times 4 = 120$ square feet.

Let Q be assumed to be, say, 84 cubic feet per second Then $i=\frac{84}{1\cdot 2}=70$ From table i $h=0076$ Let $n=1\ 3$ Then $nh=0101$, $H+nh=1\ 010$ The corresponding correction in $(H+nh)^{\frac{3}{2}}$ and in Q is 1 5 per cent, and Q is thus 83 14 cubic feet per second

To allow for i by table xiii $\frac{A}{a}=\frac{30\times 4}{25\times 1}=4\ 8$ When c is 60 c_a is about 1 015 When c is 61 c_a is about 1 016 This makes $Q=83\ 19$ cubic feet per second

Example 2 —A river 50 feet wide has a maximum discharge of 600 cubic feet per second, the depth being then 3 feet A weir with a rounded crest ($c=80$) is to be built in the river so as to raise the flood level by 1 foot What must be the height of the crest above the bed?

The discharge, q, per foot run of weir is 12 cubic feet per second, and table xii for $c=80$ gives $q_r=4\ 28$ Therefore

$(H+nh)^{\frac{3}{2}}=\frac{12}{4\ 28}=2\ 80$ From table xi $H+nh=1\ 99$ feet But $i=3\ 0$, and h (table i)$=14$ foot Therefore, n being 1 0, H is 1 85 feet, and the crest must be 2 15 feet above the bed The result is quite accurate, supposing that the channel downstream of the weir is altered for a long distance so as to give a free fall over the weir Otherwise the weir will be drowned, H_2 being 85 foot, but judging from Bazin's results (art 14) with weirs having a moderate top width and sloping back and face, the discharge will hardly be affected, H_2 being only $46H_1$ Actually H would perhaps be 1 9 or 1 95 feet

Example 3 —A river whose mean width is 50 feet, depth 10 feet, and mean velocity 3 feet per second, has a bridge built across it The piers and abutments are square, and the total width of the water-way in the bridge is 30 feet Find the heading up caused by the bridge

Let c be 60 Since Q is 1500 cubic feet per second, and a is 300, therefore $V=\frac{1500}{300\times 60}=8\ 33$ feet per second From table i $H=1\ 08$ feet nearly, but as there is high velocity of approach H will be less, say 1 0 foot Therefore $A=50\times 11\ 0=550$ square feet, and $i=\frac{1500}{550}=2\ 73$ feet per second From table i $h=115$ Let $n=1\ 0$ Then $H+nh=1\ 115$ From table i $V=8\ 47$ feet per second, which is too great by nearly 2 per cent, and H is therefore less than 1 foot by 4 per cent, that is, it is 96 foot

Example 4 —The depth of full supply in a canal is 5 feet The discharges with depths of 4 feet and 2 feet are 153 cubic feet and 46 cubic feet per second respectively Design a trapezoidal notch for a free fall in the canal The co-efficient is 66

From equation 62, page 109,

$$r=\frac{153-2\ 828\times 46}{12\ 10\times\ 66\times 2^{\frac{5}{2}}}=51$$

From equation 63, page 109,

$$l_s=\frac{2\ 262\times 4\ 6-4\times 153}{6\ 05\times\ 66\times 2^{\frac{3}{2}}}=3\ 78 \text{ feet}$$

Example 5 —A weir in a thin wall is 4 feet high and H is 1 foot The bed of the stream becomes filled up, so that the depth above the weir becomes 2 5 feet instead of 5 feet, but Q is unaltered How is H affected ?

The ratios $\frac{A}{a}$ are 5 and 2 5 nearly From table xiii, c being 60 and n being 1 33, the values of c_a are 1 013 and 1 057, so that Q is increased about 4 4 per cent if H is the same H will therefore be less than before by $\frac{2}{3}\times 4$ 4 per cent, that is, it will be 97 feet

Table XI

Values of H and $H^{\frac{3}{2}}$. (Art 1)

H	$H^{\frac{3}{2}}$	Diff 01 H	H	$H^{\frac{3}{2}}$	Diff 01 H	H	$H^{\frac{3}{2}}$	Diff 01 H
04	0080	0032	60	4648	0119	1 8	2 415	0202
05	0112	0035	62	4882	0121	1 85	2 516	0205
06	0147	0038	64	5120	0123	1 90	2 619	0208
07	0185	0041	66	5362	0125	1 95	2 723	0210
08	0226	0044	68	5607	0127	2	2 828	0214
09	0270	0046	70	5857	0129	2 05	2 935	0216
10	0316	0049	72	6109	·0130	2 1	3·043	0218
11	0365	0051	74	6366	0131	2 15	3 152	0221
12	0416	0053	76	6626	0132	2 2	3 263	0224
13	0469	0055	78	6889	0133	2 25	3 375	0226
14	0524	0057	80	7155	0135	2 3	3 488	0228
15	0581	0059	82	7426	0137	2 35	3 602	0231
16	0640	0061	84	7699	0138	2 4	3 718	0234
17	0701	0063	86	7975	0140	2 45	3 834	0237
18	0764	0064	88	8255	0142	2 5	3 053	0238
10	0828	0066	90	8538	0143	2 55	4 072	0240
20	0894	0068	92	8824	0145	2 6	4 192	0242
22	1032	0072	94	9114	0146	2 65	4 314	0244
24	1176	0075	96	·9406	0148	2 7	4 437	0246
26	1326	0078	98	9702	0149	2 75	4 560	0250
28	1482	0081	1 0	1 000	0152	2 8	4 685	0252
30	1643	0084	1 05	1 076	0156	2 85	4 811	0254
32	1810	0087	1 10	1 154	0158	2 90	4 939	0255
34	1983	0089	1 15	1 233	0163	2 95	5 066	0260
36	2160	0091	1 2	1 315	0166	3 0	6 196	0262
38	2342	0094	1 25	1 398	0168	3 1	5 455	0266
40	2530	0096	1 3	1 482	0172	3 2	5 724	0271
42	2722	0099	1 35	1 568	0176	3 3	5 995	0275
44	2919	0101	1 4	1 657	0178	3 4	6 269	0279
46	3120	0103	1 45	1 746	0182	3 5	6 548	0283
48	3326	0106	1 5	1 837	0186	3 6	6 831	0287
50	3536	·0109	1 55	1 930	0188	3 7	7 117	0291
52	3750	0112	1 6	2 024	0190	3 8	7 408	0294
54	3968	0113	1 65	2 119	0191	3 9	7 702	0298
56	4191	0116	1 7	2·217	0197	4 0	8 000	0302
58	4417	0117	1 75	2 315	0200			

TABLE XII — VALUES OF q_r OR $\frac{2}{3}c\sqrt{2g}$ OR 5 35c. (Art 1)

c	q_r	c	q_r	c	q_r
001	00535	61	3·264	81	4 334
002	0107	62	3 317	82	4 387
·003	·01605	63	3 371	83	4 441
·004	·0214	64	3 424	84	4 494
·005	0268	65	3 478	85	4 548
·006	0321	66	3 531	86	4·601
007	0375	67	3 581	87	4 655
008	0428	68	3 638	88	4 708
·009	·0482	69	3 692	89	4 762
5	2 675	7	3 745	9	4 815
51	2 729	71	3 799	91	4 869
52	2 782	72	3 852	92	4 922
53	2 836	73	3 906	93	4·976
54	2 889	74	3 959	94	5·029
55	2 943	75	4 013	95	5 083
56	2 996	76	4 066	96	5 136
57	3 050	77	4 120	97	5 190
58	3 103	78	4 173	98	5 243
59	3 157	79	4 227	99	5 297
6	3 21	8	4 28	1	5 35

TABLE XIII — CO-EFFICIENTS OF CORRECTION, c_a, FOR VELOCITY OF APPROACH (Art 5)

$\frac{A}{a}$	$c = 60$						[illegible]		
	Values of n.						Values of n		
	1 4	1 33	1				1 4	1 33	1
2	1078	1093	1067				1198	1159	1129
2·2	1079	1075	1055				1176	1149	1105
2 5	1060	1057	1042				1115	1110	1079
3	1041	1039	1028				1074	1071	1050
4	1022	1021	1015				1011	1039	1028
5	1014	1013	1009				[illegible]	1024	1017
7	1007	1007	1005				1012	1011	1008
10	1003	1003	1001				1006	1006	1004

TABLES XIV AND XV—COEFFICIENTS OF DISCHARGE, c, FOR WEIRS IN THIN WALLS WITH COMPLETE CONTRACTION (Art 6)

XIV—Ordinary Weirs

Head in Feet	Length of Weir in Feet						
	66	1 (*)	2	3	5	10	19
1	632	639	646	652	653	655	656
15	619	625	634	638	640	641	642
2	611	618	626	630	631	633	634
25	605	612	621	624	626	628	629
3	601	608	616	619	621	624	625
4	595	601	609	613	615	618	620
5	590	596	605	608	611	615	617
6	587	593	601	605	608	613	615
7	585	590	598	603	606	612	614
8			595	600	604	611	613
9			592	598	603	609	612
1			590	595	601	608	611
1 2			585	591	597	605	610
1 4			580	587	594	602	609
1 6				582	591	600	607
1 7						599	607
2			585(*)				

XV—Short Weirs

Head in Feet	Length of Weir in Feet						
	·033	·066	·099	164	246	3 9	654
03				634			[illegible]
05			620	618			
10				60	608	618	6[illegible]4
13				613	60	60[illegible]	618
16			629	614	604	98	611
25		6 3	628	612	602		[illegible]94
33		648	627	612			91
39	[illegible]79	64[illegible]	627	612		[illegible]89	590
60	608	640	[illegible]28	614		593	91
80	·606	642	[illegible]28	615		594	

TABLE XVI.—CO-EFFICIENTS OF DISCHARGE, c, FOR WEIRS IN THIN WALLS WITHOUT END CONTRACTIONS, BUT WITH FULL CREST CONTRACTION (Art. 6.)

Head in Feet.	Length of Weir in Feet.								
	1·6 to 6·6	2(?)	3(?)	4	5	7	10	15	19
	Bazin's Co-efficients	Smith's Co-efficients							
·1				·	·639	·638	·638	·637	·637
·15		·652	·649	·647	·645	·645	·644	·644	·643
·2	·662	·645	·642	·641	·638	·637	·637	·636	·635
·25	·655	·641	·638	·636	·634	·633	·632	·631	·630
·3	·652	·639	·636	·633	·631	·629	·628	·627	·626
·4	·646	·636	·633	·630	·628	·625	·623	·622	·621
·5	·640	·637	·633	·630	·627	·624	·621	·620	·619
·6	·637	·638	·634	·630	·627	·623	·620	·619	·618
·7	·635	·640	·635	·631	·628	·624	·620	·619	·618
·8	·633	·643	·637	·633	·629	·625	·621	·620	·618
·9	·633	·645	·639	·635	·631	·627	·622	·620	·619
1	·632	·648	·641	·637	·633	·628	·624	·621	·619
1·2	·631	·	·646	·641	·636	·632	·626	·623	·620
1·4	·630			·644	·640	·634	·629	·625	·622
1·6	·627			·647	·642	·637	·631	·626	·623
1·7	·626					·638	·632	·626	·623
1·8	·625	·							

TABLE XVII.—CORRECTIONS FOR WIDE CRESTS. (Art. 10.)
(The correction is always minus except when marked plus.)

Head in Feet.	Width of Crest in Inches.								
	1	2	3	4	6	8	10	12	[illegible]
·10	·007	·016	·018	·018	·017	·017	·017	[illegible]	[illegible]
·15	+·002	·017	·023	·024	·025	[illegible]	·025	[illegible]	[illegible]
·20		·012	·024	·029	·031	·032	·033	·033	·034
·30		+·005	·017	·031	·041	[illegible]	[illegible]	·045	[illegible]
·40			·010	·022	·045	[illegible]	[illegible]	[illegible]	[illegible]
·45			+·009						
·50				[illegible]	·041	[illegible]	·069	[illegible]	[illegible]
·60					·031	[illegible]	[illegible]	·083	[illegible]
·70					·015	[illegible]	[illegible]	[illegible]	112
·80					[illegible]	[illegible]	·071	[illegible]	[illegible]
·90					+·019	[illegible]	[illegible]	[illegible]	[illegible]
1·0						[illegible]	[illegible]	[illegible]	[illegible]
1·2						+[illegible]	·021	[illegible]	[illegible]
1·4							·013	[illegible]	[illegible]
1·5								[illegible]	

TABLES XIV AND XV.—COEFFICIENTS OF DISCHARGE, c, FOR WEIRS IN THIN WALLS WITH COMPLETE CONTRACTION (Art 6)

XIV.—Ordinary Weirs

Head in Feet	Length of Weir in Feet						
	66	1()	[illegible]	3	5	10	19
1	[illegible]	639	646	652	653	655	656
15	619	[illegible]	634	638	640	641	642
2	611	618	626	630	631	633	634
25	[illegible]	012	621	624	[illegible]	[illegible]	[illegible]
3	601	608	616	619	621	624	[illegible]
4	595	001	609	613	615	618	[illegible]
5	590	596	[illegible]	608	611	615	617
6	587	593	601	605	608	613	615
7	585	590	598	603	606	612	614
8			595	600	604	611	613
9			592	598	603	609	612
1			590	595	601	608	611
1 2			585	591	597	605	610
1 4			580	587	594	602	609
1 0				582	591	600	607
1 7						599	607
2			585(*)				

XV.—Short Weirs

Head in Feet	Length of Weir in Feet						
	·033	·066	[illegible]	·164	[illegible]	[illegible]	·654
03					634		
05			620		618		
10				60	608	618	[illegible]
13				613	60	[illegible]	618
16			629	[illegible]	604	598	611
25		6 3	628	612	[illegible]		94
33		·648	[illegible]	612			91
39	[illegible]	64	[illegible]	[illegible]		[illegible]	[illegible]
60	[illegible]	·640	[illegible]	[illegible]		593	91
80	[illegible]	642	[illegible]	[illegible]		[illegible]	

TABLE XVI.—CO-EFFICIENTS OF DISCHARGE, c, FOR WEIRS IN THIN WALLS WITHOUT END CONTRACTIONS, BUT WITH FULL CREST CONTRACTION (Art. 6.)

Head in Feet.	Length of Weir in Feet.								
	1·6 to 6·6	2(?)	3(?)	4	5	7	10	15	19
	Bazin's Co-efficients	Smith's Co-efficients.							
·1					659	658	658	657	657
·15		652	649	647	645	645	644	644	·643
·2	662	645	642	641	638	637	637	636	635
·25	653	641	638	·636	634	633	632	631	·630
·3	652	639	636	633	631	629	628	627	626
·4	646	636	613	630	628	625	623	622	621
·5	640	637	633	630	627	624	621	·620	619
·6	637	638	634	630	627	623	620	619	618
·7	635	640	635	631	628	624	620	619	·618
·8	633	643	637	633	629	625	621	·620	618
·9	633	·645	639	635	631	627	622	·620	·610
1	632	648	641	637	633	628	624	621	619
1·2	631		646	641	636	632	626	623	620
1·4	630			644	640	·634	629	625	·622
1·6	627			647	642	637	631	626	·623
1·7	620					638	632	626	623
1·8	625								

TABLE XVII.—CORRECTIONS FOR WIDE CRESTS (Art. 10.)
(The correction is always minus except when marked plus.)

Head in Feet.	Width of Crest in Inches								
	1	2	3	4	6	8	10	12	24
·10	·007	·016	018	·018	017	·017	017	·017	017
·16	+ ·002	·017	·023	·024	025	·025	025	·025	026
·20		·012	·024	029	·031	·032	·033	·033	034
·30		+ *·003*	*017*	*·030*	*·041*	*·045*	*047*	*·048*	*050*
·40			*·010*	*·022*	·045	·055	·060	·062	·066
·45			+ ·009						
·50				·006	041	·060	·069	074	·082
·60					·031	·059	·075	·083	·097
·70					·017	·052	075	·089	112
·80					·000	040	071	·091	125
·90					+ ·019	·027	·062	·089	137
1·0						·056	·050	082	149
1·2						+ ·025	·021	·061	168
1·4							+ ·013	·032	180
1·5								·015	

TABLES XVIII TO XXII—INCLUSIVE CO EFFICIENTS, *C*, FOR WEIRS 6 56 FEET LONG WITHOUT END CONTRACTIONS

XVIII—Weirs in Thin Walls (Art 6)

Height of Weir in Feet	Head in Feet												
	164	23	33	53	66	39	98	1 15	1 31	1 48	1 64	1 80	1 97
66													
98													
1 31													
1 64													
1 97													
2 62													
3 28	674	660	651	644	642	643	645	648	650	653	656	658	66…
4 92	672	660	650	641	638	636	636	636	636	638	639	640	641
6 56	672	659	650	641	635	633	632	632	632	632	632	632	632

XIX—Weirs with Flat Tops and Vertical Face and Back (Art 10)

Dimensions of Weirs		Heads in Feet							Remarks
Width in Feet	Height in Feet	3		1 0	1 4	2 3	4	6	
6 56	4 57 2 46	 45	48 (?) 48	49 (?) 48	48 (?) 50	47 (?)	50 (?)	51 (?)	When the upstream edge was rounded to a ra…
2 62	4 57 2 46	 48							
1 31	2 46	50	51	54	59				
66	2 46 1 15	52 53	58 60	65 67	70				
33	2 46 1 15	57 57	65 72	80* 77*	 71*				
16	2 46 1 15	63 [illegible]	80* [illegible]*	77* 72*					

* These are for sheets drowned underneath All other figures are for free sheets and the corresponding figures for sheets depressed or drowned underneath are the same to within generally 4 per cent.

XX.—*Weirs with Rounded Tops* (Art. 12.)

Sections of Weirs.	Dimensions of Weirs.		Head in Feet.					
	Radius of Crest.	Height in Feet.	·3	[illegible]	1·4	[illegible]3	4·0	6·0
Fig. 69, p. 82,	34 ft. up-stream, 40 ft. down stream	1·64	·67	79	86			
Fig. 78, p. 99, .	·26 ft.	1·64	72	84	84	.		
Fig. 78A, p. 99, .	3 37 ft.	5 3		57 (*)	62 (*)	66 (*)	68 (*)	69 (*)
Fig. 78B, p. 99,		1 64	57	59	65			

XXI.—*Weirs with Steep Back-slopes* (Art. 11.)

Top Width of Weir in Feet	Height of Weir in Feet.	Slope of Face of Weir	Back Vertical.						Back ½ to 1			Back ¾ to 1		
			Head in Feet.						Head in Feet.			Head in Feet.		
			·3	[illegible]	1·4	[illegible]3	4	6	3	[illegible]	1·4	3	[illegible]	1·4
0·00	1·64	Vertical							65	76	71	75	78	71
	1 64	½ to 1	·68	78	74									
	1 64	2 to 1	75	79	77									
33	1 64	Vertical							56	73	72	56	73	71
	1 64	1 to 1	59	72	80				57	73	·78	60	73	80
	1 64	2 to 1	61	71	77									
	4 7	2 to 1		63 (*)	·66 (*)	·68 (*)	69 (*)	69 (*)						
66	1 64	2 to 1	58	65	73									
	4 9	2 to 1		62 (*)	66 (*)	69 (*)	69 (*)	·69 (*)						
	4 9	3 to 1		70 (*)	72 (*)	68 (*)	66 (*)	66 (*)						
	4 9	5 to 1		63 (*)	63 (*)	63 (*)	63 (*)	63 (*)						

XXII — Weirs with Flat Back-slopes (Art 11)

Slope of Back of Weir	Top Width of Weir in Feet	Height of Weir in Feet	Face Vertical — Head in Feet			Face 1 to 1 — Head in Feet			Face 2 to 1 — Head in Feet					
			3		1 4	3	7	1 4	3	~	1 4	2 3	4	6
1 to 1	00	1 64				81	80	76	79	78	77			
	00	2 46	72	73	75									
	00	4 9								79 (*)	77 (*)	73 (*)	70 (*)	68 (*)
	33	1 64	57	71	78	60	72	82	61	71	79			
	66	1 64							58	65	74			
2 to 1											78			
											78			
	60	1 64				58	63	74	58	64	73			
	60	2 46	48	56	68	51	61	71	55	62	70			
	60	4 9								63 (*)	66 (*)	68 (*)	69 (*)	69 (*)
	1 31	2 46	49	51	58									
3 to 1	00	1 04	55	04	68									
5 to 1	00	1 64				66	66	69	68	69	71			
	00	2 46	58	58	60									
	33	1 64	54	68	63	50	64	70	58	65	69			
	33	4 9							5	66 (*)	68 (*)	66 (*)	66 (*)	66 (*)
	66	1 64							58	62	68			
10 to 1	00	2 46	52	54	56									

CHAPTER V

PIPES

[For preliminary information see chapter II articles 8 to 21]

Section I.—Uniform Flow

1. **General Information.**—In a uniform pipe, AB (Fig. 84), let the length AC, amounting to two or three times the diameter, be termed

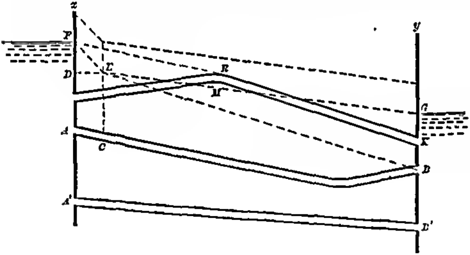

FIG 84

the mouthpiece of the pipe. At the entrance of the pipe a head $\frac{V^2}{2g}$ must be spent in imparting momentum to the water. This causes a loss of pressure head only, and not of total head. In exchange for the loss of pressure the water obtains a velocity head $\frac{V^2}{2g}$, but this is finally lost in the receiving reservoir, where the energy possessed by the water is wasted in eddies. There is also a loss in the mouthpiece depending on the co-efficient of resistance (chap. III art. 5), and varying from about $\cdot 06 \frac{V^2}{2g}$ in a bell-

mouthed, to about $50\frac{V^2}{2g}$ in a cylindrical mouthpiece This last occurs if the pipe simply stops short flush with the side of the reservoir without being splayed out If the pipe projects into the reservoir, and ends without a flange, the loss of head is about $93\frac{V^2}{2g}$ The total loss of pressure head at the entrance of a pipe is thus $(1+z_a)\frac{V^2}{2g}$ where z_a varies from 06 to 93 This loss of head is the height *FD* The line of hydraulic gradient is *FEG*

In equal lengths L, L, etc, the falls in the line of gradient or losses of head by friction are equal If the inclination of the pipe is uniform, as in *AB'*, the line of virtual slope is straight, but not otherwise Generally, however, the variations in the inclination of the pipe in lengths L_1, L, etc, are not enough to cause great differences in the lengths of their horizontal projections, and the line of virtual slope is practically straight Sometimes the length of a pipe is so great that the loss of head at the entrance may be neglected in estimating H, and the length of the mouthpiece in estimating L S is then found more easily The actual position of the pipe is of no consequence The virtual slopes and discharges of the pipes *AB*, *AB*, etc, are all equal, provided the roughnesses, diameters, and lengths are equal If the pipe discharges freely into air, the virtual slope is *FB* Pipes are always assumed to be circular in section unless the contrary is stated

If at any point *K* the line of the pipe rises above the line of the hydraulic gradient, the pressure is less than the atmospheric pressure At such a point air may be disengaged from the water and the flow impeded, the line of gradient being shifted to *FI* (loss of head at entrance not considered) and the pipe *IK* running only partly full If the height *MK* is more than 34 feet the pressure becomes negative and flow impossible The above refers to cases in which the water is subjected throughout to ordinary atmospheric pressure If the pressures on the two reservoirs are unequal the heads must be calculated (chap ii art 1) and the gradient *FJ* drawn accordingly Arrangements must be made for periodically drawing off the air which accumulates at 'summits' such as *K* lying above the gradient line

With small pipes a great increase in the temperature of the water increases the discharge The following results have been found —

Diameter of Pipe	Increase in Temperature of Water		Increase of Discharge
	From	To	
Inches. 1	60°	212°	25 per cent.
1.5	77°	120°	8 per cent. (V about 8.5) 10 per cent. (V about 6.7).
2	72°	[illegible]	Discharge was perceptibly increased

The pressure in a pipe, after allowing for difference in head, decreases somewhat in going from the circumference to the centre.

Let D be the diameter of a pipe. Then R is $\frac{D}{4}$ or half the actual radius. Since the sectional area is as D^2, $\sqrt{R}$ as $\sqrt{D}$, and since C also increases with D, the discharge increases more rapidly than $D^{\frac{5}{2}}$. If two pipes are nearly equal in diameter, their discharges will be nearly as $D^{\frac{5}{2}}$. Allowing for increase of C, a pipe of 2 feet diameter will discharge nearly as much as six pipes of 1 foot diameter. To double the discharge of a pipe it is only necessary to increase the diameter by about 30 per cent. Since V increases as $\sqrt{S}$, and C also increases slightly with S, the discharge increases rather more rapidly than $\sqrt{S}$. In order to double the discharge S must be more than trebled. Doubling the slope increases the discharge by perhaps 50 per cent. For a given head H the slope is inversely as L, and Q therefore increases more rapidly than $\frac{1}{\sqrt{L}}$. It is clear that slight errors in measuring the diameter of a pipe, or an insufficient number of measurements when the diameter varies—as it nearly always does—may cause considerable errors in discharges or co-efficients.

All the ordinary problems connected with flow in uniform pipes can be solved by means of equations 14 and 15 (p. 21), some directly and some by the tentative process. The problems referred to are those in which one of the quantities Q, S and D has to be found, the others being given. V can, of course, always be found from D and Q without difficulty, or either of those quantities from V and the other. Pipes are generally manufactured of certain fixed sizes, and when the theoretical diameter has been calculated the most suitable of these sizes can be adopted, unless a special size

is to be made To facilitate calculations various tables have been prepared The method of using them and of dealing with the above problems will be clear from the examples given and the remarks which precede them

2 **Short Pipes** —When the length of a pipe is not very great the velocity may be high the co efficient C may be outside the range of experimental data, and its value then can only be estimated For cases in which L is not more than $100D$ the pipe may be treated as a short tube, and equation 7 (p 13) used The following values of c have been found —

Ratio of L to D	Co efficients of Discharge c		Remarks
	Small Metal Pipes	Pipes of Unglazed Earthenware Diameters 235 foot to 48 foot	
2	82		With the earthenware pipes the discharge of two pipes laid side by side was 2 4 times that of a single pipe So great a difference would not have been expected With long pipes no such effect would occur The co efficients for these earthenware pipes are very irregular
3	82		
5	79		
10	77		
25	71		
31		50	
37 5		52	
50	64		
53		25	
60	60		
100	55		

All the experiments were made with small heads The shorter the pipe the greater the proportionate loss of head at the entrance and the less the variation of c for a proportionate increase in L Thus when L increases from $25L$ to $50L$ c does not decrease so much as when L increases from $50L$ to $100L$

3 **Combinations of Pipes** —If a pipe does not simply connect two reservoirs, but is, say, a branch supplied from a larger pipe and itself bifurcating its discharge can only be ascertained by tapping it and attaching pressure columns

When a water main gives off branches it may undergo reductions in diameter Suppose that the conditions in such a main are to be determined when no water is being drawn off by the branches If the discharge of the main is known the loss of head and gradient in each length can be found Suppose however that only the total loss of head H is known Obviously the

gradient in any length will be flatter as D is greater, and $\sqrt{S}$ will be roughly as $\frac{1}{D^{\frac{5}{2}}}$ or $\frac{H}{L}$ as $\frac{1}{D^5}$ or H as $\frac{L}{D^5}$ Thus if the total loss of head is known the loss in each length can be roughly found, the gradient being sketched and the discharge computed When greater accuracy is required let D' be an approximation to the average diameter of the whole main With this diameter and gradient $\frac{H}{L}$ find an approximate discharge Q', and thence the velocities V_1, V_2, etc. Then for any length L_1, $C_1\sqrt{R_1}=\frac{V_1}{\sqrt{S}}$ The slopes S_1, S_2, etc, can then be found, and the losses of head are $L_1 S_1$, $L_2 S_2$, etc If these when added together are not equal to H the discharge Q' must be corrected When Q has been found accurately the diameter D of the equivalent uniform main is known It is such as gives the discharge Q with the gradient $\frac{H}{L}$. If the above problem again occurs with the same pipe, but a different value of H, there will be no difficulty, for D will be practically unaltered

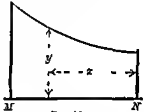

Fig 85

Let Fig 85 represent a main of uniform diameter, and let its discharge be drawn off gradually by branches If the discharges at M and N are Q and zero respectively, and if the discharge is supposed to decrease uniformly along the whole length of the pipe, then the line of gradient will be a curve If x and y are the ordinates of any point in the curve, and A and B are constants, $Q=Ax$ But if C is supposed constant, $Q=B\sqrt{S}=B\left(\frac{dy}{dx}\right)^{\frac{1}{2}}$ Therefore $\frac{dy}{dx}=\frac{A^2}{B^2}x^2$ Integrating, $y=\frac{A^2}{3B^2}x^3$

When $x=L$, $y_1=\frac{A^2}{3B^2}L^3$, and the mean gradient $\frac{y_1}{L}=\frac{A^2}{3B^2}L^2$ But when $x=L$, $\frac{dy}{dx}$ is $\frac{A^2}{B^2}L^2$, or the mean gradient is one third of the gradient at M The total loss of head is one third of what it would have been if the whole discharge Q had been delivered at N As C increases with S the fraction is really greater than one third

If in a branched pipe (Fig 86) the pressures at A, B, C are known, the discharges can be found by assuming a pressure head, H, at D, and calculating the discharges Q_1, Q_2, Q_3. If Q_1 does not

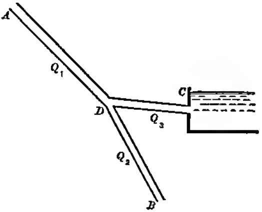

Fig 86

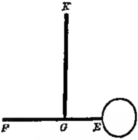

Fig 87

equal Q_2+Q_3, then H must be altered and a fresh trial made. Q_3 may be plus, zero, or minus according to the direction in which the water flows.

Let E (Fig 87) be a water main, EF a branch, and GK a pressure column, and let there be a three way cock at G. If no water is being drawn off at F the water rises to a height K, determined by the pressure in the main, whether GK or GF is open; but if water is being drawn off at F the height GK will be less when GF is open. If EF is a house service pipe and GK a pipe rising to the ground level outside the house, then by means of a pressure gauge at K an inspector can tell, without entering the house, whether water is being used in it or not.

In a system of bifurcating pipes (Fig 88) such as that used for the water supply of a town, the pressure heads sufficient to force the water to the required levels at various points, L, K, F, having been determined, the gradients corresponding to imaginary pressure columns at these points can be drawn, and the required discharges q_1, q_2, etc, being known, the diameters of the various pipes

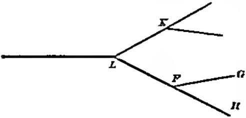

Fig 88

can be calculated. Suppose the system to be at work, then if the consumption in a branch FG is increased, the pressure head at F will be lowered and the branch FH will not be able to obtain its

estimated supply, unless its conditions are similar to those of *FG* The lowering of the pressure at *F* causes an increased discharge in *LF*, and a lowering at *L*, and thus more water is drawn in from the reservoir, but not to the same amount as the increase taken by *FG* Thus any excessive consumption tends to partially remedy itself, firstly by preventing water being forced to high levels in its neighbourhood, and secondly, by drawing more water into the main (Cf chap vii art 6)

4 Bends.—For bends in small pipes Weisbach found the loss of head to be $z_1 \frac{v^2}{2g}$. For a bend of 90° he found z_1 to be as follows,

$\frac{r}{R}=$ 1	2	3	4	5	6	7	8	9	1·0
$z_1=$ 13	14	16	21	29	44	66	98	1 41	1 98

where *R* is the radius of the bend and *r* that of the pipe For angles other than 90° the loss of head does not increase so fast as the length of the bend According to the above figures, the loss of head decreases as the radius of the bend increases, and this view has generally been accepted But recent experiments made at Detroit with large pipes by Williams, Hubbell, and Fenkell[1] show that, with large pipes at least, the resistance for a bend of 90° decreases as the radius of the bend decreases, provided it does not fall below 2 or 2½ diameters of the pipe The relative resistances seem to be somewhat as follows with a new iron pipe 30 inches in diameter —

(1) Radius of 90 Curve	(2) Length of Curve.	(3) Relative Loss of Head due to resistance in a length of 1 foot of pipe	(4) Total Relative Loss of Head in bend of 90	(5) Loss of Head in same length of straight pipe.
Feet.	Feet.			
4	6 3	7·0	44	6 3
6	9 4	4 8	45	9 4
10	15 7	4 3	68	15·7
15	23·6	4 0	94	23 6
25	39 3	3 1	122	39 3
40	62 8	2 3	144	62 8
60	94·2	2·0	188	94·2
straight.		1·0		

[1] *Transactions of the American Society of Civil Engineers*, vol xlvii

The resistance per foot run of pipe decreases, though not very rapidly, as the radius of the bend increases, but owing to the greater length of bend the total resistance increases with the radius of the bend

The results of the experiments are not as exact as could be desired It was found that the straight lengths of pipes, adopted as standards for comparison with the bends, came themselves to some extent under the influence of abnormal velocity distribution due to bends upstream of them The velocities were all measured by Pitot tubes and somewhat complex apparatus, and it is not certain that the results are quite accurate (chap viii arts 14 and 15), but any errors of this kind must have affected all observations in very much the same manner, so that the relative results are hardly affected The diameters of the pipes were not measured at as many points as desirable The figures in the above table are not quotations, but have been arrived at from figures given in the paper They are only intended to be approximations, but having regard to the great differences among the figures in column 3, and to their regularity, the view of the experimenters, that their proposition is true beyond a doubt, must be upheld Some preliminary experiments made with 12 inch and 16 inch pipes gave similar results When the radius of curvature becomes very small the law no longer holds good This is perhaps because contraction occurs (Cf chap vii art 1)

It is also proved in the paper under reference that when it is desired to connect two straight portions, *AL*, *LD* (Fig 89), of a pipe by a 90° curve, a small curve *CF* is cheaper, because, although the line is longer than *AEGD*, the cost of laying pipes is much less per foot run on the straight than on curves

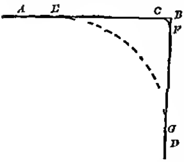

Fig 89

In a cross section a few feet down stream of the termination of a 90 degree curve of 10 feet radius in a 30 inch pipe the maximum velocity was found with low velocities to be in the centre of the pipe, but it moved when the *maximum velocity was 3 5 feet per second* to a distance from the edge of the pipe equal to about 20 of the diameter A further increase of 30 per cent in the velocity failed to shift it further With curves of 15 feet and 10 feet radius its position was about the same

5 Relative Velocities in Cross Section —The velocities at different points in the cross section of a pipe have been observed chiefly by means of the Pitot tube (chap VIII art 14) Bazin, discussing some observations made by Darcy and some by himself on pipes, finds some previously proposed formulæ to be unsuitable for general application, and arrives at two empirical equations—

$$V_c - v = V\sqrt{b}\left\{21\left(\frac{r}{R}\right)^{1} + 27\left(\frac{r}{R}\right)^{2}\left(1 - 1\ 10\frac{r}{R}\right)^{2}\right\} \qquad (68)$$

$$V_c - v = 39\ 5V\sqrt{b}\left\{1 - \sqrt{1 - 0\ 95\left(\frac{r}{R}\right)^{2}}\right\} \qquad (69)$$

where V_c is the central velocity, V the mean velocity, v the velocity at radius r, R the radius of the pipe, and $b = \frac{RS}{V^2}$ Either of these equations gives a velocity curve which nearly agrees with the observed velocities, and both give $r = 74R$ as the distance from the centre where the velocity of the water is equal to V In a 30 inch pipe the form of the velocity curve has been found by Williams, Hubbell, and Fenkell to be very nearly a semi ellipse Regarding the ratio of V to the central velocity V_c, the various experiments made show somewhat conflicting results, and are not sufficiently reliable and numerous to enable the ratio to be fixed with confidence *The general tendency* is for the ratio to increase with V and also with the diameter of the pipe The following table has been compiled The figures must be taken as showing probable and approximate values only, but are likely

Kind of Pipe	Diameter of Pipe in inches.	Mean Velocities in Feet per Second							
		75	1 5	2 5	3 5	5	8	14	6? 5
Brass	$1\frac{1}{8}$								84
Brass seamless	2	70	73	77	79	80			
Cast iron	7 5			80	81	82	83	84	
Cast iron	9 5		80	81	82	83	84	85	
Cast iron with deposit	9 5		81	81	82	82	83	83	
New iron coated with coal tar	12		83	83	84	85	85	85	
	16		82	83	84	85			
	30	75	83	84	85				
Cement,	31 5				85	86			
New iron coated with coal tar,	42				86				

to give better results than the adoption of a fixed ratio for all cases The range of velocities for which the figures are given for any pipe is approximately that for which experiments were made By means of these ratios it is possible to deduce the probable mean velocity in a pipe by an observation at the axis

In the Detroit experiments above mentioned it was found that the velocity ratios tended to become irregular with low velocities, and it was suggested, in the discussion on the experiments, that a point of 'critical velocity' (chap I art. 15) exists at velocities of about 70 feet, 30 feet, and 14 feet per second for the 30 inch, 16 inch, and 12 inch pipes respectively

Section II.—Variable Flow

6 **Abrupt Changes**.—The losses of head occurring at abrupt changes in small pipes have been found experimentally by Weisbach, and are as below

Abrupt Enlargement (Fig 4, p 5).—The loss of head is $\frac{(V_1 - V_2)^2}{2g}$ or the head due to the relative velocity, but see remarks in chap II art 18

Abrupt Contraction (Fig 3, p 5).—The loss of head (and also for a diaphragm (Fig 90) or for a contraction with a diaphragm) is chiefly caused by the enlargement from EF to MN, and is to be found as above To find the velocity at EF divide the velocity

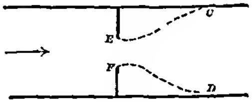

Fig 90.

at MN by c_c. For a diaphragm (Fig 90) the values of c_c were found to be as follows —

$\frac{\text{Area } EF}{\text{Area } CD} =$	1	2	3	4	5	6	7	8	9	1 0
$c_c =$	624	632	643	659	681	712	755	813	892	1·00

These may be accepted for the other cases

Elbow (Fig 91).—The loss of head is

$$c_b \frac{V^2}{2g} \text{ where } c_b = 946 \sin^2\frac{\theta}{2} + 2\ 05 \sin^4\frac{\theta}{2}$$

The values of z_e are as follows:—

$\theta=$	20°	40°	60°	80°	90°	100°	110°	120°	130°	140°
$z_e=$	·046	·139	·364	·740	·984	1·260	1·556	1·861	2·158	2·431.

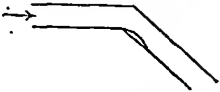

FIG. 91.

Thus at a right-angled elbow nearly the whole head due to the velocity is lost. When two right-angled elbows closely succeed each other the loss of head is double that in one elbow if the two bends are in opposite directions, but is no greater than that in a single elbow if the bends are both in one direction.

Gate-Valve (Fig. 92).—

$\frac{h}{D}=$	1·0	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{a}{A}=$	1·0	·948	·856	·740	·609	·466	·315	·159
$z_e=$	·0	·07	·26	·81	2·06	5·52	17·0	97 8.

Where A is the sectional area of the pipe and a that of the opening.

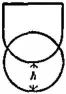

FIG. 92.

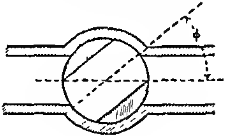

FIG. 93.

Cock (Fig. 93).—

$\phi=$	5°	10°	15°	20°	25°	30°	35°	40°	45°
$\frac{a}{A}=$	·926	·850	·772	·692	613	·525	·458	·385	·315
$z_e=$	·05	·29	·75	1·56	3·10	5·47	9·68	17·3	31·2
$\phi=$	50°	55°	60°	65°	82°				
$\frac{a}{A}=$	·250	·190	·137	·091	0 0				
$z_e=$	52·6	106	206	486.	∞				

Throttle Valve (Fig 94) —

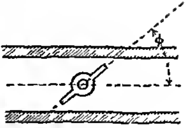

Fig 94

$\phi = 5°$ 10° 15° 20° 25° 30° 35° 40° 45° 50°
$z_t = \cdot24$ 52 90 1 54 2 51 3 91 6 22 10 8 18 7 32 6
$\phi = 55°$ 60° 65° 70°
$z_t = 58\ 8$ 118 256 751

In the last three cases z_v, z_c, and z_t are multiplied by $\frac{V^2}{2g}$ to give the loss of head

It is not at all certain that the above figures apply correctly to large pipes, and in fact it has been proved that some of them do not apply correctly For a gate in a 2 foot pipe z_v has been found to be as below.

$\frac{h}{D}$	z_v as observed	z_v by Weisbach's rule given above
$\frac{1}{12}$	41 2	43
$\frac{1}{6}$	31 35	28
$\frac{1}{4}$	22 7	17
$\frac{1}{3}$	11 9	7 92
$\frac{3}{8}$	8 63	5 52
5/12	6 33	3 77
11/24	4 58	2 87
1/2	3 27	2 06
7/12	1 55	1 11
2/3	77	57
1 0	00	·00

When loss of head due to any of the above causes occurs, the line of hydraulic gradient shows a sudden drop as at *GH*, Fig 95, its inclination is reduced, and with it the velocity and discharge of the pipe If the local loss of head did not exist the slope would be *KL* The velocity to be used in calculating the loss of head is that due to *KG* and not *KL* If a second cause operates at *M* the gradient becomes *KG*, *HM*, *NL*, and the loss of head *GH* is now less than before because the velocity is less Thus the *loss of*

head does not increase in proportion to the number of causes operating But where economy of head is desired, it is necessary to avoid abrupt changes of all kinds, using tapering 'reducers' where the diameter changes, and curves of fair radius at all bifurcations or changes in direction

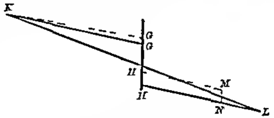

Fig 95

It appears that the disturbance of the velocity ratios due to abrupt changes may extend downstream for long distances Bazin found that the disturbance from the entrance contraction of a 32 inch pipe disappeared at 25 to 50 diameters downstream, but disturbance due to curves has been found to extend to 100 diameters In the disturbed region the pressures, as indicated by pressure columns, appear to be below normal, or at least to be unreliable In some important experiments on a 6 foot pipe[1] some of the results are doubtful and probably erroneous, owing to a piezometer being placed just downstream of an abrupt change

7 Gradual Changes —When a gradual change occurs in the sectional area of a pipe equation 16, page 22, must be used At a point where the diameter of a pipe changes a tapering piece is usually put in If the taper is gradual the loss of head in it from resistances is about the same as in a uniform pipe with the same mean velocity

The following are examples of accidental changes in the diameters of pipes —

(1)	(2)	(3—4)		(5—6)		(7)	(8)	(9)	(10)	(11)	(12)
Length of Pipe	Nominal Diameter	Actual Diameters		Velocities		$\frac{V_1^2 - V_2^2}{2g}$ or h_e	$\sqrt{\frac{V_1^2+V_2^2}{2}}$	c	Loss of Head from Resistance or h or $\frac{v^2 L}{c^2 m}$	Actual Fall in Gradient or h	Percentage of figure in column 7 to figure in column 10.
		d_1	d_2	V_1	V_2						
Feet.	Ins	Ins.	Ins	Feet	Feet.	Feet.	Feet.		Feet.	Feet.	
100	12	12 5	11 5	4·0	4 73	– ·099	4 38	113	609	708	16·2
25	30	29⅞	30⅛	4·0	3·93	+ ·0082	3 97	128	·0384	·0302	21 4
25	30	29⅞	30⅛	1·0	·983	+ ·00051	·993	113	·00312	·00261	16 3

[1] *Transactions of the American Society of Civil Engineers* vol. xxvi.

The figures in column 11 are obtained from those in columns 7 and 10 If the flow were uniform the figures in columns 10 and 11 would be the same, and the *ratio* of *these figures to one another* shows the error caused by assuming the pipe to be uniform If the fall h is observed, and V found from h and C, the value of V found will be erroneous in the ratio (neglecting the small variation in C) of $\sqrt{h}$ to $\sqrt{h}$, that is, in the first of the cases shown, by about 8 per cent of the smaller figure If h and V are observed (*V being found, say, by measuring Q in a tank*) and C is deduced the error in C will be similar to the above If h is not observed, but deduced from known values of V and C, then the percentage error is as shown in column 12 The second and third cases show the same pipe with very different velocities, and it will be noticed that the percentage of error does not vary very greatly In the first case quoted the variation of the diameter from the nominal diameter is perhaps excessive and hardly likely to occur in practice With longer lengths of pipe the percentage of error will, of course be small, but sometimes observations are made on short lengths, and it is clear that in such cases great error may arise, if the diameter is assumed to be uniform

When the diameter of a pipe is reduced (Fig 96) the velocity head in the narrow part is increased and the pressure head

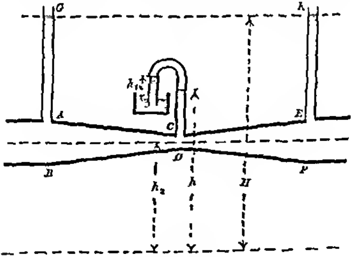

Fig 96

reduced The insertion of a portion like ACE in a pipe causes very little loss of head if the tapers are 1 The case is similar to that of a compound tube (ch ") If CD is

small enough, the pressure there will fall below the atmospheric pressure P_a, and if holes are bored in the pipe at this section no water will flow out, but air will enter The pressures on the conical surfaces $ACDB$ and $CDFE$ balance one another, and the water has no more tendency to push the pipe forward than it has in a uniform pipe

With the arrangement shown in Fig 97, the orifices being made to correspond as exactly as possible, the water flows with very little waste into the second reservoir, and the head GH is slightly less than KL The pressure in the jet KG is P_a, and it makes no practical difference whether this portion is enclosed by a pipe or not, so long as the head KL is kept the same

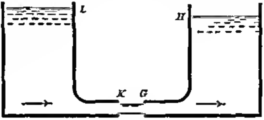

FIG 97

If at CD (Fig 96) another pipe is introduced, pumping can be effected through it, as with the case of a cylindrical or compound tube

When the hydraulic gradient of a pipe is so flat that the fall between two pressure columns would be too small to be properly observed, the 'Venturi Meter' (Fig 96) is adopted It consists of two tapering lengths of pipe with three pressure columns If the diameters, velocities, and sectional areas at AB and CD are D, v, A and d, V, a, then (chap II)

$$\frac{V^2}{2g}+h=\frac{v^2}{2g}+H$$

Also
$$\frac{V^2}{2g}=\frac{A^2}{a^2}\frac{v^2}{2g}$$

Therefore
$$\frac{v^2}{2g}\left(\frac{A^2}{a^2}-1\right)=H-h$$

$$v^2=\frac{2ga^2}{A^2-a^2}(H-h)$$

$$v=\frac{a}{\sqrt{A^2-a^2}}\sqrt{2g(H-h)}$$

To allow for loss of head in the tube a co-efficient c must be used, and

$$Q=c\frac{Aa}{\sqrt{A^2-a^2}}\sqrt{2g(H-h)} \qquad (70)$$

The figures in column 11 are obtained from those in columns 7 and 10 If the flow were uniform the figures in columns 10 and 11 would be the same, and the ratio of these figures to one another shows the error caused by assuming the pipe to be uniform If the fall h is observed, and V found from h and C, the value of V found will be erroneous in the ratio (neglecting the small variation in C) of $\sqrt{h}$ to $\sqrt{h'}$, that is, in the first of the cases shown, by about 8 per cent of the smaller figure If h and V are observed (V being found, say, by measuring Q in a tank) and C is deduced, the error in C will be similar to the above If h is not observed, but deduced from known values of V and C, then the percentage error is as shown in column 12 The second and third cases show the same pipe with very different velocities, and it will be noticed that the percentage of error does not vary very greatly In the first case quoted the variation of the diameter from the nominal diameter is perhaps excessive and hardly likely to occur in practice With longer lengths of pipe the percentage of error will, of course, be small, but sometimes observations are made on short lengths, and it is clear that in such cases great error may arise, if the diameter is assumed to be uniform

When the diameter of a pipe is reduced (Fig 96) the velocity head in the narrow part is increased and the pressure head

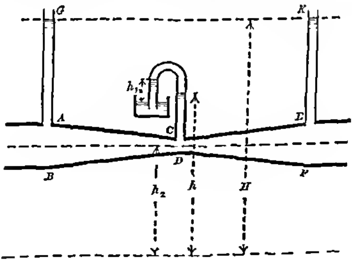

FIG 96

reduced The insertion of a portion like ACE in a pipe causes very little loss of head if the tapers are gradual The case is similar to that of a compound tube (chap III art 17) If CD is

small enough, the pressure there will fall below the atmospheric pressure P_a, and if holes are bored in the pipe at this section no water will flow out, but air will enter The pressures on the conical surfaces $ACDB$ and $CDFE$ balance one another, and the water has no more tendency to push the pipe forward than it has in a uniform pipe

With the arrangement shown in Fig 97, the orifices being made to correspond as exactly as possible, the water flows with very little waste into the second reservoir, and the head GH is slightly less than KL The pressure in the jet KG is P_a, and it makes no practical difference whether this portion is enclosed by a pipe or not, so long as the head KL is kept the same

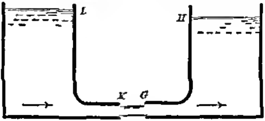

Fig 97

If at CD (Fig 96) another pipe is introduced, pumping can be effected through it, as with the case of a cylindrical or compound tube

When the hydraulic gradient of a pipe is so flat that the fall between two pressure columns would be too small to be properly observed, the 'Venturi Meter' (Fig 96) is adopted It consists of two tapering lengths of pipe with three pressure columns If the diameters, velocities, and sectional areas at AB and CD are D, v, A and d, V, a, then (chap II)

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Also $$\frac{V^2}{2g}=\frac{A^2}{a^2}\frac{v^2}{2g}$$

Therefore $$\frac{v^2}{2g}\left(\frac{A^2}{a^2}-1\right)=H-h$$

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$$v=\frac{a}{\sqrt{A^2-a^2}}\sqrt{2g(H-h)}$$

To allow for loss of head in the tube a co-efficient c must be used, and

$$Q=c\frac{Aa}{\sqrt{A^2-a^2}}\sqrt{2g(H-h)} \qquad (70)$$

The tops of the pressure columns at G and K will be practically at the same level, and one or other of them may be used If the pressure at CD is less than the atmospheric pressure, the height h_1 measures the difference and this height must be subtracted from h_1 In experiments made by Herschel with $A=$ 77 square feet, $a=$ 086 square feet, and pressures at CD less than atmospheric, c usually ranged from 96 to 1 01 With $A=57$ 8 square feet and $a=7$ 07 square feet, c varied from 95 to 99, its value being higher as the velocity was lower The highest velocity at CD was 34 5 feet per second

SECTION III —CO EFFICIENTS

8 **General Information** —The values of the co efficient C for pipes are not well known An iron pipe unprotected by an inside coating of coal tar or asphalt generally becomes in time corroded and incrusted, but occasionally it is not so Much depends on the character of the water Incrustation may occur in a coated pipe if the coating is imperfect or damaged Severe incrustation may reduce the discharge to almost any extent, say by 30 per cent in large pipes, and by still more in smaller ones, where not only is the roughness increased but the diameter greatly reduced Definite co efficients cannot be given for incrusted pipes, but only for new and clean pipes In a wooden pipe the co efficients may be reduced by organic growth, but on the other hand the wood in some cases has become smoother with use

Besides the causes given in chapter ii (arts 9 and 11) for discrepancies in C, it must be added that even if the pipe is uniform the diameter is often wrongly stated, the manufacturer s size being accepted It has been shown above that a slight difference in D has a great effect Errors in the measurement of Q D, and S may be in either direction (those in S and D, especially of S, being greatest with low values of these quantities), but error arising from unsuspected or unreported incrustation, or losses of head from bends or other causes all tend to give low values of C Hence generally C as reported is likely to be too low and to be worst determined when S is small

9 **Values of Co efficients** —Darcy obtained a set of co efficients which vary from 93 to 113, as the hydraulic radius varies from 012 foot to 1 0 foot Smith framed a much more extensive set, making C increase with both R and S Fanning s co efficients follow a similar law, his values, however, sometimes agreeing with Smith's and sometimes falling short of them by some 10 per cent

Smith's values are probably the more reliable, because of the greater attention which he gives to the subject and the care with which he eliminated faulty or doubtful experiments, and this is, very likely, why his figures are higher Fanning's co-efficients apply to cast-iron pipes, Smith's to well made cast iron pipes or riveted sheet-iron or steel, all supposed to be coated, joints smooth, and curves of fair radius The following statement gives an abstract of the co-efficients[1] —

Hydraulic Radius.	Hydraulic Gradients.				Remarks
	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$	
Feet.					
1·0		140	137	133	Smith.
		136	133	128	Fanning
		143	135	131	Mean
·25	124	116	104		Smith.
	115	110	104	100	Fanning
	120	113	104	100	Mean
10	107	101	96		Smith. Fanning
	107	101	96		Mean

Recently Tutton[2] has investigated an immense number of pipe experiments, including nearly all considered before He adopts the formula $V=C_f R^x S^y$ where C_f is constant for any one pipe The following are some of his figures —

Kind of Pipe	C_f	x	y
New cast-iron (C I.) or tarred pipe,	126 to 158	66	51
Wood stave pipe,	129 to 155	66	51
Wrought iron riveted pipe (W I),	127 to 165	62	55
Asphalt-coated pipe,	139 to 188	62	55
Tuberculated pipe,	31 to 80	66	51

[1] Fanning gives his co efficients in another form and not for the equation $V=C\sqrt{RS}$, but they have now been reduced to the above form

[2] *Journal of the Association of Engineering Societies*, vol xxiii.

The tops of the pressure columns at G and K will be practically at the same level and one or other of them may be used If the pressure at CD is less than the atmospheric pressure, the height h_1 measures the difference, and this height must be subtracted from h In experiments made by Herschel with $A=$ 77 square feet, $a=$ 086 square feet, and pressures at CD less than atmospheric, c usually ranged from 96 to 1 01 With $A=57$ 8 square feet and $a=7$ 07 square feet, c varied from 95 to 99, its value being higher as the velocity was lower The highest velocity at CD was 34 5 feet per second

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great range of diameters and slopes, but no doubt many of them are inaccurate and some perhaps worthless. On examining the diagrams of them it is seen that in many cases other indices of S would fit the results. This, combined with the preceding remarks, seems sufficient to show that the figures in the above table derived from his formula should be considered only in their general and mean aspects, and must be taken generally as being low.

The largest pipes considered by Tutton were 4 feet in diameter. Some recent co-efficients for a 6 feet W. I. pipe are some 20 per cent less than Smith's and Fanning's, while those for a 5 feet 1 in. C. I. pipe are 4 and 9 per cent in excess. From these and Tutton's figures it is reasonable to conclude that C is lower for W. I. pipe than for C. I., and this may be due to the rivet heads. For asphalt-coated pipes Tutton's formula would give results some 10 per cent higher than for W. I. For wood pipes the experiments have so far been few, but have included diameters of 6 feet. The co-efficients seem to be about the same as for C. I. pipes. The general conclusion is that, though the whole subject is in a highly unsatisfactory state, Smith's co-efficients—or Fanning's for the small pipes, which Smith did not consider—are fairly reliable for clean C. I. or wood or coated pipes, and that for W. I. pipes 5, 10, or 15 per cent. should be deducted, the deduction being smaller as D is greater. Detailed values of Smith's and Fanning's co-efficients are given in tables xxiv. and xxv.

Kutter's co-efficients (table xxix. *et seq.*) are not very suitable for pipes, C remaining unaltered when S is increased above $\frac{1}{1000}$. They agree generally, when $N = .011$, with Smith's, but give too low velocities for small pipes and too high velocities for large pipes with small slopes. (Cf. chap. vi. art. 13.)

For small tin and zinc pipes Fanning's co-efficients are fairly correct. For $2\frac{1}{2}$ inch hose they are fairly correct when the hose is of linen and unlined, but they should be increased by some 25 per cent. when the hose is of rubber or lined with rubber.

EXAMPLES

Explanation.—The problem to be solved may be either to find the discharge in a pipe for which all the data are known, or when the discharge and one of the quantities D or S are known, to find the other. In the first case the solution is direct, in the others (since R and C vary with D and S) indirect. The methods to be adopted will be clear from the following examples.

Tutton's formulæ and co-efficients have been accepted by one recent writer, but on examination they lead to somewhat curious results. When reduced to the Chézy formula, Tutton's co-efficients for iron pipes come out as follows :—

Hydraulic Radius	Kind of Pipe	Tutton's Co-efficients (C_f)		Slopes $\frac{1}{10}$	Slopes $\frac{1}{100}$	Slopes $\frac{1}{1000}$	Slopes $\frac{1}{10000}$
				Co-efficient C in Chézy formula			
Feet 1 0	C I	158	Highest values	156	151	148	145
	W I	165		147	131	116	104
	Mean			152	141	132	125
	C I	126	Lowest values	123	120	118	112
	W I	127		113	101	89	80
	Mean			118	111	104	96
2 5	C I	158	Highest values	124	121	118	116
	W I	165		125	111	99	88
	Mean			125	116	109	102
	C I	126	Lowest values	99	96	94	93
	W I	127		96	85	76	68
	Mean			98	91	85	81
10	C I	158	Highest values	107	104	102	100
	W I	165		113	99	89	79
	Mean			110	102	96	90
	C I	126	Lowest values	85	83	81	80
	W I	127		86	76	68	61
	Mean			86	80	76	71

Taking the highest values, it seems that in going from a steep to a flat slope V decreases very much faster with a smooth W I pipe than with a smooth C I pipe. This, though unlikely, is conceivable. But if both pipes are roughened (roughening is the only apparent cause for lower values of C_f), a similar law holds good, and V decreases faster for the rough W I pipe than for the rough C-I pipe. This is highly improbable. Again for tuberculated pipe, which seems to include both kinds, V, if calculated, would decrease slowly. For asphalt it decreases rapidly. For coal tarred and galvanised pipe Tutton makes V vary as $S^{.5}$, that is, C increases as S decreases, a result different from any hitherto accepted. The results considered by Tutton cover in each case a

great range of diameters and slopes, but no doubt many of them are inaccurate and some perhaps worthless On examining the diagrams of them it is seen that in many cases other indices of S would fit the results This, combined with the preceding remarks, seems sufficient to show that the figures in the above table derived from his formula should be considered only in their general and mean aspects, and must be taken generally as being low

The largest pipes considered by Tutton were 4 feet in diameter Some recent co-efficients for a 6 feet W I pipe are some 20 per cent less than Smith's and Fanning's, while those for a 5 feet 1 in C I pipe are 4 and 9 per cent in excess From these and Tutton's figures it is reasonable to conclude that C is lower for W I pipe than for C I, and this may be due to the rivet heads For asphalt-coated pipes Tutton's formula would give results some 10 per cent higher than for W I For wood pipes the experiments have so far been few, but have included diameters of 6 feet The co-efficients seem to be about the same as for C I pipes The general conclusion is that, though the whole subject is in a highly unsatisfactory state, Smith's co-efficients—or Fanning's for the small pipes, which Smith did not consider—are fairly reliable for clean C I or wood or coated pipes, and that for W I pipes 5, 10, or 15 per cent should be deducted, the deduction being smaller as D is greater Detailed values of Smith's and Fanning's co efficients are given in tables xxiv and xxv

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EXAMPLES

Explanation —The problem to be solved may be either to find the discharge in a pipe for which all the data are known or when the discharge and one of the quantities D or S are known, to find the other In the first case the solution is direct, in the others (since R and C vary with D and S) indirect The methods to be adopted will be clear from the following examples

One advantage of the system of tables here adopted as compared to some others, is that V always enters as a factor It is a distinct advantage, in designing that the value of V, and not only of Q, should constantly come to notice

Example 1 —Using Smith s co efficients, find the discharge of a W I pipe whose diameter is 3 feet and slope 1 in 1000

From table xxiv, C is about 123 5 and V about 3 4 Smith's co efficient for this value of V is 130, so that V will be about 3 6 and C about 130 From table xxiii $\sqrt{R}=866$ From table xxvi $C\sqrt{R}=112\ 5$ From table xxviii $V=3\ 56$, which agrees nearly with the value assumed, and confirms the co efficient 130 From table xxiii $A=7\ 07$ Then $Q=7\ 07\times3\ 56=25\ 17$ c ft per second Since the pipe is W I a deduction should be made As the pipe is rather large deduct 10 per cent, making $Q=22\ 65$ c ft per second

Example 2 —Using Smith's co efficients, design a pipe to carry 20 c ft per second, the fall being 10 ft in 5000

Assume $D=2$ ft From table xxiii $A=3\ 142$ sq ft and $\sqrt{R}=707$ Also $V=\frac{20}{3\ 14}=6\ 37$ ft per second From table xxv $C=129$ From table xxvi $C\sqrt{R}=91\ 2$ This value does not appear in table xxviii, look out 182 4, which gives (for $S=\frac{1}{100}$) $V=8\ 16$, V is 4 08, which is too low, that is, the assumed diameter was too small

Let $D=2\ 5$ ft From table xxiii $A=4\ 91$ and $\sqrt{R}=791$ Also $V=\frac{20}{4\ 91}=4\ 07$ ft per second From table xxv $C=128$ From table xxvi $C\sqrt{R}=101$ From table xxviii $V=4\ 52$ ft per second which is too high The diameter 2 5 ft is thus slightly in excess of what is required To find the actual discharge, C (for $V=4\ 5$) is 129 5, $C\sqrt{R}$ is 102 4, V is 4 58, and Q is $4\ 58\times4\ 91=22\ 49$ c ft per second

Since $\left(\frac{2'\ 4''}{2'\ 6''}\right)^{\frac{5}{2}}=\left(\frac{14}{15}\right)^{\frac{5}{2}}=\frac{12\ 5}{15}$ nearly, a 2 ft 4 in pipe would be too small

Example 3 —A $1\frac{1}{2}$ ft C I pipe has to carry a discharge of 18 c ft per second What will the gradient be? Fanning's co efficient to be used From table xxiii $A=1\ 77$ Then $V=\frac{18}{1\ 77}=10\ 2$ ft per second From table xxiv $C=117$ and $S=020$ nearly From table xxvii $\sqrt{S}=1414$ From table xxiii

$\sqrt{R}=$ 612 From table xxvi $C\sqrt{R}=71$ 6 and 71 6 $\times$ 1414 $=10$ 23 Therefore $S=$ 020 is correct.

Example 4 —A pipe 2 in in diameter and 20 ft long connects two reservoirs, the head being 1 ft and the pipe projecting into the upper reservoir Find the discharge, using Fanning s coefficients

The pipe being short, the loss of head at entrance must be allowed for This (art 1) is $z_e=1\ 93\frac{V^2}{2g}$ Suppose V to be 4 ft per second Then from table i $\frac{V^2}{2g}=$ 25 and z_e is 48 This loss occurs in the length of, say, 4 ft, so that $L=19$ 6 ft and $S=\frac{1\cdot 0-\ 48}{19\ 6}=$ 027 From table xxiv $S=$ 040 is the slope which gives $V=4$ 0, so that V has been assumed too high

Let V be 3 5 ft per second Then $\frac{V^2}{2g}=$ 19, and z_e is 37, and $S=\frac{1\ 0-\ 37}{19\ 6}=$ 032 Table xxii does not give this slope exactly, but evidently C is about 97 From table xviii $\sqrt{R}$ is 204 In table xxvi look out 408 Then $C\sqrt{R}$ is $\frac{39\ 6}{2}=19$ 8 The slope $S=$ 032 is steeper than those in the tables Therefore calculate $\sqrt{S}$, which is 18, and $C\sqrt{RS}$, which is 19 8 $\times$ 18, or 3 56 ft per second, which is near enough

Example 5 —An open stream discharging 16 c ft per second is passed under a road through a syphon or tunnel of smooth plastered brickwork of section 2 ft $\times$ 2 ft, which first descends 10 ft vertically, then travels 80 ft horizontally, and again rises 10 ft vertically, the bends being right-angled and sharp What is the loss of head in the tunnel?

Here $V=\frac{16}{4}=4$ ft per second There are 4 elbows of 90° each That at the entrance to the tunnel is opposite in direction to the second ~~Here~~ Hence *the total loss of head from the elbows is*

$4\times 984\times\frac{V^2}{2g}=984$ ft

To find the approximate loss of head from friction let Fanning s co efficients be used Then $R=$ 5 $C=117$, $S=$ 0024 The fall in 100 ft is 24 ft The total loss of head is thus $98+\ 24=1\ 22$ ft

Table XXIII.—Values of A and R for Circular Pipes

Diameter (D) Feet	Diameter (D) Inches	Sectional Area (A). Square Feet	Hydraulic Radius (R). Feet.	$\sqrt{R}$	Remarks
	½	00136	0104	102	
	¾	00307	0156	125	
	1	00545	0208	144	
	1¼	·00852	0260	161	
	1½	0123	0312	177	
	1¾	0167	0364	191	
	2	·0218	0417	204	
	2½	0341	0521	228	
	3	·0491	0625	250	
	4	0873	0833	289	
	5	136	104	323	
	6	196	125	354	
	7	267	146	382	
	8	349	166	408	
	9	442	187	433	
	10	545	208	456	
	11	660	229	479	
1	0	785	250	50	
1	1	922	271	520	
1	2	1 395	292	540	
1	3	1 227	313	559	
1	4	1 396	333	577	
1	5	1 576	354	595	
1	0	1 767	375	612	
1	7	1 969	396	629	
1	8	2 181	417	646	
1	9	2 405	437	662	
1	10	2 640	458	677	
2	0	3 142	500	707	
2	2	3 687	542	736	
2	4	4 276	583	764	
2	6	4 909	625	791	
2	8	5 585	667	817	
2	10	6 305	708	841	
3	0	7 067	750	866	
3	3	8·296	812	901	
3	6	9 621	875	935	
3	9	11 05	937	967	
4	0	12 57	1 0	1 0	
4	6	15 90	1 125	1 061	
5	0	19 64	1 25	1 118	
5	6	23 76	1 375	1 173	
6	0	28 27	1 50	1 225	
6	6	33 18	1 625	1 275	
7	0	38 48	1 75	1 323	
7	6	44 18	1 875	1 370	
8	0	50 26	2 0	1 414	
8	6	56 74	2 125	1 458	
9	0	63 62	2 25	1 5	
9	6	70 88	2 375	1 541	
10	0	78 54	2 50	1 581	

Diameters not given in Table. To find A for a larger diameter look out A for half the diameter and multiply by 4. For a smaller diameter, look out A for double the diameter and divide by 4. To find $\sqrt{R}$ for a larger diameter, look out $\sqrt{R}$ for one fourth the diameter and multiply by 2. For a smaller diameter, look out $\sqrt{R}$ for 4 times the diameter and divide by 2.

Circular Channels not full. For a channel of circular section running half full, A is one half of the value in the table, and $\sqrt{R}$ is the same as in the Table.

TABLES XXIV. AND XXV.—CO-EFFICIENTS FOR PIPES CORRESPONDING TO GIVEN DIAMETERS AND VELOCITIES (Art 9)

The small figures in table xxiv show, nearly, the slopes which give the velocities entered in the heading, and they can be used to show the approximate slopes when the co efficients in table xxv are used

XXIV.—Fanning's Co-efficients

Diameter of Pipe.	Velocities in Feet per Second.									
	1	·5	1	2	3	4	6	10	15	20
Inches.										
½	43	51	76 ·016[illegible]	87 ·049	93 ·098	94 167	96 ·371	100 ·93	102	103
¾	50	75	79 ·00[illegible]	88 ·032	93 ·066	96 112	98 ·213	101 ·6[illegible]	103	104
1	73	77	81 ·00[illegible]3	89 ·0[illegible]4	94 ·045	94 ·063	98 180	102 47	104	105
1½	77	81	86	00 ·016	94 ·033	96 ·0[illegible]	100 140	102 ·32	104	103
2	85	88	90	04 ·010	96 ·0[illegible]	98 ·040	101 ·0[illegible]	104 ·[illegible]3	106	106
3	89	92	03	06 ·007	98 ·016	100 ·0[illegible]7	102 056	105 143	106	106
4	93	93	95	97 ·0019	100 ·011	102 ·019	103 ·039	106 107	108	108
6	94	95	97	100 0032	102 ·00[illegible]0	103 ·012	106 ·0[illegible]6	108 0 0	109	111
8	96	97	99	102 ·00[illegible]4	101 0050	105 ·0087	107 ·019	110 ·050	112	113
Feet. 1	98	100	102	105 ·0015	106 ·003[illegible]	108 ·0054	110 ·012	114 ·032	115	116
1 5		104	106	109 ·009[illegible]	111 0019	113 ·0034	114 ·00 4	117 ·020	118	
2		109	111	114 ·000[illegible]3	116 ·0014	117 ·00[illegible]4	118 ·0051	121 ·014	122	
3		117	118	121 ·00038	123 0008	124 ·0014	127 ·0030	128 ·008[illegible]	129	
4		127	128	129 00024	131 0005[illegible]	132 00094	135 ·0020	135 ·005	136	
5		134	135	136 ·00017	137 ·00040	137 00067	138 ·0014	142 ·0040	142	
6		137	137	137 ·00014	140 ·00032	141 ·00053	143 ·001[illegible]	147 ·0031	147	
7		141	143	143 ·00011	146 ·000[illegible]5	147 ·00045	148 ·00097	151 ·00 4	151	
8		149	150	151 ·00009	151 ·000[illegible]0	152 ·00034	155	158	158	

TABLE XXVI.—*Continued*.—VALUES OF $C\sqrt{R}$ FOR VARIOUS VALUES OF C AND $\sqrt{R}$

For a value of C lower than 90 look out double the value and halve the result

For a value of C over 140 look out half the value and double the result

Values of C	Values of $\sqrt{R}$								
	559	577	595	612	629	646	662	677	707
90	50 3	51 9	53 6	55 1	56 6	58 1	59 6	60 9	63 6
91	50 9	52 5	54 2	55 7	57 2	58 8	60 2	61 6	64 3
92	51 4	53 1	54 7	56 3	57 9	59 4	60 9	62 3	65 0
93	52 0	53 7	55 3	57 9	58 5	60 1	61 6	63 0	65 8
94	52 5	54 2	55 9	57 6	59 1	60 7	62 2	63 6	66 4
95	53 1	54 8	56 5	58 1	59 8	61 4	62 9	64 3	67 2
96	53 7	55 4	57 1	58 8	60 4	62 0	63 6	65 0	67 9
97	54 2	56 0	57 7	59 4	61 0	62 7	64 2	65 7	68 6
98	54 8	56 5	58 3	60 0	61 6	63 3	64 9	66 3	69 3
99	55 3	57 1	58 9	60 6	62 3	64 0	65 5	67 0	70 0
100	55 9	57 7	59 5	61 2	62 9	64 6	66 2	67 7	70 7
101	56 5	58 3	60 1	61 8	63 5	65 3	66 9	68 4	71 4
102	57 0	58 9	60 7	62 4	64 2	65 9	67 5	69 1	72 1
103	57 6	59 5	61 3	63 0	64 8	66 5	68 2	69 7	72 8
104	58 1	60 0	61 9	63 6	65 4	67 2	68 8	70 4	73 5
105	58 7	60 6	62 5	64 3	66 0	67 8	69 5	71 1	74 2
106	59 3	61 2	63 1	64 9	66 7	68 5	70 2	71 8	74 9
107	59 8	61 7	63 7	65 5	67 3	69 1	70 8	72 4	75 7
108	60 4	62 3	64 3	66 1	67 9	69 8	71 5	73 1	76 4
109	60 9	62 9	64 9	66 7	68 6	70 4	72 2	73 8	77 1
110	61 5	63 5	65 5	67 3	69 2	71 1	72 8	74 5	77 8
111	62 1	64 1	66 1	67 9	69 8	71 7	73 5	75 2	78 5
112	62 6	64 6	66 6	68 5	70 4	72 4	74 1	75 8	79 2
114	63 7	65 8	67 8	69 8	71 7	73 6	75 5	77 2	80 6
116	64 8	66 9	69 0	71 0	73 0	74 9	76 8	78 5	82 0
118	66 0	68 1	70 2	72 2	74 2	76 2	78 1	79 9	83 4
120	67 1	69 2	71 4	73 4	75 5	77 5	79 4	81 2	84 8
122	68 2	70 4	72 6	74 7	76 7	78 8	80 8	82 6	86 3
124	69 3	71 5	73 8	75 9	78 0	80 1	82 1	83 9	87 7
126	70 4	72 7	75 0	77 1	79 3	81 4	83 4	85 3	89 1
128	71 6	73 9	76 2	78 3	80 5	82 7	84 7	86 7	90 5
130	72 7	75 0	77 4	79 6	81 8	84 0	86 1	88 0	91 9
132	73 8	76 2	78 5	80 8	83 0	85 3	87 4	89 4	93 3
134	74 9	77 3	79 7	82 0	84 3	86 6	88 7	90 7	94 7
136	76 0	78 5	80 9	83 2	85 5	87 9	90 0	92 1	96 2
138	77 1	79 6	82 1	84 5	86 8	89 1	91 4	93 4	97 6
140	78 3	80 8	83 3	85 7	88 1	90 4	92 7	94 8	99 0

TABLE XXVI.—*Continued*.—VALUES OF $C\sqrt{R}$ FOR VARIOUS VALUES OF C AND $\sqrt{R}$

For a value of C lower than 100 look out double the value and halve the result.

For a value of C over 160 look out half the value and double the result

Values of C	Values of $\sqrt{R}$						
	·736	·764	·791	·817	·841	·866	·901
100	73·6	76·4	79·1	81·7	84·1	86·6	90·1
101	74·3	77·2	79·9	82·5	84·9	87·5	91·0
102	75·1	77·9	80·7	83·3	85·8	88·3	91·9
103	75·8	78·7	81·5	84·2	86·6	89·2	92·8
104	76·5	79·5	82·3	85·0	87·5	90·1	93·7
105	77·3	80·2	83·1	85·8	88·3	90·9	94·6
106	78·0	81·0	83·8	86·6	89·1	91·8	95·6
107	78·8	81·7	84·6	87·4	90·0	92·7	96·4
108	79·5	82·5	85·4	88·2	90·8	93·5	97·3
109	80·2	83·3	86·2	89·1	91·7	94·4	98·2
110	81·0	84·0	87·0	89·9	92·5	95·3	99·1
111	81·7	84·8	87·8	99·7	93·4	96·1	100·0
112	82·4	85·6	88·6	91·5	94·2	97·0	100·9
113	83·2	86·3	89·4	92·3	95·0	97·9	101·8
114	83·9	87·1	99·2	93·1	95·9	98·7	102·7
115	84·6	87·9	91·0	94·0	96·7	99·8	103·6
116	85·4	88·6	91·8	94·8	97·8	100·4	104·5
118	86·8	90·2	93·3	96·4	99·2	102·1	108·3
120	88·3	91·7	94·9	98·0	100·9	103·9	108·1
122	89·8	93·2	96·5	99·7	102·6	105·6	109·9
124	91·3	94·7	98·1	101·3	104·2	107·3	111·7
126	92·7	96·3	99·6	102·9	105·9	109·0	113·5
128	94·2	97·8	191·2	104·5	107·6	110·8	115·3
130	95·7	99·3	102·8	106·2	109·3	112·5	117·1
132	97·2	100·8	104·4	107·8	111·0	114·2	118·9
134	98·6	102·4	106·0	109·4	112·7	115·9	120·7
136	100·0	103·9	107·5	111·1	114·3	117·7	122·5
138	101·6	105·4	109·1	112·7	116·0	119·4	124·3
140	103·0	106·9	110·7	114·3	117·7	121·2	126·1
142	104·5	108·4	112·3	116·0	119·4	122·9	127·9
144	105·9	110·0	113·9	117·6	121·1	124·7	129·7
146	107·4	111·5	115·5	119·2	122·7	126·4	131·5
148	108·9	113·0	117·0	120·9	124·4	128·1	133·3
150	110·4	114·6	118·6	122·5	126·1	129·8	135·1
152	111·8	116·1	120·2	124·1	127·8	131·6	136·9
154	113·3	117·6	121·8	125·7	129·4	133·3	138·7
156	114·8	119·1	123·3	127·4	131·1	135·0	140·5
158	116·3	120·7	124·9	129·1	132·8	136·7	142·3
160	117·7	122·2	126·5	130·7	134·5	138·5	144·1

Table XXVI —*Continued*—Values of $C\sqrt{R}$ for various Values of C and $\sqrt{R}$

For a value of C lower than 100 look out double the value and halve the result

For a value of C over 160 look out half the value and double the result

Values of C	Values of $\sqrt{P}$						
	935	967	1·00	1·061	1·118	1·173	1 [illegible]
100	93·5	96·7	100·0	106·1	111·8	117·3	122·5
101	94·4	97·7	101·0	107·1	112·9	118·5	123·7
102	95·4	98·6	102·0	108·2	114·0	119·6	124·9
103	96·3	99·6	103·0	109·3	115·1	120·8	126·1
104	97·2	100·6	104·0	110·3	116·2	121·9	127·4
105	98·2	101·6	105·0	111·4	117·3	123·1	128·6
106	99·1	102·6	106·0	112·4	118·5	124·3	129·8
107	100·1	103·5	107·0	113·5	119·6	125·5	131·0
108	100·9	104·4	108·0	114·5	120·7	126·6	132·3
109	101·8	105·4	109·0	115·6	121·8	127·8	133·5
110	102·8	106·3	110·0	116·7	122·9	129·0	134·7
111	103·7	107·3	111·0	117·8	124·0	130·2	135·9
112	104·7	108·3	112·0	118·8	125·1	131·3	137·1
113	105·6	109·2	113·0	119·9	126·2	132·5	138·3
114	106·5	110·2	114·0	120·9	127·3	133·6	139·6
115	107·5	111·2	115·0	122·0	128·4	134·8	140·8
110	108·4	112·1	116·0	123·0	129·6	136·0	142·0
118	110·3	114·0	118·0	125·1	131·8	138·3	144·4
120	112·2	110·0	120·0	127·3	134·1	140·7	147·0
122	114·1	117·9	122·0	129·4	136·3	143·0	149·4
124	115·9	119·8	124·0	131·5	138·5	145·3	151·0
126	117·8	121·7	126·0	133·6	140·7	147·6	154·7
128	119·6	123·7	128·0	135·7	143·0	150·0	156·8
130	121·5	125·6	130·0	137·8	145·2	152·3	159·2
132	123·8	127·6	132·0	140·0	147·5	154·7	161·6
134	125·2	129·5	134·0	142·1	149·7	157·0	164·0
136	127·1	131·5	136·0	144·2	152·0	169·4	166·5
138	129·0	133·4	138·0	146·3	154·2	161·7	168·9
140	130·9	135·3	140·0	148·5	156·5	164·2	171·5
142	132·8	137·2	142·0	150·6	158·7	166·5	173·9
144	134·6	139·1	144·0	152·7	160·9	168·8	176·4
146	136·5	141·0	146·0	154·8	163·1	171·1	178·8
148	138·3	143·0	148·0	156·9	165·4	173·5	181·7
150	140·2	144·9	150·0	159·0	167·6	175·8	183·7
152	142·0	146·9	152·0	161·2	169·9	178·2	186·1
154	143·9	148·8	154·0	163·3	172·1	180·5	188·5
156	145·8	150·8	156·0	165·4	174·4	182·9	191·0
158	147·7	152·7	158·0	167·5	176·6	185·2	193·4
160	149·6	154·7	160·0	169·7	178·8	187·6	196·0

TABLE XXVII.—VALUES OF S AND $\sqrt{S}$

(For steep slopes not included in Table XXVIII.)

To find $\sqrt{S}$ for a steeper slope, look out a slope 4 times as flat and multiply $\sqrt{S}$ by 2. Thus, for 1 in 50, $\sqrt{S}$ is ·07071 × 2 = ·14142.

Slope 1 in	Fall per Foot or S	$\sqrt{S}$	Slope 1 in	Fall per Foot or S	$\sqrt{S}$
100	·010	1	230	004348	·06594
105	·0095238	09759	240	004167	06455
110	·009091	095346	250	004000	06325
115	·008696	093250	260	003847	06202
120	·008333	091287	270	003704	·06086
125	008	·089442	280	003571	05976
130	007692	08771	290	003448	·05872
135	·007407	08607	300	·003333	05774
140	007143	08452	310	003226	05680
145	·006897	·08305	320	003125	05590
150	006067	·08165	330	·003030	05505
155	·006452	08032	340	002941	05423
160	·00625	·07906	350	002857	05345
165	006061	·07785	360	·06278	05271
170	·005882	·07670	370	002703	05199
175	005714	07559	380	002632	05130
180	005556	·07454	390	002564	05064
185	·005405	·07352	400	0025	05
190	005263	07255	420	002381	04880
195	·005128	·07161	440	·002273	04767
200	·005	07071	460	002174	·04663
210	004762	06901	480	002083	04564
220	004545	06742	500	002	04472

Note to table XXVIII.—This table shows values of V for given values of $C\sqrt{R}$ and $\sqrt{S}$.

The first line of the heading shows $\frac{1}{S}$, the third line $\sqrt{S}$. The figures in brackets show the amount by which $\frac{1}{S}$ must be altered to alter $\sqrt{S}$ and V by 1 per cent. Thus for $S=$ [illegible] the slopes [illegible] and [illegible] give V 1 per cent more or less than in the table. For $C\sqrt{R}=103$, V is 2·32 and 2·28 feet per second.

TABLE XXVIII (See note on preceding page)

Values of $C\sqrt{P}$	500 (10) ·04472	550 (11) 04264	600 (12) 04083	650 (13) 03922	700 (14) 03780	750 (15) ·03652	800 (16) 03536	900 (18) ·03333
100	4 47	4 26	4 08	3 92	3 78	3 65	3 54	3 33
102	4 56	4 35	4 17	4 00	3 66	3 73	3 61	3 40
104	4 65	4 44	4 25	4 06	3 93	3 60	3 68	3 47
106	4 74	4 52	4 33	4 16	4 01	3 67	3 75	3 53
106	4 63	4 61	4 41	4 24	4 08	3 94	3 82	3 60
110	4 92	4 69	4 49	4 31	4 16	4 02	3 89	3 67
112	5 01	4 76	4 57	4 39	4 23	4 09	3 96	3 7[illegible]
114	5 10	4 66	4 66	4 47	4 31	4 16	4 03	3 80
116	5 19	4 95	4 74	4 55	4 39	4 24	4 10	3 87
116	5 26	5 03	4 82	4 63	4 46	4 31	4 17	3 93
120	5 67	5 12	4 90	4 71	4 54	4 36	4 24	4 00
123	5 50	5 25	5 02	4 62	4 65	4 49	4 35	4 10
126	5 63	5 37	5 15	4 94	4 76	4 60	4 46	4 20
129	5 77	5 50	5 27	5 06	4 68	4 71	4 56	4 30
132	5 90	5 63	5 39	5 18	4 99	4 82	4 67	4 40
135	6 04	5 76	5 51	5 30	5 10	4 96	4 77	4 50
138	6 17	5 66	5 64	5 41	5 22	5 04	4 66	4 60
141	6 31	6 01	5 76	5 53	5 83	5 15	4 99	4 70
144	6 44	6 14	5 66	5 65	5 44	5 26	5 09	4 60
147	6 57	6 26	6 00	5 77	5 56	6 37	5 20	4 90
150	8 71	6 40	6 13	5 66	5 67	5 46	5 30	5 00
153	6 64	8 52	6 25	6 00	4 76	5 59	5 [illegible]	5 10
156	8 96	6 85	6 37	6 12	5 90	5 70	[illegible]	[illegible] 20
160	7 16	6 62	8 53	6 28	6 05	5 84	5 66	[illegible] 33
164	7 33	8 99	8 70	6 43	6 20	5 99	8 60	[illegible] 17
166	7 51	7 16	6 86	6 59	6 85	6 14	[illegible]	[illegible] 60
172	7 89	7 33	7 02	8 75	6 50	8 2[illegible]	[illegible]	[illegible] 73
178	7 87	7 51	7 19	6 90	6 65	6 43	[illegible]	[illegible] 87
180	6 05	7 66	7 35	7 06	8 80	6 57	[illegible]	[illegible] 00
185	8 27	7 69	7 55	7 26	6 99	6 76	6 51	6 17
190	8 50	8 10	7 76	7 45	7 18	6 94	6 72	6 83
195	8 72	8 32	7 96	7 65	6 37	7 12	6 90	6 50
200	8 94	0 53	8 17	7 84	7 56	7 30	7 07	6 67
205	8 17	8 74	8 37	8 04	7 75	7 49	7 25	6 83
210	9 39	8 95	8 57	8 24	7 94	7 67	7 43	7 00
215	9 62	9 17	8 78	8 43	8 13	7 85	7 60	7 17
220	9 84	9 38	8 98	8 63	6 32	8 03	7 78	7 33
225	10 1	9 59	9 19	6 82	6 51	8 22	7 96	7 50
230	10 3	9 81	9 39	9 02	6 69	8 40	8 13	7 67
235	10 5	10 0	9 60	9 22	8 88	8 58	8 31	7 83
240	10 7	10 2	9 80	9 41	9 07	8 77	8 49	8 00
246	11 0	10 5	10 0	9 65	9 30	8 98	8 70	8 20
252	11 3	10 8	10 3	9 88	9 53	9 20	8 91	8 40
258	11 5	11 0	10 5	10 1	9 75	9 42	9 12	8 60
264	11 8	11 3	10 8	10 4	9 98	9 64	9 34	8 80
270	12 1	11 5	11 0	10 6	10 2	9 86	9 55	9 00
276	12 3	11 8	11 3	10 8	10 4	10 1	9 76	9 20
282	12 6	12 0	11 5	11 1	10 7	10 3	9 97	9 40
288	12 9	12 3	11 8	11 3	10 9	10 5	10 2	9 60
294	13 2	12 5	12 0	11 5	11 1	10 7	10 4	9 80
800	13 4	12 8	12 3	11 8	11 3	11 0	10 6	10 0

TABLE XXVIII.—*Continued.*

Values of $C\sqrt{r}$	1,000 (25) ·02162	1,100 (27) ·02013	1,200 (34) [illegible]	1,300 [illegible]	1,400 (29) [illegible]	1,500 (30) [illegible]	1,600 (33) [illegible]	1,800 [illegible]	2,000 (39·41) [illegible]
100	3·16	3·02	2·89	2·77	2·67	2·58	2·50	2·36	2·24
102	3·23	3·08	2·95	2·83	2·73	2·63	2·55	2·40	2·28
104	3·29	3·14	3·00	2·89	2·78	2·68	2·60	2·45	2·33
106	3·35	3·20	3·06	2·94	2·84	2·74	2·65	2·50	2·37
108	3·42	3·26	3·12	3·00	2·89	2·79	2·70	2·55	2·42
110	3·48	3·32	3·18	3·05	2·94	2·84	2·76	2·60	2·46
112	3·54	3·38	3·23	3·11	2·99	2·89	2·80	2·64	2·51
114	3·60	3·44	3·29	3·16	3·05	2·94	2·85	2·69	2·55
116	3·67	3·50	3·35	3·22	3·10	3·00	2·90	2·73	2·59
118	3·73	3·56	3·41	3·27	3·15	3·05	2·95	2·78	2·64
120	3·79	3·62	3·46	3·32	3·21	3·10	2·00	2·83	2·68
123	3·89	3·71	3·55	3·41	3·29	3·18	3·08	2·90	2·75
126	3·98	3·80	3·64	3·50	3·37	3·25	3·16	2·97	2·82
129	4·09	3·89	3·71	3·58	3·45	3·33	3·23	3·01	2·88
132	4·17	3·98	3·81	3·66	3·53	3·41	3·30	3·11	2·95
135	4·27	4·07	3·90	3·74	3·61	3·49	3·38	3·18	3·02
138	4·36	4·16	3·98	3·83	3·69	3·56	3·45	3·25	3·09
141	4·46	4·25	4·07	3·91	3·77	3·64	3·53	3·32	3·16
144	4·55	4·34	4·16	4·00	3·85	3·72	3·60	3·39	3·22
147	4·65	4·43	4·24	4·08	3·93	3·80	3·68	3·47	3·29
150	4·74	4·52	4·33	4·16	4·01	3·87	3·75	3·54	3·35
153	4·84	4·61	4·42	4·24	4·09	3·95	3·83	3·61	3·42
156	4·93	4·70	4·50	4·33	4·17	4·03	3·90	3·68	3·49
160	5·06	4·82	4·62	4·44	4·28	4·13	4·00	3·77	3·58
164	5·19	4·95	4·73	4·55	4·38	4·23	4·10	3·87	3·67
168	5·31	5·07	4·85	4·66	4·49	4·31	4·20	3·96	3·76
172	5·44	5·19	4·97	4·77	4·60	4·44	4·30	4·05	3·85
176	5·56	5·31	5·08	4·88	4·70	4·54	4·40	4·15	3·94
180	5·69	5·43	5·20	4·99	4·81	4·61	4·50	4·21	4·03
185	5·85	5·58	5·34	5·13	4·95	4·74	4·63	4·36	4·14
190	6·01	5·73	5·49	5·27	5·08	4·91	4·75	4·48	4·25
195	6·17	5·88	5·63	5·41	5·21	5·01	4·88	4·60	4·36
200	6·32	6·03	5·77	5·55	5·35	5·16	5·00	4·71	4·47
205	6·48	6·18	5·93	5·69	5·48	5·29	5·13	4·83	4·58
210	6·64	6·33	6·06	5·83	5·61	5·42	5·25	4·95	4·70
215	6·80	6·48	6·21	5·96	5·75	5·55	5·38	5·07	4·81
220	6·96	6·63	6·35	6·10	5·88	5·68	5·50	5·19	4·92
225	7·11	6·78	6·50	6·24	6·02	5·81	5·63	5·30	5·03
230	7·27	6·94	6·64	6·38	6·16	5·94	5·75	5·42	5·14
235	7·43	7·09	6·78	6·52	6·28	6·07	5·88	5·54	5·26
240	7·59	7·24	6·93	6·66	6·42	6·20	6·00	5·66	5·37
246	7·78	7·42	7·10	6·82	6·58	6·35	6·15	5·80	5·50
252	7·97	7·60	7·29	6·99	6·74	6·51	6·30	5·94	5·64
258	8·16	7·78	7·45	7·16	6·90	6·66	6·45	6·08	5·77
264	8·35	7·96	7·62	7·32	7·06	6·82	6·60	6·22	5·90
270	8·54	8·14	7·79	7·49	7·22	6·97	6·75	6·36	6·04
276	8·73	8·32	7·97	7·66	7·39	7·13	6·90	6·51	6·17
282	8·92	8·50	8·14	7·82	7·55	7·28	7·05	6·65	6·31
288	9·11	8·68	8·32	7·99	7·80	7·44	7·20	6·79	6·44
294	9·30	8·86	8·49	8·16	7·96	7·59	7·35	6·93	6·57
300	9·49	9·05	8·66	8·32	8·02	7·75	7·50	7·07	6·71

TABLE XXVIII.—*Continued.*

Values of $C\sqrt{R}$	2 200 (43·45) ·02132	2 400 (47·49) ·02041	2 700 (53·55) ·01925	3 000 (59·61) ·01826	3 300 (65·67) ·01741	3 600 (71·73) ·01667	4 000 (79·81) ·01581	4 500 (89·91) ·01491	5 000 (99·101) ·01414
100	2.13	2.04	1.93	1.83	1.74	1.67	1.58	1.49	1.41
102	2.18	2.08	1.96	1.86	1.78	1.70	1.61	1.52	1.44
104	2.22	2.12	2.00	1.90	1.81	1.73	1.64	1.55	1.47
106	2.26	2.16	2.04	1.94	1.85	1.77	1.68	1.58	1.50
108	2.30	2.20	2.08	1.97	1.88	1.80	1.71	1.61	1.53
110	2.35	2.25	2.12	2.01	1.92	1.83	1.74	1.64	1.56
112	2.39	2.29	2.16	2.05	1.95	1.87	1.77	1.67	1.58
114	2.43	2.33	2.20	2.08	1.99	1.90	1.80	1.70	1.61
116	2.47	2.37	2.23	2.12	2.02	1.93	1.83	1.73	1.64
118	2.52	2.41	2.27	2.16	2.05	1.97	1.87	1.76	1.67
120	2.56	2.46	2.31	2.19	2.09	2.00	1.90	1.79	1.70
123	2.62	2.51	2.37	2.24	2.14	2.05	1.44	1.83	1.74
126	2.69	2.57	2.43	2.30	2.19	2.10	1.99	1.88	1.78
129	2.75	2.63	2.48	2.36	2.25	2.15	2.04	1.92	1.82
132	2.82	2.69	2.54	2.41	2.30	2.20	2.09	1.97	1.87
135	2.88	2.78	2.60	2.47	2.85	2.25	2.13	2.01	1.91
138	2.94	2.82	2.86	2.52	2.40	2.30	2.18	2.08	1.95
141	3.01	2.88	2.72	2.58	2.45	2.35	2.23	2.10	1.99
144	3.07	2.94	2.77	2.63	2.51	2.40	2.28	2.15	2.04
147	3.18	3.00	2.83	2.68	2.56	2.45	2.82	2.19	2.08
150	3.20	3.08	2.89	2.74	2.81	2.50	2.37	2.24	2.12
153	3.26	3.12	2.95	2.79	2.66	2.55	2.42	2.28	2.16
156	3.88	3.18	3.00	2.85	2.72	2.60	2.47	2.83	2.21
160	3.41	3.27	3.08	2.92	2.79	2.67	2.53	2.89	2.26
164	3.50	3.35	3.16	3.00	2.86	2.73	2.59	2.45	2.31
168	3.58	3.43	3.23	3.07	2.93	2.80	2.66	2.51	2.38
172	3.67	3.51	3.31	3.14	3.00	2.87	2.72	2.57	2.43
176	3.75	3.59	3.89	3.21	3.07	2.93	2.78	2.68	2.49
180	3.84	3.87	3.47	2.29	3.13	3.00	2.85	2.88	2.55
185	3.95	3.78	3.56	3.88	3.22	3.08	2.93	2.76	2.62
190	4.05	3.88	3.66	3.47	3.31	3.17	3.00	2.83	2.69
195	4.16	3.98	3.75	3.56	3.50	3.25	3.08	2.91	2.76
200	4.26	4.08	3.85	3.55	3.48	3.33	3.16	2.98	2.83
205	4.37	4.18	3.95	3.74	3.57	3.42	3.24	2.06	2.90
210	4.48	4.29	4.04	3.84	3.66	3.50	3.32	3.13	2.97
215	4.58	4.39	4.14	3.93	3.74	3.58	3.40	3.21	3.04
220	4.69	4.49	4.24	4.02	3.83	3.67	3.48	3.28	3.11
225	4.80	4.59	4.33	4.11	3.92	3.75	3.56	3.36	3.18
230	4.90	4.69	4.43	4.20	4.00	3.83	3.64	3.43	3.25
235	5.01	4.80	4.52	4.29	4.09	3.92	3.72	3.50	3.32
240	5.12	4.90	4.62	4.38	4.19	4.00	3.79	3.68	3.39
246	5.25	5.02	4.74	4.49	4.28	4.10	3.89	3.67	3.48
252	5.37	5.14	4.85	4.60	4.39	4.20	3.99	3.76	3.56
258	5.50	5.27	4.97	4.71	4.49	4.30	4.08	3.85	3.65
264	5.63	5.39	5.08	4.82	4.60	4.40	4.17	3.94	[illegible]
270	5.76	5.51	5.20	4.93	4.70	4.50	4.27	4.03	3.82
276	5.88	5.63	5.31	5.04	4.81	4.60	4.36	4.12	3.90
282	6.01	5.76	5.43	5.15	4.91	4.70	4.46	4.20	[illegible]
288	6.14	5.88	5.51	5.26	5.01	4.80	4.55	4.29	4.07
294	6.27	6.00	5.66	5.37	5.12	4.90	4.65	4.38	4.16
300	6.40	6.12	5.78	5.48	5.22	5.00	4.74	4.47	4.21

TABLE XXVIII —*Continued*

Values of $C\sqrt{F}$	5 500 (108 112) ·01349	6 000 (118 1[illegible]) ·01[illegible]1	6 500 (1[illegible]8 13[illegible]) ·01240	7 000 (138 142) ·01195	7 500 (148 15[illegible]) ·01155	8 000 (158 162) ·01118	8 500 (16[illegible] 1[illegible]3) ·01085	9 000 (177 183) ·01054	10 000 (1[illegible] 203) ·0100
100	1 35	1 29	1 24	1 20	1 16	1 12	1 09	1 05	1 00
102	1 38	1 32	1 47	1 22	1 18	1 14	1 11	1 08	1 02
104	1 40	1 34	1 29	1 24	1 20	1 16	1 13	1 10	1 04
106	1 43	1 37	1 32	1 27	1 22	1 19	1 15	1 12	1 06
108	1 46	1 39	1 34	1 29	1 25	1 21	1 17	1 14	1 08
110	1 48	1 42	1 36	1 32	1 27	1 23	1 19	1 16	1 10
112	1 51	1·45	1 39	1 34	1 29	1 25	1 22	1 18	1 12
114	1 54	1 47	1 41	1 36	1 32	1 27	1 23	1 20	1 14
116	1 57	1 50	1 44	1 39	1 34	1 30	1 26	1 22	1 16
118	1 59	1 52	1 46	1 41	1 36	1 32	1 28	1 24	1 18
120	1 62	1 55	1 49	1 43	1 39	1 34	1 30	1 27	1 20
123	1 66	1 59	1 53	1 47	1 42	1 38	1 34	1 30	1 23
126	1 70	1 63	1 56	1 51	1 46	1 41	1 37	1 33	1 26
129	1 74	1 67	1 60	1 54	1 49	1 44	1 40	1 36	1 29
132	1 78	1 70	1 64	1 58	1 53	1 48	1 43	1 39	1 82
135	1 83	1 74	1 66	1 61	1 56	1 51	1 47	1 42	1 85
138	1 86	1 78	1 71	1 65	1 59	1 54	1 50	1 45	1 88
141	1 90	1 82	1 75	1 69	1 63	1 58	1 53	1 49	1 41
144	1 04	1 86	1 79	1 72	1 66	1 61	1 56	1 52	1 44
147	1 98	1 90	1 82	1 76	1 70	1 64	1 60	1 56	1 47
150	2 02	1 94	1 86	1 79	1 73	1 68	1 63	1 58	1 50
153	2 06	1·98	1 90	1 83	1 77	1 71	1 66	1 61	1 53
156	2 11	2·01	1 93	1 87	1 80	1 74	1 69	1 64	1 56
160	2 16	2 07	1 98	1 91	1 85	1 79	1 74	1 69	1 60
164	2 21	2 12	2 03	1 96	1 89	1 83	1 78	1 78	1 64
168	2 2[illegible]	2 17	2 08	2 01	1 94	1 88	1 82	1 77	1 68
172	2 82	2 22	2 13	2·06	1 09	1 92	1 87	1 81	1 72
176	2 87	2 27	2 18	2 10	2 03	1 97	1 91	1 86	1 76
180	2 43	2 32	2 23	2 15	2 08	2 01	1 95	1 90	1 80
185	2 50	2 39	2 80	2 21	2 14	2 07	2 01	1 95	1 85
190	2 56	2 45	2 36	2 27	2 20	2 12	2 06	2 00	1 90
195	2 63	2 52	2 42	2 33	2 25	2 18	2 12	2 06	1 95
200	2 70	2 58	2 48	2 39	2 31	2 24	2 17	2 11	2 00
205	2 77	2 65	2 54	2 45	2 37	2 29	2 22	2 16	2 05
210	2 83	2 71	2 60	2 51	2 43	2 35	2 28	2 21	2 10
215	2 90	2 78	2 67	2 57	2 48	2 40	2 33	2 27	2 15
220	2 97	2 84	2 73	2 63	2 54	2 46	2 39	2 32	2 20
225	2 04	2 91	2 79	2 69	2 60	2 52	2 44	2 37	2 25
230	3 10	2 97	2 85	2 75	2 66	2 57	2 50	2 42	2 30
235	3 27	3 03	2 91	2 81	2 72	2 63	2 55	2 48	2 35
240	3 24	3 10	2 98	2 87	2 77	2 68	2 60	2 53	2 40
246	3 32	3 18	3 05	5 94	2 84	2 75	2 67	2 59	2 46
252	3 40	3 25	3 13	3 01	2 91	2 82	2 74	2 66	2 52
258	3 48	3 33	3 20	3 08	2 98	2 88	2 80	2 72	2 58
264	3 56	3 41	3 27	3 16	3 05	2 95	2 86	2 78	2 64
270	3 64	3 48	3 35	3 23	3 12	3·02	2 93	2 85	2 70
276	3 72	3 56	3 42	3 30	3 19	3 09	3 00	2 91	2 76
282	3 80	3 64	3 50	3 37	3 26	3 15	3·06	2 97	2 82
288	3 89	3 72	3 57	3 44	3 33	3 22	3 13	3 04	2 88
294	3 97	3 80	3 65	3 51	3 40	3 29	3 19	3 10	2 94
300	4 05	3 87	3 72	3 59	3 47	3 35	3 26	3 16	3·00

CHAPTER VI

OPEN CHANNELS—UNIFORM FLOW

[For preliminary information see chapter ii articles 8 16 and 22 24]

Section I —Open Channels in General

1 **General Remarks** —Uniform flow can take place only in a uniform channel Strictly speaking, a uniform channel is one which has a uniform bed slope, and all its cross sections equal and similar, but if the cross sections, though differing somewhat in form, as in Fig 98, are of equal areas and equal wet borders, the channel is to all intents and purposes uniform, provided the form of the section changes gradually The term 'uniform channel' will be used in this extended sense [1] Breaches of uniformity in a channel may be frequent, and the reaches in which the flow is variable may be of great length The flow in a uniform channel is thus by no means everywhere uniform Bends are for convenience treated of in chap vii, but flow round a bend may be uniform Thus a uniform stream need not be assumed to be straight It will be seen hereafter (chap vii art 16) that nearly everything which is true for uniform flow is true, with some modifications, for variable flow

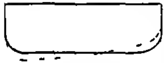

Fig 98

The mean depth D (Fig 99) of a stream is the sectional area A divided by the surface width W' Since $A = DW' = RB$, therefore the hydraulic radius is less than the mean depth in the same ratio as the surface width is less than the border This will often assist in forming an idea of the hydraulic radius The greater the width of a stream in proportion to its depth, and the

Fig 99

[1] If R varies in the opposite manner to S the flow may be uniform in a variable channel, but this is very rare

fewer the undulations in the border, the more nearly will the surface-width approach to the border and the hydraulic radius to the mean depth If the depth of water in a channel alters, the hydraulic radius alters in the same manner When the water-level rises A increases faster than W, and R therefore increases, but $\frac{W}{B}$ decreases (unless the side slopes are flat), so that R increases less rapidly than D For small changes of water level R and D both change at about the same rate

2 **Laws of Variation of Velocity and Discharge**—For orifices, weirs, and pipes it was possible to describe in a few words the general laws according to which the velocities and discharges vary, but for open stream it is not so One law is simple, and that is, that for any channel whatever V and Q are nearly as $\sqrt{S}$ To double V or Q it is necessary to quadruple S For other factors it is necessary to consider the shape of the cross section

For a stream of 'shallow section,' that is, one in which W greatly exceeds D, a change in W has hardly any effect on R or on V, while Q is directly as W Also R is very nearly as D For depths not very small C is approximately as $D^{\frac{1}{6}}$, so that V is as $D^{\frac{2}{3}}$ In this case, if D is doubled, V is increased in the ratio 1 59 to 1 On comparing velocities, taken from tables, for channels from 8 to 300 feet wide with sides vertical, or 1 to 1, and with various velocities, the actual ratio is found to vary from 1 52 to 1 73 If the sides are steep A is nearly as D, and Q therefore as $D^{\frac{5}{3}}$ or thereabouts For a stream of 'medium section'—that is, one in which W is 2 to 6 times D—with vertical sides A is as D, and for moderate changes of water level and depths not very small V is nearly as $D^{\frac{1}{2}}$, so that Q is as $D^{\frac{3}{2}}$ Both these kinds of section are extremely common A flattening of the side slopes may make Q vary as D^2 If a stream has vertical sides and a depth far exceeding its width—a rare case—the effects of W and D are reversed For a triangular section—used for small drains—R is as D, A as D^2, C probably as $D^{\frac{1}{2}}$, and Q as D^3

For other kinds of section no definite laws can be framed, but the effect of D is nearly always greater than that of W, so that D is the most important factor in the discharge, especially if the side slopes are flat, and S is always the least important factor

If two streams have equal discharges, and have one factor in the discharge equal, the approximate relation between the other two factors can be found Let two streams of shallow section have equal slopes, and let one be twice as deep as the other The

latter must be $(2)^{\frac{5}{3}}$ or 3 2 times as wide as the first This law is nearly the same as for weirs When two reaches of a canal have different bed slopes, but equal and similar cross sections, the depth of water is, of course, less in the reach of steeper slope If the discharge is approximately as $S^{\frac{1}{2}}D^{*}$, the depths in the two reaches will be inversely as the fourth roots of the slopes The velocities are inversely as the depths, and are, therefore, as the fourth roots of the slopes A change of 40 per cent in the slope will cause a change of only about 10 per cent in the velocity, and a change of the same proportion, but of opposite kind, in the depth of water When the changes in the two factors are relatively small they are inversely as the indices in the formulæ Suppose a stream of shallow section with depth D and slope S gives a certain discharge Q Let D be increased by a small amount $\frac{D}{n}$ Then the compensatory change in S will be $\frac{3S}{n}$ This principle may be applied in designing a channel to carry a given discharge, whenever for any reason it becomes necessary to make a slight change in the value first assumed for any factor

The discharging power of a stream can be increased by increasing the depth of water, the width or the slope, the last being often effected by cutting off bends The efficiencies of these processes are in the order named In any channel having sloping sides both V and Q are more increased by raising the surface level than by deepening the bed by the same amount It follows that embanking a river is more effective than deepening it for increasing its discharging power and enabling it to carry off floods It is in fact the most effective plan that can be devised

In clearing out the head reaches of Indian inundation canals—so called because they flow only for a few months, when the rivers are swollen—it used to be the custom to place the bed rather high, at the off take, in order to obtain a good slope Of late years it has been the custom to lower the bed giving a flatter slope but a greater depth of water The velocity is about the same in both cases, the increase in depth making up for the decrease in slope, but the lowered bed of course gives a greatly augmented discharge On the other hand, the lowered bed must cause the introduction of water more heavily charged with silt Moreover, the ratio of depth to velocity in the canal is greater than before, and this (chap II art 22) tends to cause increased deposit Under the old system of high beds the heads of the canals silted more or less It has been impossible to find out whether more silt has actually

deposited since the introduction of the low level system, because, owing to changes in the course of the river, the same head channel is seldom cleared for several years in succession, and also because the quantity of silt deposited depends on other factors, such as the position of the head, a canal taken off from the highly silt laden main stream silting more than one taken from a side channel Although the tendency of the low bed is to silt more than the high one, but the worst that can happen is its silting up till it assumes the level of the high one This takes time, and while it is going on an increased discharge is obtained

Section II.—Special Forms of Channel

3 **Section of 'Best Form.'**—A stream is of the 'best form' when for a given sectional area the border is a minimum, and the hydraulic radius, therefore, a maximum The velocity and discharge are greater than in any other stream of the same sectional area, slope, and roughness The form which complies with this condition is a semicircle whose diameter coincides with the line of water surface This form is used in concrete channels but not often in others, because of the difficulty of constructing curved surfaces Of rectilineal figures the best form is half a regular polygon. The greater the number of sides the better, but in practice the form of section is usually restricted to that having a bed level across and two sides vertical or sloping The best form for vertical sides is the half square (Fig 100), and for sloping sides the semi hexagon (Fig 101) If the angle of the side slopes is fixed (as it generally is) at some angle other than 60°,

Fig 100

Fig 101

Fig 102

the best form is that in which the bed and sides are all tangents to a semicircle (Fig 102) The bed width is $D(\sqrt{n^2+1}-n)$ where n is the ratio of the side slopes In every channel of the best form the hydraulic radius is half the depth of water, and if the section is rectilineal, the surface width is equal to the sum of the two slopes, so that the border is the sum of the surface and bed widths

The following statement shows the sectional areas of various channels of the best form All the channels have the same central depth D, the same hydraulic radius $\frac{D}{2}$ and therefore the same velocity

Description of Cross section	Sectional Area	Ratio of Sectional Area to that of the Inscribed Semicircle
Semicircle,	$1\ 57\ D^2$	1 00
Half square,	$2\ D^2$	1 27
Semi hexagon,	$1\ 732 D^2$	1 10
Trapezoid, side slopes ½ to 1,	$1\ 736 D^2$	1 11
,, , 1 ,, 1,	$1\ 818 D^2$	1 16
,, ,, 1½ ,, 1,	$2\ 106 D^2$	1 34
,, ,, 2 , 1	$2\ 472 D^2$	1 57
, ,, 3 , 1,	$3\ 334 D^2$	2 12

A channel of the best form is not usually the cheapest If made of iron, wood, or masonry the cost will probably be reduced by somewhat increasing the width and reducing the depth, thereby enabling the sides to be made lighter, though the length of border is slightly increased In an excavated channel, where the water surface is to be at the ground level, the best form will give the minimum quantity of work and will be the cheapest if the material excavated is rock, but if it is earth an increase of width and decrease of depth will reduce the lift of the earth, and therefore the cost If the water surface is not to be at the ground level the cheapest form may differ greatly from the best form

If it is desired simply to deliver a stream of water of given discharge with as high a velocity as possible, the best form is suitable If the object is to obtain high silt-supporting power, so that the channel may not silt or may scour and enlarge itself, the question of ratio of depth to velocity must be taken into account, and even when the object is to discourage the growth of weeds the question of depth comes in

If the depth of water in a channel fluctuates, the section can, of course, be of the best form for only one water level Sewers are often made of oval sections in order that the stream may be of the best form, or nearly so, when the water level is low, the object being to prevent deposits In Fig 103 (Metropolitan Ovoid) the radius of the invert is half that of the crown, and in Fig 104 (Hawkesley's Ovoid) nearly three-fifths There is also a form known as Jackson s Peg top Section In each case the velocity with the sewer one third full is about three fourths of the velocity when it is two thirds full.

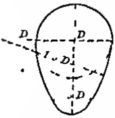

Fig 103

Fig 104

4 *Irregular Sections*—The cross section of a stream may be called 'irregular' when the border contains undulations or saliences of such a character as to divide the section into well marked divisions (Fig 105) In this case the water in each division has a velocity of its own, and in order to calculate the discharge of the whole stream by the use of the formula $V=C\sqrt{RS}$, it is necessary to consider each division separately, finding its hydraulic radius from its area and border The length AB is not included in the border of either division, since if there is any friction along it, it accelerates the motion in one division and retards it in the other If A_1 A_2 are the sectional areas, and R_1 R_2 the hydraulic radii,

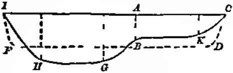

Fig 105.

$$Q_1=C_1A_1\sqrt{R_1S}$$
$$Q_2=C_2A_2\sqrt{R_2S}$$

The discharge of the whole channel calculated from the equation $Q=CA\sqrt{RS}$, equals Q_1+Q_2 only when $R_1=R_2$, otherwise it is less The more R_1 and R_2 differ, the more Q differs from Q_1+Q_2, and for given values of R_1 and R_2 the difference is greatest when $A_1=A_2$ If either A_1 or A_2 is relatively very small, the difference between Q and Q_1+Q_2 will be small It may happen that R_1 and R_2 differ greatly with low supplies, and not much with high supplies If without altering either the length of the border or the sectional area of the stream the border be changed to $CDEF$, the section is no longer irregular, and the equation $V=C\sqrt{RS}$ is the proper one to use There are thus two cross sections with equal values of R and different mean velocities, that is, different values of C Even in a regular section the same principle holds good The discharge is the sum of the discharges of a number of parts and may be affected by a change in the form of the border alone (See also art 13)

An instance of an irregular section occurs when a stream over flows its banks (Fig 106) As the overflow occurs the border of the whole stream may increase far more rapidly than the sectional area, and Q, if calculated as a whole, would diminish with rise of the water-level The velocity and discharge of the main

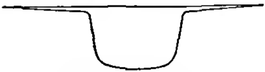
Fig 106.

body and of the overflow must be considered separately, and both will increase as the water level rises Similarly, if there are longitudinal grooves or ruts in the bed of a stream, such, for instance, as those caused by longitudinal battens, the water in the grooves has a separate velocity of its own and the velocity of the main body cannot be reduced indefinitely by increasing the number and depth of the grooves, although the border can be increased in this manner to any extent If the river is winding, the spill water, which flows straight, may have a slope greater than that in the river channel, but its velocity may still be very low, especially if the country is covered with crops or vegetation Some of the spill water, however, disappears by absorption, and it is clear that in every case it takes off some of the discharge of the river Thus the embanking of a river, so as to shut off spills, must necessarily, to start with, raise the flood level Whether scour of the channel subsequently reduces the level is another matter

5 Channels of Constant Velocity or Discharge—Let A be the area, B the border, and W the surface width of any stream whose water level is RS (Fig 107), and let the water level rise to TU, the increase in depth being a small quantity d and the increase in the surface width being $2w$ Then if the slopes RT, SU be made such that

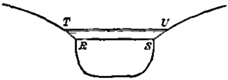

FIG 107

$\dfrac{(W+w)d}{2\sqrt{d}+w}=\dfrac{A}{B}$, the border will have increased in the same ratio as the area, and R will be unaltered By using the new values of A and B, corresponding to the raised surface, the process can be continued, but the slope becomes rapidly flatter If the *surface falls below RS, R is no longer constant, but decreases* It is impossible to design a section such that R will remain constant as the depth decreases to zero And even within the limits in which R is constant, the *mean velocity* is not constant The channel is irregular, and the velocity, both in the main body of water and in the minor ones, increases as the water level rises The investigations which have at times been made to find the equation to the curve of the border when R is constant are useful only as mathematical exercises

The velocity as the water level rises is *nearly constant* in a very deep, narrow channel with vertical sides, and it may be kept

quite constant by making the sides overhang—as in a sewer running nearly full—but the process is speedily terminated by the meeting of the two sides

To keep the discharge constant for different water levels is still more difficult, but would be of great practical use, especially in irrigation distributaries It could be effected by making the sides overhang, but they would have to project almost horizontally and would very soon meet, thus giving only a small range of depth Any form of section adopted for giving either constant velocity or constant discharge must be continuous along the channel from its head for a great distance If of short length the slope or hydraulic gradient in it would be liable to vary greatly, and with it the velocity (Cf chap ii art 14)

6 **Circular Sections**—A channel of circular section is an open channel when it is not running full In such a channel the hydraulic radius, and therefore the velocity, is a maximum when the angle subtended by the dry portion of the border is 103°, or the depth is 81 of the full depth If the depth is further increased R decreases, but at first the increase of area more than compensates for this, and the discharge goes on increasing When the angle above mentioned is about 52°, or the depth is 95 of the full depth, the expression $AC\sqrt{R}$ is a maximum, and Q is then about 5 per cent more than when the channel is flowing full

SECTION III—RELATIVE VELOCITIES IN CROSS SECTION

7 **General Laws**—Except near abrupt changes the water at every point of a cross section of a stream has its chief velocity parallel to the axis of the stream and in the direction of flow, and the velocity varies gradually from point to point Although the velocity at any point in a cross section is affected to some extent by its distance from every part of the border, it depends chiefly on its distance from that part of the border which is near to it Those portions of the border which are remote from the point have a small, often an inappreciable effect In

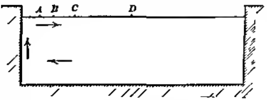

FIG 108.

Fig 108 the velocity at A is less than at B because of the effect of the neighbouring side At all points between C and D the

velocities are nearly equal because both sides are remote Given the cross section of a stream, the forms of the velocity curves are known in a general way but not with accuracy In other words, their equations are not known

The law that the velocity is greatest at points furthest from the border is subject to one important exception The maximum velocity in any vertical plane parallel to the axis of the stream is generally at a point somewhat below the surface and not at the surface If D is the depth of water and D_m the depth of the point of maximum velocity, the ratio $\frac{D_m}{D}$ in a stream of shallow section at points not near the sides may have any value from zero to 30, and if the side slopes are not steep the same ratio may be maintained right across the channel When the sides are very steep or vertical the ratio $\frac{D_m}{D}$ close to the side is about 50 or 60, and it decreases towards the centre of the stream, attaining its normal value in a shallow section at a distance from the side equal to about $2D$ or $2 5D$, and thereafter remains constant or nearly so

The depression of the maximum velocity has been sometimes attributed to the resistance of the air, but this theory is now quite discredited Air resistance could cause only a very minute depression, and it cannot account for the variation of the depression at different parts of a cross section It is true that wind acting on waves and ripples may produce some effect The water level in the Red Sea at Suez is raised during certain seasons of the year when the wind blows steadily up the Red Sea On the Mississippi, with depths ranging from 45 to 110 feet, an upstream wind was found to reduce the surface velocity and increase the ratio $\frac{D_m}{D}$ A downstream wind produced opposite effects, but even with a downstream wind the maximum velocity was below the surface, and the same thing has been observed elsewhere Wind acting on ripples is a different thing to simple air resistance The depression is attributed by Thomson to the eddies which rise from the bed to the surface The water of which the eddies are composed is slow moving, and though the eddies retard the velocity at all points which they traverse, they have most effect at the surface, because they spread out and accumulate there This explanation seems to be the true one at least as regards the central portions of a stream *When no*

depression exists there, it is because the eddies are weak relatively to the other factors The increased depression of the maximum velocity near the sides when these are steep or vertical is clearly connected with certain currents which circulate transversely in a stream Near the side there is an upward current (Fig 108), at least in the upper portion of the section, and there is a surface current from the side outwards It is this current which causes floating matter to accumulate in mid stream At a lower level there must be an inward current which brings quick moving water towards the sides, while the slow moving water near the surface travels outwards and reduces the surface velocity at all points which it reaches

As to the cause of the currents, Stearns, who has investigated the subject,[1] considers that they are due to eddies produced at the sides The eddies from the side tend on the average to move at right angles to it, but they also tend to move chiefly in the direction of the least resistance, that is, towards the surface

8 Horizontal Velocity Curves—A horizontal 'mean velocity curve' is one whose ordinates are the mean velocities on different verticals extending from surface to bed The general forms of these curves for a rectangular section are shown in Fig 109 for two water levels When the section is shallow the velocities on different verticals, at a distance from the side exceeding $2D$ or $3D$, become nearly equal Fig 110 shows a channel with *sloping sides* The length in which the velocity is practically constant is somewhat greater than before, and the curves in this portion are nearly as before, but the part in

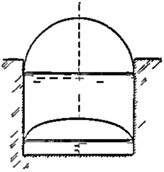

Fig 109

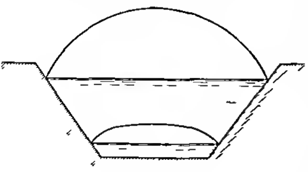

Fig 110

[1] *Transactions of the American Society of Civil Engineers*, vol XII.

which the velocity varies is longer, both actually and relatively to the whole width If the bed is not level across (Fig 105, p 159) the velocity is greater where the depth is greater If there are, at a distance from the sides, divisions of considerable width and constant depth, as *HG* and *BK*, the velocity in each such division is nearly constant The rough rule for a channel of shallow section considered as a whole, that V is approximately as $D^{\frac{2}{3}}$ where D is the mean depth, probably applies to any two divisions such as those under consideration and to the same division for different water levels But if a division is of small width its velocity is affected by those adjoining it Thus the velocity curve is one which tones down the irregularities of the bed On the South American rivers with depths of 9 to 73 feet, gradually increasing from the bank to the centre of the stream, Revy found the velocity to vary as D^n where n is greater than unity This may be the law in very deep rivers, but Revy's observations were not numerous, and in most of them the flow was unsteady owing to tidal influences Where the change of depth is small the variation of $D^{\frac{2}{3}}$ is not very different from that of D The form of the velocity curves in a channel of irregular sections changes, as it does in regular channels, with the water level Irregularities which have a marked effect at low water may have no perceptible effect at high water

The nature of the horizontal mean velocity curve depends on the shape of the cross section, and not on its size From observations made by Bazin on small artificial channels lined with plaster, plank, or gravel, with widths of about 6 5 feet, and depths up to 1 5 feet, and observations made by Cunningham on the Ganges Canal in an earthen channel about 170 feet wide and 5 feet deep, and in a masonry channel 85 feet wide, with depths of 2 feet to 3 5 feet, it is also proved that if the velocity is altered by altering the surface slope (and in the case of Bazin's channels by altering the roughness), the velocities on different verticals all alter in about the same proportion It is probable, considering the complications arising from eddies and transverse currents, that the actual size of the channel has some effect, but it is negligible, at least in streams of shallow section, and under the conditions which occur in practice

Let U be the mean velocity on the central vertical, and V that in the whole cross section Let $\frac{V}{U} = a$ The values of the coefficient a are as follows —

Ratio of mean width to depth, .	1	1·5	2	3	4	5	6	7	10	20	30	50	90
Value of a,	·86	·87	·88	·89	·90	·91	·92	·93	·94	·95	·96	·97	·98

These co-efficients are applicable to rectangular and trapezoidal channels, but may not be very accurate for the latter when the ratio of the mean width to the depth is small, especially if the side slopes are flat. In other cases they are probably correct to within 1 or 2 per cent. for the deeper sections, and to within 5 per cent. for shallower sections. The co-efficients have been found chiefly from the observations above mentioned. Bazin did not work out this particular co-efficient, but his figures enable it to be found. In any particular channel the co-efficient increases as the water level falls.

The co-efficient *a* was determined in the observations on the Solani aqueduct in the Roorkee experiments. In the aqueduct there is a central wall which divides the canal into two channels, each 85 feet wide. The aqueduct is 932 feet long, and the observations were made in the middle, that is, only 466 feet from the upper end. Upstream of the aqueduct the canal consists of one undivided channel, and the greatest velocities are in the centre. Owing to this fact the maximum velocities at the observation sites in the aqueduct at times of high supply are not in the centres of the channels, but nearer the central wall. The velocities observed to determine *a* were, however, made in the centres of the channels, and the resulting values of *a* were therefore too high. The depth varied from 4 to 10 feet, and the ratio of width to depth therefore from 21 to 8·5. The values of *a* were nearly constant at ·95 or ·96. For the lower depths the co-efficient agrees with that in the above table. For the higher depths it was overestimated for the reason just given. (See chap. ii. art. 21.)

The co-efficients are strictly applicable only when the bed, as seen in cross section, is a straight and horizontal line, but practically they are applicable whenever the central depth is the mean depth (not counting the sections over the side slopes), and does not differ much from the others. If there is a shallow in the centre the co-efficient may exceed 1·0, and may increase greatly at low water. For some particular sections somewhat hollow in the centre the co-efficient may not vary as the water level changes.

The above refers to horizontal mean velocity curves. The properties of horizontal curves at particular levels, for instance at the surface, mid-depth, or bed, are, generally speaking, similar to the above. In the central portions of the stream the curves are

probably all parallel projections of one another Near to vertical or very steep sides, owing to the greater depression of the line of maximum velocity, the mid depth velocity curve, and to some extent the bed velocity curve, become more protuberant and the surface curve less so Fig 111 shows the distribution of velocities

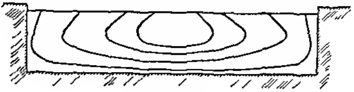

FIG 111

found by Bazin in a channel 6 feet wide and 1 5 feet deep, lined with coarse gravel Each line passes through points where the velocities are equal

9 **Vertical Velocity Curves**—The general forms of the curves are shown in Figs 112 and 113[1] Many attempts have been made

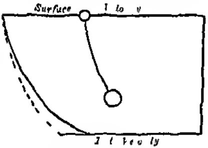

FIG 112

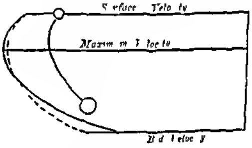

FIG 113

to find the equations to the curves and it is sometimes said that the curve is a parabola with a horizontal axis corresponding to the line of maximum velocity This is improbable The transverse curve is certainly not a parabola The bed of a channel retards the flow in the same manner as the side retards it and the velocity probably decreases very rapidly close to the bed just as it does close to a vertical or steep bank Except near the bed almost any geometric curve can be made to fit the velocity curve The equation to the curve is not nearly of so much practical importance as the ratios of the different velocities to one another If these are known, the observation of surface velocities enables the bed velocities and mean velocities to be ascertained A slight difference in the ratios may make a great difference in the equation Even the *information regarding the ratios* is very imperfect, and

[1] The floats and dotted lines are referred to in chap vii

until it is improved it is useless to discuss the equation When the depths on adjoining verticals are not equal, the curves are probably of a highly complex nature, since each must influence those near it

Let U_s, U_m, U, and U_b be the surface, maximum, mean and bed velocities on any vertical not near a steep side of a channel, then the ratios which are of most practical importance are those of D_m to D, and of U to each of the other velocities The results as to these ratios furnished by experiments show great discrepancies The fact seems to be that the ratios are easily disturbed A change in depth, roughness, or surface slope may cause the eddies to rise in greater or less proportion, and so alter the ratios The quantity of silt or drift perhaps affects them, since some of the work of the eddies is expended in lifting or moving the materials Wind may affect the surface velocities and unsteadiness in the flow may affect the ratios The depth D_m is seldom accurately observed This is because the velocities above and below the line of U_m differ very slightly from U_m, and also because the velocities are not generally observed at close intervals A greater defect is in the observation of bed velocities They are seldom observed really close to the bed When so observed a rapid decrease of velocity has been noticed

Generally the different ratios roughly follow one another When the eddies reach the surface in greater proportion the ratio $\frac{D_m}{D}$ increases At the same time U_m is diminished and U_b is increased, because more quickly moving water takes the place of that which rises. Thus the different velocities tend to become equal and the ratios to approach unity It will be sufficient to consider for the present only the ratios $\frac{D_m}{D}$ and $\frac{U}{U_s}$ On examining the results of experiments no clear connection between these ratios and the quantities U and D is apparent, but by considering the two separate elements on which, for any given depth, U depends, namely N and S, some more definite, though not very satisfactory results are obtained The following table contains an abstract of the results of some of the chief observations Each group consists generally of several series, each series having a separate value of D and U, and sometimes of N or S The table is a mere abstract, and is intended to show only what experiments have been considered and their general results On the Mississippi and Irrawaddy and Ganges Canal the observations were made with

Abstract of Results of Observations on Verticals not near the Sides of the Channels

Serial Number of Group	Channel	Observer	Depth Roughness and Velocity on Vertical			Ratios	
			D	N	U	$\frac{D_n}{D}$	$\frac{U}{U_n}$
Division I — Great Rivers							
1	Mississippi	Humphreys and Abbott	76	027	3 5	38	98
2	,,	,,	79	027	2 1	13	94
3	,,	,,	65	031	5 3	27	97
4	,,	,,	27	025	4 7	28	97
5	Irrawaddy	Gordon	50		5 4	03	95
6	,,	,,	29		1 8	zero	93
7	Parana de las Palmas	Revy	50		2 4	zero	83
8	La Plata	,,	24		1 3	zero	69
Division II — Ordinary Streams							
9	Saone	Leveille	14	028	2 2	15	90
10	Garonne	Baumgarten	11	0275	5 0	10	90
11	Seine	Emmery	9	026	2 5	05	89
12	Rhine	International Commission	7	030	7 1	zero	85
13	Branch of Rhine	Defontaine	5	0275	3 5	zero	87
14	Ganges Canal	Cunningham	0	025	3 5	12	88
15	,,	,,	6 5	013	4 2	19	93
Division III — Small Streams							
16	Artificial Channels	Bazin	1 3	020	5 9	05	84
17	,,	,,	1 1	015	6 6	zero	89
18	,,	,,	1	012	6 5	zero	91
19	,,	,,	9	010	9 1	zero	92

the double float, and the ratio $\frac{U}{U_m}$ was thus seriously vitiated (chap viii art 9), the values of U obtained being too high On the Ganges Canal U was, however, observed separately by means of rod floats, and by making certain corrections for the length of rod used, corrected values of U have been found and used In Revy's observations the flow was unsteady

By considering the figures of each separate series in divisions

II and III it is quite clear that the ratio $\frac{U}{U_m}$ increases as N decreases This result had previously been found by Bazin for his small channels It also seems probable that the ratio generally increases with the depth In division I the figures are unreliable, as above explained, but to some extent they confirm the above laws From a consideration of the various results the following table has been prepared, but the figures given are only probable and approximate The only law that seems to be well established is that of change of the ratio with change of N, the rest being somewhat doubtful The figures are, however, an advance on the present rough rule that the ratio is '85 to 90' The blanks in the table may be filled in according to judgment In some small and rough channels the ratio has been found to be as low as 60 The ratio $\frac{U}{U_s}$ may be designated β

Probable Ratios of Mean to Surface Velocities $\left(\beta \text{ or } \frac{U}{U_s}\right)$ on Verticals not near the Sides of a Channel.

Depth on Vertical.	Values of N								
	030	·0 5	0·0	·0 25	·0 0	·01 5	015	013	010
Feet									
90					83	86	88	89	91
1 10					84	87	89	90	91
1·25					85	87	89	·91	91
1 50					87	88	90	91	92
2 00			87						
3 00			88						
5·0	85	87	89					93	
7 0			90						
10 0	86	89	90					·92	
13·0		91	89						
18 0	89	92	88		89		90	·91	
23·0		93							
28 0		95							

After the preparation of the above table for depths up to 18 feet the author's attention was drawn to an extensive and careful series of observations made with current-meters by Marr on the Mississippi[1] The results worked up and abstracted are as follows —

[1] Report on Current meter Observations in the Mississippi near Burlington

Abstract of Results of Observations on Verticals not near the Sides of the Channels

Serial Number of Group	Channel	Observer	Depth Roughness and Velocity on Vertical			Ratios	
			D	N	L	$\frac{D_n}{D}$	$\frac{L}{L_m}$
	Division I—Great Rivers						
1	Mississippi	Humphreys and Abbott	76	027	3 5	38	98
2	,,	,,	79	027	2 1	13	94
3	,,	,,	65	031	5 3	27	97
4	,,	,,	27	025	4 7	28	97
5	Irrawaddy	Gordon	50		5 4	03	95
6	,,	,,	29		1 8	zero	93
7	Parana de las Palmas	Revy	50		2 4	zero	83
8	La Plata	,,	24		1 3	zero	69
	Division II—Ordinary Streams						
9	Saone	Leveille	14	028	2 2	15	90
10	Garonne	Baumgarten	11	0275	5 0	10	90
11	Seine	Emmery	9	026	2 5	05	89
12	Rhine	International Commission	7	030	7 1	zero	85
13	Branch of Rhine	Defontaine	5	0275	3 5	zero	87
14	Ganges Canal	Cunningham	9	025	3 5	12	88
15	,,	,,	6 5	013	4 2	19	93
	Division III—Small Streams						
16	Artificial Channels	Bazin	1 3	020	5 9	05	84
17	,,	,,	1 1	015	6 6	zero	89
18	,,	,,	1	012	6 5	zero	91
19	,,	,,	9	010	9 1	zero	92

the double float, and the ratio $\frac{U}{U_m}$ was thus seriously vitiated (chap viii art 9), the values of U obtained being too high On the Ganges Canal U was, however, observed separately by means of rod floats, and by making certain corrections for the length of rod used, corrected values of U have been found and used In Revy's observations the flow was unsteady

By considering the figures of each separate series in Divisions

n and m it is quite clear that the ratio $\frac{U}{U_m}$ increases as N decreases This result had previously been found by Bazin for his small channels It also seems probable that the ratio generally increases with the depth In division 2 the figures are unreliable, as above explained, but to some extent they confirm the above laws From a consideration of the various results the following table has been prepared, but the figures given are only probable and approximate The only law that seems to be well established is that of change of the ratio with change of N, the rest being somewhat doubtful The figures are, however, an advance on the present rough rule that the ratio is '·85 to ·90' The blanks in the table may be filled in according to judgment In some small and rough channels the ratio has been found to be as low as ·60 The ratio $\frac{U}{U_s}$ may be designated β

PROBABLE RATIOS OF MEAN TO SURFACE VELOCITIES $\left(\beta \text{ OR } \frac{U}{U_s}\right)$ ON VERTICALS NOT NEAR THE SIDES OF A CHANNEL.

Depth on Vertical.	Values of N								
	·030	·0275	·0250	·0225	·020	·0175	·015	·013	·010
Feet.									
·90					83	86	88	89	91
1·10					84	87	89	90	91
1·25					85	87	89	91	91
1·50					87	88	90	91	92
2·00			87						
3·00			88						
5·0	85	87	89					93	
7·0			90						
10·0	86	89	90					92	
13·0		91	89						
18·0	89	92	88		89		90	·91	
23·0		93							
28·0		95							

After the preparation of the above table for depths up to 18 feet the author's attention was drawn to an extensive and careful series of observations made with current-meters by Marr on the Mississippi [1] The results, worked up and abstracted, are as follows —

[1] Report on Current meter Observations in the Mississippi near Burlington

Abstract of Results of Observations on Verticals not near the Sides of the Channels

Serial Number of Group	Channel	Observer	Depth Roughness and Velocity on Vertical			Ratios	
			D	N	U	$\frac{D_v}{D}$	$\frac{U}{U_m}$
Division I —Great Rivers							
1	Mississippi	Humphreys and Abbott	76	027	3 5	38	J8
2	,,	,,	79	027	2 1	13	94
3	,,	,,	65	031	5 3	27	97
4	,,	,,	27	025	4 7	28	97
5	*Irrawaddy*	*Gordon*	*50*		*5 4*	*03*	95
6	,,	,,	29		1 8	zero	93
7	Parana de las Palmas	Revy	50		2 4	zero	83
8	La Plata	,,	24		1 3	zero	60
Division II —Ordinary Streams							
9	Saone	Leveille	14	028	2 2	15	90
10	Garonne	Baumgarten	11	0275	5 0	10	90
11	Seine	Emmery	9	026	2 5	05	89
12	Rhine	International Commission	7	030	7 1	zero	85
13	Branch of Rhine	Defontaine	6	0275	3 5	zero	87
14	*Ganges Canal*	*Cunningham*	9	025	3 5	12	98
15	,,	,,	6 5	013	4 2	19	J3
Division III —Small Streams							
16	Artificial Channels	Bazin	*1 3*	020	*5 9*	*05*	*84*
17	,,	,,	1 1	015	6 6	zero	89
18	,,	,,	1	012	6 5	zero	91
19	,,	,,	9	010	9 1	zero	92

the double float, and the ratio $\frac{U}{U_m}$ was thus seriously vitiated (chap viii art 9), the values of U obtained being too high On the Ganges Canal U was, however, observed separately by means of rod floats, and by making certain corrections for the length of rod used, corrected values of U have been found and used In Revy's observations the flow was unsteady

By considering the figures of each separate series in divisions

n and m it is quite clear that the ratio $\frac{U}{U_m}$ increases as N decreases. This result had previously been found by Bazin for his small channels. It also seems probable that the ratio generally increases with the depth. In division 1 the figures are unreliable, as above explained, but to some extent they confirm the above laws. From a consideration of the various results the following table has been prepared, but the figures given are only probable and approximate. The only law that seems to be well established is that of change of the ratio with change of N, the rest being somewhat doubtful. The figures are, however, an advance on the present rough rule that the ratio is ·85 to ·90. The blanks in the table may be filled in according to judgment. In some small and rough channels the ratio has been found to be as low as ·60. The ratio $\frac{U}{U_s}$ may be designated β.

PROBABLE RATIOS OF MEAN TO SURFACE VELOCITIES $\left(\beta \text{ or } \frac{U}{U_s}\right)$ ON VERTICALS NOT NEAR THE SIDES OF A CHANNEL.

Depth on Vertical.	Values of N								
	·030	·0275	·0250	·0225	·0200	·0175	·015	·013	·010
Feet.									
·90					·83	·86	·88	·89	·91
1·10					·84	·87	·89	·90	·91
1·25					·85	·87	·89	·91	·91
1·50					·87	·88	·90	·91	·92
2·00			·87						
3·00			·88						
5·0	·85	·87	·89					·93	
7·0			·90						
10·0	·86	·89	·90					·92	
13·0		·91	·89						
18·0	·89	·92	·88		·89		·90	·91	
23·0		·93							
28·0		·95							

After the preparation of the above table for depths up to 18 feet the author's attention was drawn to an extensive and careful series of observations made with current-meters by Marr on the Mississippi.[1] The results, worked up and abstracted, are as follows:—

[1] Report on Current meter Observations in the Mississippi, near Burlington.

	Feet	Feet	Feet	Feet	Feet
Depth =	11 2	13 2	20 4	21 6	27 6
$V =$	2 0	2 6	1 9	2 2	2 2
$U - U_1 =$	89	91	93	93	945
$D_m - D =$	09	09	26	21	·09

The values of N and S are not stated, but N is judged to have been about 0275, and the above table has been accordingly extended to depths of 28 feet The velocities were not observed near enough to the bed to enable U_b to be found

When the maximum velocity is at the surface the ratio $\frac{U}{U_m}$ is the same as $\frac{U}{U_s}$ Otherwise it is 1 to 3 per cent lower

No law for the variation of $\frac{D_m}{D}$ can be traced, except that in small streams the ratio is greater the rougher the channel The ratio never exceeds 20 except on the Mississippi On the Irrawaddy, with not dissimilar depths and velocities, it is very small or zero The difference may possibly be due to differences in N and S It appears that in very deep rivers all the ratios are more sensitive

The ratio $\frac{U_b}{U_m}$ or $\frac{U_b}{U_s}$ generally follows the ratio $\frac{U}{U_m}$ In the detailed series of division III of the table on page 168 both ratios attain maximum and minimum values together Values ranging from 58 to 63 have been found for the ratio on the Lower Rhine, Meuse, Oder, Worth, and Messel It is probable that in nearly all experiments the ratios found are too high because the velocities are hardly ever observed close to the bed and also because of the rapid decrease of velocity near the bed On the Stone the current-meter was placed as near to the bed as possible, and the ratio comes out very low The following table shows such probable values of this ratio as it has been possible to arrive at —

N	[illegible] [illegible]	[illegible]	[illegible]	·015	·010
Depth =	Feet 5 to 18		Feet 1 to 1 5	Feet 1 to 1½	Feet 1
$U_b - U_m$	50 to [illegible]		50 to [illegible]	[illegible]	[illegible]

When the various ratios are known the vertical velocity curve

can be drawn The curves are, of course, sharper the less the depth of water The depth at which the velocity is equal to the mean velocity on the vertical varies somewhat, being generally deeper as D_m is deeper It has been found to vary from 55D to 67D On the average it is at about 60D or 625D The mid-depth velocity is greater than the mean, but generally by only 1 or 2 per cent On the Mississippi it was found to remain constant while U was constant, even though U_s was increased or decreased by wind, a compensating change occurring near the bed The mean velocity can be found approximately by an observation at 60 or 625 of the full depth It can be found very nearly, as has been shown by Cunningham, by observing the velocities at 21 and 79 of the full depth and taking the mean of the two

10 **Central Surface Velocity Co-efficients** —Sometimes the mean velocity V in a cross section is inferred from an observation in the centre of a stream If U is the velocity on the central vertical $V=aU$ Sometimes U_s the central surface velocity, is observed and multiplied by a co efficient δ It is clear that δ must be $a\times\beta$ It has been seen that a depends on the shape of the section, and is practically independent of the size, roughness, and slope, while β, at least in streams of shallow section, seems to depend on these three factors In a given stream of shallow section and fairly level bed a decreases as D increases, but β increases Hence δ does not in ordinary cases show any very great fluctuation On the Ganges Canal, with earthen channels 190 to 60 feet wide, and masonry channels 85 feet wide, and with depths of water from 2 to 11 feet, δ varied from 84 to 89 Neither a nor β varied much With widths of 10 to 20 feet, and depths of 1 to 3 feet, a was somewhat reduced, and δ was also less, its values being 81 to 85 At one site, where there was a shallow in the middle, a rose at low water to 1 07 and δ to 95 Ordinarily δ is seldom below 80

Bazin found for small channels the values of a co-efficient Δ, giving the ratio of U_m to V Its values do not differ very much from those of δ Bazin, however, assumed that Δ depended only on N and R, and on this assumption he worked out values of the co-efficient for values of R, extending up to 20 feet or far beyond the limits of his experiments It has been the custom to use these co-efficients as values of δ, that is, to use them for obtaining V from U_s This in itself would not cause any very large error, but the values of the co efficients, when applied to channels of slopes, sizes, and roughnesses, differing greatly from those used by Bazin,

are entirely wrong Neither δ nor Δ can depend only on R and N, but must depend on the values of a and β

Other general expressions for δ have been proposed by Prony and others, but they, in common with those of Bazin, are almost useless as general formulæ

Section IV.—Co efficients

11 **Bazin's and Kutter's Co efficients**.—Setting aside obsolete and discarded figures, the first important set of co efficients for open channels is that obtained by Darcy and Bazin from experiments on artificial channels, whose width did not exceed 6 56 feet in masonry and wood and 21 feet in earth Bazin from these experiments, framed tables of C (connecting them by an empirical formula and extending them *far outside the range of the experiments*) for four classes of channel, namely, earth, rubble masonry, ashlar or brickwork, and smooth cemented surfaces It has been found that these co efficients, though correct enough for small channels, often fail for others More recently two Swiss engineers, Ganguillet and Kutter, went thoroughly into the subject, and after investigating the results of the principal observations, and making some themselves, arrived at various sets of co-efficients for channels of different degrees of roughness, the roughness being defined by a 'rugosity co efficient' N The following statement shows some selected values of Bazin's and Kutter's co efficients The last three columns will be referred to below —

Hydraulic Radius (R)	Bazin's Co-efficients			Kutter's Co efficients for Channels having a Slope of 1 in 5000			Bazin's New Co efficients		
	Cement etc	Rubble Masonry	Earth	Cement Plaster etc	Earthen Channels in Good Order	Earthen Channel in Bad Order	Cement etc.	Regular Channels.	Very Rough Channels
				$N = \cdot 010$	$N = \cdot 020$	$N = \cdot 030$	$\gamma = \cdot 109$	$\gamma = 1$ [illegible]	γ [illegible]
·5	135	72	36	132	57	35	136	50	[illegible]
1·0	141	87	48	152	69	43	142	60	[illegible]
2·0	144	98	62	170	82	53	146	[illegible]	49
4·0	146	106	76	185	94	63	149	89	61
6·0	147	110	84	193	101	69	151	97	69
10·0	147	112	91	201	108	76	152	106	[illegible]

It will be seen that C always increases with R and that the increase is less rapid as R becomes greater, and that as R increases

C becomes less affected by the degree of roughness Also that, with change of R, Kutter's co efficient varies more than Bazin's for smooth channels, and less than Bazin's for rough channels

Bazin's co efficients are independent of S, but Kutter's depend to some extent on S, as will appear from the following statement —

Value of R	Kutter's Co efficients for different Slopes			
	N = ·010		N = ·030	
	Slope 1 in 10 000	Slope 1 in 1000 and Steeper Slopes	Slope 1 in 10 000	Slope 1 in 1000 and Steeper Slopes.
5	126	138	33	36
1 0	148	156	42	45
2 0	168	172	52	54
4 0	186	185	64	63
6 0	195	191	70	68
10 0	206	197	78	74

When R is about 3 2 C is independent of S It increases or decreases with S according as R is below or above 3 2, but it varies only slightly for a great change of S, the variation being greatest when S is between 1 in 2500 and 1 in 5000 For slopes steeper than 1 in 1000 the variation is negligible For all values of N the variation of C with S is very similar in relative amount

Kutter's co efficients for flat slopes are based on the Mississippi observations of Humphreys and Abbott The fall here was so small (sometimes 02 foot per mile) that the deduced slopes are absolutely unreliable This is the opinion of Bazin and also of Smith There is really no proper evidence that C increases as S decreases Bazin, who has recently reviewed the whole question and considered all the best known experiments, has arrived at a new set of co efficients, some of whose general values are given in the last three columns of the first of the above tables As before he makes C independent of S, and his different sets of co-efficients correspond to certain values of γ which is analogous to Kutter's N The rate at which C varies with change of R conforms more nearly than before to that of Kutter's co-efficients Bazin in his discussion includes some results which are known to be wrong, such as those obtained on the Irrawaddy (art. 9) and in the Solani aqueduct, Ganges Canal (chap. vii art. 5), but the rejection of these would not appreciably alter his figures.

The experimental values of C when plotted form a mass of irregularly placed dots and Bazin's co efficients seem to suit them as well as Kutter's, while the law of their variation is more simple It has, however, been seen that for pipes C undoubtedly increases with S, and it is unlikely that a different law holds good for small open channels It is quite likely that C always increases with S, but that for large values of R the increase is negligible

Engineers have now become familiar with Kutter's values of N, and it is desirable to continue their use It is possible to expand Bazin's new sets of co efficients so as to give values corresponding to all Kutter's values of N, but it seems undesirable to do this, having regard to the doubt as to what the real law is Complete sets of both Kutter's and Bazin's co efficients are given in tables xxix to xlii

The empirical formulæ connecting the different values of the co efficients are as follows —

Bazin's original co efficients

$$C=\frac{1}{\sqrt{\alpha\left(1+\frac{\beta}{R}\right)}}$$

Kutter's co efficients

$$C=\frac{41.6+\frac{1.811}{N}+\frac{.00281}{S}}{\sqrt{R}+N\left(41.6+\frac{.00281}{S}\right)}\sqrt{R}$$

Bazin's new co efficients

$$C=\frac{157.6}{1+\frac{\gamma}{\sqrt{R}}}$$

The quantities α, β, N and γ are all constants depending on the nature of the channel

12 **Rugosity Co-efficients** —The kinds of *materials for which* various values of N have been generally accepted are as follows Unless otherwise stated all are supposed to be in good order If in bad order the next higher value of N may be used, if extra smooth, the next lower

·009	Timber planed and perfectly continuous		
·010	Timber planed	Glazed and enamelled materials	Cement and plaster

·011			Plaster and cement with one third of sand	Pipes of iron, cement, or terra cotta, well joined and in best order.	
·012	Timber unplaned and perfectly continuous				New brickwork
·013		Unglazed stoneware and earthenware			Good brickwork and ashlar
·015	Wooden frames covered with canvas Wooden troughs with battens inside, ½ in apart			Foul and slightly tuberculated iron.	Rough faced brickwork Well-dressed stonework
·017	Fine gravel well rammed.		Rubble in cement	Tuberculated iron	Brickwork, stonework and ashlar in inferior condition
·020	Coarse gravel well rammed Wooden troughs with battens inside, 2 ins apart		Coarse rubble laid dry Rubble in inferior condition		

For earthen channels the following are the general values —

·017	Channels in	very good order
·020	,,	good order
·0225	,,	order above the average
·025	,,	average order
·0275	,,	order below the average
·030	,,	bad order
·035	,,	very bad order

A channel in very good order is free from irregularities, lumps, hollows, snags, or other obstructions, weeds and overhanging growth. A channel having all the above irregularities (or even a few of them in excess) would be in very bad order. If a channel is choked by weeds, N may rise even higher than ·035. The channels are all supposed to be free from bends. In the Punjab canals, when the channel has been worn very smooth and even,

The experimental values of C when plotted form a mass of irregularly placed dots and Bazin's co-efficients seem to suit them as well as Kutter's, while the law of their variation is more simple. It has, however, been seen that for pipes C undoubtedly increases with S, and it is unlikely that a different law holds good for small open channels. It is quite likely that C always increases with S, but that for large values of R the increase is negligible.

Engineers have now become familiar with Kutter's values of N, and it is desirable to continue their use. It is possible to expand Bazin's new sets of co-efficients so as to give values corresponding to all Kutter's values of N, but it seems undesirable to do this, having regard to the doubt as to what the real law is. Complete sets of both Kutter's and Bazin's co-efficients are given in tables xxix. to xlii.

The empirical formulæ connecting the different values of the co-efficients are as follows:—

Bazin's original co-efficients

$$C=\frac{1}{\sqrt{a\left(1+\frac{\beta}{R}\right)}}$$

Kutter's co-efficients

$$C=\frac{41.6+\frac{1.811}{N}+\frac{.00281}{S}}{\sqrt{R}+N\left(41.6+\frac{.00281}{S}\right)}\sqrt{R}$$

Bazin's new co-efficients

$$C=\frac{157.6}{1+\frac{\gamma}{\sqrt{R}}}$$

The quantities a, β, N and γ are all constants depending on the nature of the channel.

12. **Rugosity Co-efficients.**—The kinds of materials for which various values of N have been generally accepted are as follows. Unless otherwise stated all are supposed to be in good order. If in bad order the next higher value of N may be used; if extra smooth, the next lower.

·009	Timber planed and perfectly continuous		
·010	Timber planed	Glazed and enamelled materials	Cement and plaster

.011		Plaster and cement with thin coat of sand	Unplaned iron, concrete, terra cotta, well jointed and laid in good order	
.012	Timber, well planed and perfectly continuous			New brick work
.013		Unglazed stoneware and earthenware		Good brick work and ashlar
.015	Wooden frame covered with canvas. Wooden triangle with battens breadth, ½ in apart		Rough and slightly tuberculated iron	Rough faced brickwork. Well dressed stonework
.017	Fine gravel well rammed	Rubble in cement	Tuberculated iron	Brick work, stonework and rubble in bad condition
.020	Coarse gravel well rammed. Wooden triangles with battens at sides, 2 in apart	Coarse rubble laid dry. Rubble in inferior condition		

For earth channels the following are the general values:

.017	Channels in very good order
.020	good order
.0225	order above the average
.025	average order
.0275	order below the average
.030	bad order
.035	very bad order

A channel in very good order is free from large obstructions, large boulders, stumps, or other obstructions, weeds and overhanging growth. A channel having all the above large obstructions or even a few of them in excess would be in very bad order. If a channel is choked by weeds, n may rise even higher than .035. The channels are all supposed to be free from curves. In the larger canals, when the channel has been worn very smooth and even,

N has sometimes been found to be as low as 016 There are of course channels requiring values of N intermediate to the above, and the proper value to be adopted, in any given case, is a matter of experience and judgment The deposits which occur in brick sewers increase the roughness somewhat The deposit of silt in an earthen channel frequently reduces the roughness

The kinds of channels corresponding to Bazin's γ are as follows —

109 Cement, planed wood
290 Planks, bricks, cut stone
833 Rubble masonry
1 54 Earth if very regular, stone revetments
2 35 Ordinary earth
3 17 Exceptionally rough (beds covered with boulders, sides with grass, etc)

13 **Remarks** —Besides the causes of discrepancies among the values of C mentioned in chapter ii (arts 9 and 11) there are others On the Mississippi and Irrawaddy V was obtained by the double float which gives erroneous results (chap viii art 9) The results of over a hundred discharges observed near the head of a large canal in India, when arranged into groups according to the depth of silt in the canal, show the average value of N to be 025 when there is little or no silt, but 013 when the depth of silt is from 5 feet upwards Silt generally deposits in a wedge, the depth being greatest near the head of the canal It is therefore probable that the want of uniformity of the flow gave a somewhat enhanced value to C, and consequently too low a value to N This would, however, account only partially for the low value of N, and it is probable that its correct value is not more than 016 in the silted channel The above values are the average ones In individual discharges N varies enormously For one particular depth of silt it varies from 009 to 030 These variations may be accounted for partly by real variations in the roughness of the channel, which often becomes very irregular when scouring is going on actively, partly by errors in the observations of the individual surface slopes and partly by variations in the degree of the variability of the flow

For two channels equal as regards roughness of surface and value of R, N is less when the profile of the section is semicircular or curved than when it is angular In Bazin's experiments on small channels C is 5 to 9 per cent less for a rectangular section, even though the depth was only $\frac{1}{12}$ to $\frac{1}{2}$ of the width, than for a

semicircular channel The difference is probably due to the effect of the eddies produced at the sides (art 7) The co efficients in the tables may be taken to be for average sections, the section being neither a segment of a circle nor a rectangle (See also art 4)

For smooth channels of small hydraulic radius Kutter's co efficients give results which are too low For iron or very smooth masonry conduits of diameters less than a foot, if N is taken to be 011, V comes out much too low For such cases Smith's or Fanning's pipe co efficients should be used (See also chap v art 9) Similarly for brick sewers, if N is taken to be 013, V is too low and Bazin's co-efficients may be used These facts point to the greater accuracy of Bazin's co-efficients for small and smooth channels

In earthen channels N seems to be particularly low when the ratio of width to depth is great On the river Ravi at Sidhnai the value of N, deduced from a long series of observations, is often 008 or 010, and never very much higher The bed is often silted, but not always The flow is practically uniform, and the slope observations were checked with a view to discovering any error The river is straight, very regular, about 800 feet wide, and 6 feet to 10 feet deep The case was specially investigated, and it seems to be proved that N at this site is not above 010 It is possible that the low value is due to the small effect of eddies from the sides, as compared with narrower streams and to the regularity of the flow Generally streams as wide as the Ravi are irregular [1]

Section V—Movement of Solids by a Stream

14 **Formulæ and their Application**—The observations made by Kennedy, and referred to in chap ii (art 22), were made in India on the Bari Doab Canal and its branches, the widths of the channels varying from 8 feet to 91 feet, and the depths of water from 2 3 feet to 7 3 feet The beds of these channels have, in the course of years adjusted themselves by silting or scouring, so that there is a state of permanent *regime*, each stream carrying its full charge of silt, and the charges in all being equal It was found that the relation between D and V in any channel was nearly given by the equation

$$V = 84D^{64} \qquad (71)$$

Put in a general form, this equation is

$$V = cD^{m} \qquad (72)$$

[1] See also Appendix C

The theory advanced in the paper quoted is that the silt supported per square foot of bed is P_1D where P_1 is the charge of silt, and the force of the eddies as V^2, so that P_1D is as V^2. If the solids consisted only of silt m would be $\frac{1}{2}$, but there is also drift. The silt discharge is $BDVP_1$, or is as BV^3. The drift discharge is supposed to be as BV, and relatively small, and the total solid discharge is thus as a function of V, varying less rapidly than V^3, say as V^n. On the Bari Doab Canal n was 2.56. For, since $D^{.61}$ is as V, D is as $V^{1.6}$, and $BDVP$ as $BPV^{2.56}$.

Let V_b be the bottom velocity and v the drift velocity. If it be assumed that P_1D is as V_b^2—the ratio $\frac{V_b}{V}$ increasing with V and D—the force of the current on the drift *as* $(V_b-v)^2$, and the friction of the drift on the channel as v, and that the depth of the *drift may increase with* V, then m *may come out lower*, say less than $\frac{1}{2}$ for silt alone, or $\frac{1}{3}$ for all solids. The equation

$$V=1.05D^{\frac{1}{3}} \qquad (73)$$

agrees nearly as closely as equation 67 with the observed results.

All the above equations are partly empirical, and obviously apply only to cases in which the silt and drift bear some sort of proportion to each other. In theoretical equations of general application silt and *drift* would have to be considered separately. If there is silt and no drift, equation 72 may be of the true form for all cases, m being probably $\frac{1}{2}$ or less. If there is drift and no silt, as in a clear stream rolling gravel or boulders, the moving force depends on V_b, and D will be absent from the equation or will enter into it only in so far as the ratio $\frac{V_b}{V}$ may depend on D. *Regarding equation 71 as a semi-empirical working equation*—and no more has been claimed for it—applicable to canal systems and streams carrying silt and fine sand, its practical importance is very great. It is now known that in order to prevent, say, a deposit in any reach or branch, V must not be kept constant, but be altered in the same manner as D. Whether it be altered as $D^{.61}$ or $D^{\frac{1}{3}}$ does not, for moderate changes, make very much difference. *The exact figures will in time be better known.* In designing a channel the proper relation of depth to velocity can be arranged for, or, at least, *one* quantity or the other kept in the ascendant according as scouring or silting is the evil to be guarded against.

The old idea was that an increase in V, even if accompanied by an increase in D, gave increased silt transporting power. In a stream of shallow section this is probably *correct for V increases as D*

that is, as fast as required by equation 71, and faster than required by equation 73 In a stream of deep section a decrease in D gives increased silt-transporting power If the discharge is fixed, a change in D or W must be met by a change of the opposite kind in the other quantity In this case widening or narrowing the channel may be proper according to circumstances In a deep section widening will decrease the depth of water, and may also increase the velocity, and it will thus give increased scouring power In a shallow section narrowing will increase the velocity more than it increases $D^{\frac{1}{2}}$ In a medium section it is a matter of exact calculation to find out whether widening or narrowing will improve matters

If the water entering a canal has a higher silt-charge than can be carried in the canal some of it must deposit Suppose an increased discharge to be run, and that this gives a higher silt-carrying power and a smaller rate of deposit per cubic foot of discharge, it does not follow that the deposit will be less because the quantity of silt entering the canal is now greater than before, Owing to want of knowledge regarding the proportions of silt and drift, and to want of exactness in the formulæ, reliable calculations regarding proportions deposited cannot be made

Assuming equation 71 to be correct, Kennedy has determined the following 'critical velocities,' or velocities below which silting will occur in channels supplied with turbid water, such as that of the Indian rivers, and has also published diagrams giving details

$D=$	1	2	3	4	5	6	7	8	9	10
$V=$	·84	1·30	1·70	2·04	2·35	2·64	2·92	3·18	3·43	3·67

15 **Remarks**—The channels in which the observations above referred to were made have all, as stated, assumed nearly rectangular cross sections, the sides having become vertical (Fig 114) by the deposit on them of finer silt, but the equations probably apply approximately to any channel if D is the mean depth from side to side, and V the mean velocity in the whole section

Fig 114

If the ratio of V to D^m, say V to $D^{\cdot 64}$, differs in different parts of a cross section, there is a tendency towards deposit in the parts where the ratio is least, or to scour where it is greatest There is of course, a tendency for the silt-charge to adjust itself to the circumstances of each part of the stream, that is to become less where the above ratio is less, but the irregular movements of the stream cause a transference of water transversely as well as vertically, and this tends to equalise the silt-charge In a channel with not very steep side slopes the angles at M N (Fig 115)

frequently silt up—the velocity there being relatively low—and the sides become steep or vertical. Sometimes, even when the sides are vertical, fine silt adheres to them, and the channel contracts, even though there may be no deposit in the bed. When the bed is level across there frequently occurs a shoaling near the sides, or a scour in the middle, and a marked rounding off at the lower angles. The section thus tends to assume the form shown in Fig. 116. When the bed is of sand, as in the Bari Doab Canal channels, it remains nearly level, because the sand at the sides rolls towards the centre.

FIG. 115.

FIG. 116.

It is clearly impossible to answer, in a general manner, questions such as whether the embanking of a river, or confining it by training walls, will cause its bed to rise or to scour; whether silt will deposit on flooded land; whether the minor arm of a stream will tend to silt and become obliterated. Everything depends on the charge of silt originally carried, on the hardness of the channels, and on the relations between D and V.

On some Indian canals the bed, when the water is shut off, forms a succession of steps, each about 1 foot or less in height, and 20 to 30 feet apart. From one step to the next the bed slopes upwards. This condition seems to occur when the material is sandy and scour is going on. The sand seems to be rolled up the long slope, and to fall over the step.

Some rivers in the northern hemisphere which flow in a southerly direction have a tendency to shift their channels westwards. This is especially noticeable in some of the Indian rivers. The revolution of the earth has been ascribed as a cause. As the water approaches the equator its velocity of rotation about the earth's axis increases. In latitude 30° a stream flowing south at 2 miles an hour has its velocity of rotation increased in one hour from about 1300 feet per second to 1300·37 feet per second, or by ·37 feet per second. This is not a large amount in an hour, and the pressure due to it must be a negligible quantity.

EXAMPLES[1]

Explanation.—The explanation given under examples in Chapter V applies also to open channels. If only one factor, say S, is fixed an infinite number of channels can be designed to carry a given discharge, but usually other factors are determined by practical considerations: the ratio of the side slopes, say, by the nature of the soil, and the ratio of W to D, say, by the velocity

[1] When these Examples were worked out Bazin's new co-efficients had not come to notice. They can be used in the same way as Kutter's.

desirable or the solid moving power required If V must not fall below a certain minimum this can be arranged by keeping R large enough, or if this cannot be done, by altering S, N, or Q If V is not to exceed a certain maximum R can be kept down, or S can be reduced to any extent by placing falls in the channel

Example 1 —Find the discharge of a stream with vertical sides and 15 ft wide when $D=5\ 5$ ft, $N=\ 017$, and $S=1$ in 5225

From table xliv $A=75$ and $\sqrt{R}=1\ 74$ From table xxxv $C\sqrt{R}=183$ From table xxviii a slope of $\frac{1}{5000}$ gives $V=2\ 59$, and the percentage to be deducted is $\frac{225}{10}=2\ 2$, making $V=2\ 53$ Then $Q=75\times 2\ 53=189\ 8$ c ft per second

Example 2 —Design a channel with side slopes 1 to 1 to discharge 1000 c ft per second, S being $\frac{1}{5000}$ and $N=\ 0225$ The figures in the annexed statement show the results of successive trials, the bed width being 40 ft It is clear that a depth of 7 13 ft gives the requisite discharge

	1st trial	2nd trial	3rd trial
Bed width,	40	40	40
Depth,	7 5	7 25	7 0
A from table xlv	356 3	342 6	329
$\sqrt{R}$ from table xlv ,	2 41	2 38	2 34
$C\sqrt{R}$ from table xxxvii ,	216	212	208
V from table xxviii	3 05	3 00	2 94
$Q=AV$,	1087	1028	967

Example 3 —In the preceding example let V be limited to 2 5 ft per second Find the minimum bed width

A must be 400 From table xxviii $C\sqrt{R}$ is 176, and this in table xxxvii gives $\sqrt{R}=2\ 08$ From table xlv a bed width of 80 ft and depth 4 75 ft gives practically the required values of A and $\sqrt{R}$

Example 4 —A channel 20 ft wide with side slopes ½ to 1 and depth 5 ft has to discharge 240 c ft per second N being 025 Find S

From table xliv $A=112\ 5$ and $\sqrt{R}=1\ 90$ Then $V=\frac{240}{112\ 5}=$ 2 13 ft per second Assume S to be $\frac{1}{5000}$ Then table xxviii gives $C\sqrt{R}=151$, which corresponds in table xxxvii to $\sqrt{R}=2\cdot0$ Therefore S has been assumed too low Assume it to be $\frac{1}{4500}$, then $C\sqrt{R}=142\ 8$ and $\sqrt{R}=1\ 92$ To be exact $\sqrt{S}$ must be

increased in the ratio $\frac{1\ 92}{1\ 90}$, or by 1 per cent nearly, that is, $S=\frac{1}{4411}$

Example 5—Keeping Q the same, alter D and S in the last case so as to give the necessary ratio of V to D to prevent silting according to the rules of art 14

The statement given below shows that if D is reduced to 3 25 ft S will be as before (1 in 4410), but W must be increased to 40 ft If W is left unaltered D can be 4 75, but S must be increased to about 1 in 3572 In a short channel, or one containing falls, it would be easiest to increase S, but otherwise it would be necessary to widen

Depth of water,	5 0	4 5	4 0	3 5	3 0
Velocity according to above rule,	2 35	2 20	2 01	1 87	1 70
Mean width of channel to make $Q=240$ c ft per second,	20 5	24 2	29 1	36 6	47
Bed width of channel to nearest foot,	18	22	27	35	45
$\sqrt{R}$ from table xlii,	1 87	1 83	1 79	1 73	1 61
$C\sqrt{R}$ from table xxxviii,	137	135	129	123	111
S (from table xxxviii) to give V as above, 1 in	3380	3761	1000	1320	1[illegible]00

Example 6—In a channel A is found to be 48 sq ft, $\sqrt{R}$ is 1 4 ft, Q is known to be 100 c ft per second, and S is $\frac{1}{3000}$ Find C and N

V is $\frac{100}{48}=2\ 08$ ft per second From table xxxviii, if $S=\frac{1}{3061}$, $C\sqrt{R}=114$ An addition of 61 to 3000 decreases V by 1 per cent, an addition of 100 decreases V by 1 6 per cent, and $C\sqrt{R}$ must be increased by 1 6 per cent, that is, it is 115 8 Then $C=\frac{115\ 8}{1\ 4}=82\ 7$, which (table xxxvi) corresponds very nearly to $N=020$

Example 7—In a channel with vertical sides, 70 ft wide and 5 ft deep, the central surface velocity is 3 ft per second, N is 025 What is V?

From the table on page 169 β is 89 From the table on page 165 α is 915 Then $V=3\times 89\times 915=2\ 72$ ft per second

Tables of Kutter's and Bazin's Co-efficients

These are given to three figures, and the engineer who uses them will be fortunate if the actuals come out so as to agree with the third figure or even come near it To add a fourth figure is useless, and it would render the tables bulky and less convenient The values of $C\sqrt{R}$ have been obtained from the four figure values of C, and the figures in excess of three struck off

As N increases the difference in C becomes less in proportion to the change in N Hence it is not necessary to give C for $N = \cdot 0325$

Table XXIX.—Kutter's Co-efficients ($N = \cdot 009$)

$\sqrt{r}$	1 in 20 000		1 in 15 000		1 in 10 000		1 in 5 000		1 in 2 500		1 in 1 000	
	C	$C\sqrt{r}$	C	$C\sqrt{R}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$
·4	93.4	37.4	98.9	39.5	106	42.2	114	45.6	119	47.7	123	49.1
·45	101	45.6	107	48	113	51.0	122	54.7	127	57	130	58.5
·5	108	54.2	114	56.8	120	60.1	128	64.2	133	66.6	137	68.2
·55	115	63.3	120	66.2	127	69.6	134	73.9	139	76.5	142	78.1
·6	121	72.7	120	75.8	132	79.4	140	84	145	86.7	147	88.4
·65	127	82.5	132	85.8	138	89.5	145	94.2	149	97	152	98.8
·7	133	92.7	137	96.1	143	100	150	105	154	108	156	109
·8	142	114	147	117	152	122	158	126	162	129	164	131
·9	151	136	155	140	160	144	165	149	168	151	170	153
1	159	159	163	163	167	167	171	171	174	174	175	175
1·1	166	183	169	186	173	190	177	194	179	197	180	198
1·2	173	207	175	210	178	214	181	218	183	220	184	221
1·3	178	232	180	235	183	238	185	241	187	243	188	244
1·4	184	257	185	259	187	262	189	265	190	267	191	267
1·5	188	283	190	285	191	287	193	289	193	290	194	291
1·6	193	309	194	310	195	311	196	313	196	314	197	314
1·7	197	335	197	336	198	336	198	337	199	338	199	338
1·8	201	362	201	362	201	362	201	362	201	362	201	362
1·9	204	388	204	388	204	387	203	386	203	386	203	386
2	208	415	207	414	206	413	205	411	205	410	205	409
2·1	211	443	210	440	209	438	207	436	207	434	206	433
2·2	214	470	212	467	211	464	209	460	208	459	208	457
2·3	216	497	215	494	213	490	211	485	210	483	209	481
2·4	219	525	217	520	215	516	213	510	211	507	211	506
2·5	221	553	219	547	217	541	214	535	213	532	212	529
2·6	223	581	221	574	218	568	215	560	214	556	213	554
2·7	226	609	223	601	220	594	217	585	215	581	214	578
2·8	228	637	224	629	221	620	218	610	216	605	215	602
2·9	229	665	226	656	223	646	219	635	217	630	216	626
3	231	694	228	683	224	673	220	660	218	654	217	650

TABLE XXX —KUTTER'S COEFFICIENTS (N= 01)

$\sqrt{r}$	1 in [illegible]0 000		1 in 15 000		1 in 10 000		1 in 5 000		1 in 2 500		1 in 1 000	
	C	$C\sqrt{r}$	C	$C\sqrt{R}$	C	$C\sqrt{r}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
4	81	32 4	85 7	34 3	91 4	36 6	99	39 6	104	41 [illegible]	107	42 7
45	87 9	39 6	92 6	41 3	98 3	44 3	106	47 6	110	49 [illegible]	114	51 1
5	94 4	47 2	99 1	49 6	105	52 4	112	56	117	58 2	120	[illegible]
55	100	55 2	105	57 8	111	60 8	118	64 7	122	67	125	68 6
6	106	63 6	111	66 3	116	69 6	123	73 7	127	76 1	130	77 7
65	111	72 3	115	75 3	121	73 7	128	82 9	131	85 4	134	87
7	116	81 4	121	84 5	126	88	132	92 3	136	94 9	138	96 [illegible]
8	126	100	130	104	134	107	140	112	143	114	145	116
9	134	120	137	124	141	127	146	132	149	134	151	136
1	141	141	144	144	148	148	152	132	155	155	156	[illegible]
1 1	148	162	150	166	154	169	157	173	159	175	161	177
1 2	154	184	156	187	159	190	162	194	164	196	165	198
1 3	159	207	161	209	163	212	166	216	167	217	168	219
1 4	164	230	166	232	167	234	169	237	171	239	171	240
1 5	169	253	170	255	171	257	173	259	174	260	174	261
1 6	173	277	174	278	175	280	176	281	176	282	177	282
1 7	177	301	178	302	178	302	178	303	179	304	179	304
1 8	181	325	181	325	181	[illegible]	181	326	181	326	181	326
1 9	184	350							[illegible]		[illegible]	347
2	187	375									[illegible]	[illegible]
2 1	190	400									[illegible]	391
2 2	193	425									[illegible]	413
2 3	196	450									[illegible]	435
2 4	198	476									[illegible]	[illegible]
2 5	201	501									[illegible]	479
2 6							[illegible]	507	194	503	193	501
2 7								[illegible]	[illegible]	[illegible]	194	523
2 8												
2 9												
3												

Table XXXI.—Kutter's Coefficients ($N = \cdot 011$)

$\sqrt{r}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$
·4	71·1	28·4	77·3	30·9	80·3	32·1	87·1	34·8	91·3	36·5	94·1	37·6
·45	77·4	34·8	81·6	36·8	86·6	39	93·1	42	97·3	43·8	100	45·1
·5	83·3	41·6	87·4	43·7	92·4	46·2	99	49·5	103	51·5	106	52·8
·55	88·6	48·8	92·9	51·1	97·9	53·8	101	55·7	105	57·9	111	60·8
·6	94	56·4	98	58·8	103	61·7	109	65·4	113	67·7	115	69·1
·65	95·9	61·2	103	66·8	108	69·8	113	73·7	117	76·1	119	77·6
·7	104	72·4	107	75·1	112	78·2	118	82·3	121	84·7	123	86·2
·8	112	89·5	115	92·3	120	95·6	125	99·8	128	102	130	104
·9	121	108	123	110	127	114	131	118	134	120	136	122
1	126	126	129	129	133	133	137	137	139	139	140	140
1·1	133	146	135	149	138	152	142	156	144	158	145	159
1·2	138	166	141	169	143	172	146	175	148	177	149	178
1·3	144	187	145	189	147	192	150	195	151	197	152	198
1·4	148	208	150	210	151	212	153	215	154	216	155	217
1·5	153	229	154	231	155	233	156	235	157	236	158	237
1·6	157	251	158	252	158	253	159	255	160	256	160	256
1·7	161	273	161	274	162	275	162	275	162	276	162	276
1·8	164	296	164	296	164	296	164	296	165	296	164	296
1·9	168	318	167	318	167	319	167	317	167	316	166	316
2	171	341	170	340	169	339	169	337	168	337	168	336
2·1	174	364	173	363	172	361	171	358	170	357	170	356
2·2	176	388	175	385	174	382	172	379	172	378	171	376
2·3	179	411	177	408	176	404	174	400	173	398	172	397
2·4	181	435	179	431	178	426	176	421	175	419	174	417
2·5	184	459	182	454	179	448	177	442	176	439	175	437
2·6	186	483	183	477	181	471	178	464	177	460	176	458
2·7	188	507	185	500	183	493	180	485	178	481	177	478
2·8	190	531	187	523	184	515	181	506	179	502	178	498
2·9	191	554	188	547	185	537	182	528	180	522	179	519
3	193	580	190	570	187	560	183	549	181	543	180	539

TABLE XXXII.—BAZIN'S AND KUTTER'S CO-EFFICIENTS.

√R	Bazin γ = 109		Kutter N = 012 1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	C√R	C	C√R	C	C√R	C	C√R	C	C√R	C	C√R	C	C√I
·4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
·5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
·5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
·6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
·6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
·8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
·9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1·9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2·9	152	440	177	514	174	505	171	497	168	487	166	482	165	479
3	152	456	179	536	176	527	173	518	169	507	167	501	166	498
3·1	152	472												
3·2	152	487												
3·3	152	503												
3·4	153	519												
3·5	153	535												
3·6	153	550												
3·7	153	566												
3·8	153	582												
3·9	153	598												
4	153	614												

Bazin's co-efficients for higher values of $\sqrt{R}$.

$\sqrt{R}$ =	5·0	7·0	8·0
C =	154	155	155

TABLE XXXIV.—KUTTER'S CO-EFFICIENTS ($N = .015$)

$\sqrt{R}$	1 in 20 000		1 in 15 000		1 in 10 000		1 in 5 000		1 in 2 500		1 in 1 000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
4	46 8	18 7	49 4	19 8	52 7	21 1	57 1	22 9	60	24	62	24 8
5	55 5	27 8	58 3	29 2	61 6	30 8	66 1	33	68 9	34 4	70 8	35 4
6	63 4	38	66 1	39 7	69 4	41 7	73 8	44 3	76 4	45 9	78 3	47 0
7	70 4	49 3	73	51 2	76 2	53 4	80 3	56 2	82 8	58	84 6	59 2
8	77 1	61 7	79 4	63 7	82 5	66	86 3	69 1	88 6	70 9	90 1	72 1
9	83 1	74 8	85 4	76 8	88 1	79 3	91 5	82 4	93 6	84 2	94 9	85 4
1	88 6	88 6	90 6	90 6	93 1	93 1	96 1	96 1	97 9	97 9	99 1	99 1
1 1	93 6	103	95 5	105	97 7	107	100	110	102	112	103	113
1 2									105	126	106	127
1 3									109	141	109	142
1 4									111	156	112	167
1 5	111	166	111	166	112	168	113	170	114	171	114	172
1 6	114	182	115	183	115	184	116	185	116	186	117	187
1 7	117	199	118	200	118	201	118	201	119	202	119	202
1 8	120	217	120	217	120	217	121	217	121	217	121	217
1 9	123	234	123	234	123	233	123	233	122	233	122	232
2	126	252	126	251	125	250	125	249	124	248	124	248
2 1	129	270	128	269	127	267	126	265	126	264	125	263
2 2									27	2[illegible]0	127	279
2 3												
2 4												
2 5												
2 6												
2 7												
2 8												
2 9	145	421	143									
3	147	440	144	4[illegible]								

TABLE XXXVI.—KUTTER'S CO-EFFICIENTS ($N = 020$)

$\sqrt{R}$	1 in 20 000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	32	12 8	33 6	13 4	35 7	14 3	38·7	15 5	40 6	16 2	41 9	16 8
·												
·												
1												
1												
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Table XXXVII.—Bazin's and Kutter's Co-efficients

$\sqrt{R}$	Bazin $\gamma = 1.54$		Kutter $N = \cdot 0225$											
			1 in 20 000		1 in 15,000		1 in 10 000		1 in 5,000		1 in 2500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$

Bazin's co efficients for higher values of R $\left\{ \begin{array}{llllll} \sqrt{R} = & 4.5 & 5 & 6 & 7 & 8 \\ C = & 117 & 121 & 125 & 129 & 132 \end{array} \right.$

TABLE XXXVIII.—BAZIN'S AND KUTTER'S CO EFFICIENTS

$\sqrt{R}$	Bazin $\gamma=2\ 35$		Kutter $N=\ 025$											
			1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1 000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$

Bazin's co-efficients for higher values of $R \begin{cases} \sqrt{R}= 4\ 5 & 5 & 6 & 7 & 8 \\ C=103 & 107 & 113 & 118 & 122 \end{cases}$

TABLE XXXIX.—KUTTER'S COEFFICIENTS ($N = \cdot 0275$).

$\sqrt{R}$	1 in 20,000		1 in 15,000		1 in 10,000		1 in 5,000		1 in 2,500		1 in 1,000	
	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$	C	$C\sqrt{R}$
·4	21·2	8·5	22·2	8·9	23·4	9·4	25·2	10·1	26·4	10·5	27·2	10·9
·5	25·6	12·8	26·7	13·3	28	14	29·9	15	31·2	15·6	32	16
·6	29·8	17·9	30·9	18·5	32·3	19·4	34·2	20·5	35·5	21·3	36·3	21·8
·7	33·8	23·7	34·9	24·5	36·3	25·4	38·1	26·7	39·3	27·5	40·2	28·1
·8	37·5	30	38·6	30·9	39·9	31·9	41·7	33·4	42·8	34·3	43·6	34·9
·9	41	36·9	42·1	37·9	43·2	39	45	40·5	46	41·4	46·8	42·1
1	44·4	44·4	45·3	45·3	46·5	46·5	48	48	49	49	49·9	49·6
1·1	47·5	52·2	48·4	53·2	49·4	54·4	50·8	55·9	51·7	56·8	52·3	57·5
1·8	65·6	118	65·6	118	65·6	118	65·7	118	65·7	118	65·7	118
1·9	67·8	129	67·6	128	67·5	128	67·4	128	67·2	128	67·1	128
2	69·8	140	69·3	136	69·2	138	68·8	138	68·6	137	68·5	137
2·1	71·7	151	71·3	150	70·9	149	70·3	148	69·9	147	69·7	146
2·2	73·6	162	73·1	161	72·4	159	71·6	158	71·1	157	70·9	156
2·3	75·4	174	74·8	172	73·9	170	72·9	168	72·4	167	72	166
2·4	77·2	185	76·3	183	75·4	181	74·2	178	73·6	177	73·1	175
2·5	78·8	197	77·8	195	76·7	192	75·4	188	74·6	187	74·1	185
2·6	80·4	209	79·3	206	78	203	76·5	199	75·6	197	75	195
											75·9	203
											76·8	213
											77·6	225
											78·4	235
											79·1	245
											79·8	255
											80·5	266
											81·1	276
											81·8	286
											82·3	296
3·7											82·9	307
3·8											83·4	317
3·9											84	328
4											84·5	338

TABLE XL.—BAZIN'S AND KUTTER'S CO EFFICIENTS.

$\sqrt{r}$	Bazin $\gamma - 3\ 1^{-}$		Kutter N – 030											
			1 in 20 000		1 in 15,000		1 in 10 000		1 in 5 000		1 in 2 500		1 in 1 0	
	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	$C\sqrt{r}$	C	C
4	17 9	71 6	19	7 6	19 8	7 9	20 9	8 4	22 4	9	23 5	9 4	24 2	[illegible]
5														
6														
7														
8														
9														
1														
1 1														
1 2														
1 3														
1 4														
1 5														
1 6														
1 7														
1 8														
1 9														
2														
2 1														
2 2														
2 3														
2 4														
2 5														
2 6														
2 7														
2 8														
2 9														
3														
3 1														
3 2														
3 3														
3 4														
3 5														
3 6														
3 7														
3 8														
3 9														
4														

Bazin's co efficients for higher values of $R \begin{cases} \sqrt{r} = 4\ 5 & 5 & 6 & 7 & 8 \\ C = 92 & 97 & 103 & 108 & 113 \end{cases}$

TABLE XLII.—BAZIN'S OLD CO-EFFICIENTS

These co-efficients have been superseded by Bazin's New Co-efficients, but are given here because they may be still considered suitable in some cases, or may be required for reference.

r	Very Smooth Channels (Cement.)	Smooth Channels (Ashlar or Brickwork.)	Rough Channels (Rubble Masonry.)	Very Rough Channels (Earth.)	Excessively Rough Channels (Encumbered with Detritus.)
α	·00316	00401	·00 0	·005[illegible]	00846
β –	1	23	82	4 1	8
25	125	95	57	26	18 5
5	135	110	72	36	25 6
75	139	116	81	42	30 8
1	141	119	87	48	34 9
1 5	143	122	94	56	41 2
2	144	124	98	62	46
2 5	145	126	101	67	
3	145	126	104	70	53
3 5	146	127	105	73	
4	146	128	106	76	58
4 5	146	128	107	78	
5	146	128	108	80	62
5 5	146	129	109	82	
6	147	129	110	84	65
6 5	147	129	110	85	
7	147	129	110	86	67
7 5	147	129	111	87	
8	147	130	111	88	69
8 5	147	130	112	89	
9	147	130	112	90	71
9 5	147	130	112	90	
10	147	130	112	91	72
11	147	130	113	92	
12	147	130	113	93	74
13	147	130	113	94	
14	147	130	113	95	
15	147	130	114	96	77
16	147	130	114	97	
17	147	130	114	97	
18	147	130	114	98	
20	147	131	114	98	0
25	148	131	115	100	
30	148	131	115	102	83
40	148	131	116	103	85
50	148	131	116	104	86
Infinity	148	131	117	108	91

TABLES OF SECTIONAL DATA

Rectangular and Trapezoidal Sections

For a bed width intermediate to those given it is only necessary, in order to find A, to multiply D by the difference in width and add or subtract the result. Thus, for bed 43 ft., slope $\frac{1}{2}$ to 1, and depth 3.75 ft., $A = 175.8 - 3.75 \times 2 = 168.3$. $\sqrt{R}$ changes so slowly that the correct figure can be interpolated by inspection.

Widths outside the range ... idth W and depth D, look out $\sqrt{R}$ for ... by 2, or for $\frac{W}{9}$ and $\frac{D}{9}$ and multiply by 3. Interpolations can also be made on this principle. For instance, the figures for a bed of 12.5 feet can be found from those for a 50 feet bed.

For side slopes of 4 to 3 and 3 to 4.—A and $\sqrt{R}$ are the same respectively as for a rectangular section and a $\frac{1}{2}$ to 1 section of the same mean width. Thus for a channel of bed 21 feet, side slope 4 to 3, and depth 3 feet, the mean width is 25 feet, and $A = 75$, $\sqrt{R} = 1.56$. For a bed width of 11 feet, side slopes 3 to 4, and depth 4 feet, the mean width is 14 feet, which is the same as for a channel with bed 12 feet, side slopes $\frac{1}{2}$ to 1, and depth 4 feet $A = 56$ and $\sqrt{R} = 1.64$. These rul

ratio of the side slopes, and depends

Table XLIII.—Sectional Data for Open Channels

Rectangular Sections

Depth of Water	Bed 1 foot		Bed 2 feet		Bed 3 feet		Bed 4 feet		Bed 5 feet	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet										
.5	.5	.5	1	.56	1.5	.61	2	.63	2.5	.65
.75	.75	.55	1.5	.66	2.25	.71	3	.74	3.75	.76
1	1	.58	2	.71	3	.77	4	.82	5	.85
1.25	1.25	.6	2.5	.74	3.75	.83	5	.88	6.25	.91
1.5	1.5	.61	3	.78	4.5	.87	6	.93	7.5	.97
1.75	1.75	.62	3.5	.8	5.25	.9	7	.97	8.75	1.01
2	2	.63	4	.82	6	.93	8	1	10	1.05
2.25	2.25	.64	4.5	.83	6.75	.95	9	1.03	11.25	1.09
2.5	2.5	.65	5	.84	7.5	.97	10	1.05	12.5	1.12
2.75	2.75	.65	5.5	.86	8.25	.99	11	1.08	13.75	1.14
3	3	.66	6	.87	9	1	12	1.1	15	1.17
3.25			6.5	.87	9.75	1.01	13	1.11	16.25	1.19
3.5			7	.88	10.5	1.02	14	1.13	17.5	1.21
3.75			7.5	.89	11.25	1.03	15	1.14	18.75	1.23
4			8	.89	12	1.04	16	1.15	20	1.24
4.25					12.75	1.05	17	1.17	21.25	1.25
4.5					13.5	1.07	18	1.19	22.5	1.27
4.75					14.25	1.07	19	1.19	23.75	1.28
5					15	1.07	20	1.2	25	1.29

TABLE XLIII.—*Continued* (*Rectangular.*)

Depth of Water	Bed 6 feet		Bed 7 feet		Bed 8 feet		Bed 10 feet		Bed 12 feet	
	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{R}$	A	$\sqrt{r}$	A	$\sqrt{r}$
Feet										
.5	3	.63	3.5	.66	4	.67	5	.67	6	.68
.75	4.5	.78	5.25	.79	6	.8	7.5	.81	9	.82
1	6	.87	7	.88	8	.8	10	.91	12	.93
1.25	7.5	.94	8.75	.96	10	.83	12.5	1	15	1.02
1.5	9	1	10.5	1.03	12	1.04	15	1.07	18	1.1
1.75	10.5	1.05	12.25	1.08	14	1.1	17.5	1.14	21	1.17
2	12	1.1	14	1.13	16	1.15	20	1.2	24	1.22
2.25	13.5	1.13	15.75	1.17	18	1.2	22.5	1.25	27	1.28
2.5	15	1.17	17.5	1.21	20	1.24	25	1.29	30	1.33
2.75	16.5	1.2	19.25	1.24	22	1.28	27.5	1.33	33	1.37
3	18	1.23	21	1.27	24	1.31	30	1.37	36	1.41
3.25	19.5	1.25	22.75	1.3	26	1.34	32.5	1.4	39	1.45
3.5	21	1.28	24.5	1.32	28	1.37	35	1.43	42	1.49
3.75	22.5	1.3	26.25	1.35	30	1.39	37.5	1.46	45	1.52
4	24	1.31	28	1.37	32	1.41	40	1.49	48	1.55
4.25	25.5	1.33	29.75	1.39	34	1.44	42.5	1.52	51	1.58
4.5	27	1.34	31.5	1.4	36	1.46	45	1.54	54	1.6
4.75	28.5	1.36	33.25	1.42	38	1.47	47.5	1.56	57	1.63
5	30	1.37	35	1.44	40	1.49	50	1.58	60	1.65
5.25	31.5	1.39	36.75	1.45	42	1.51	52.5	1.6	63	1.67
5.5	33	1.39	38.5	1.46	44	1.52	55	1.62	66	1.69
5.75	34.5	1.4	40.25	1.48	46	1.54	57.5	1.64	69	1.71
6	36	1.41	42	1.49	48	1.55	60	1.65	72	1.73
6.25							62.5	1.67	75	1.75
6.5							65	1.68	78	1.77
6.75							67.5	1.69	81	1.79
7							70	1.71	84	1.8
7.25							72.5	1.72	87	1.81
7.5							75	1.73	90	1.83
7.75							77.5	1.74	93	1.84
8							80	1.75	96	1.85

TABLE XLIII.—*Continued* (*Rectangular*)

Depth of Water	Bed 14 feet.		Bed 16 feet		Bed 18 feet.		Bed 20 feet		Bed 25 feet.	
	A	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$
Feet.										
·5	7	·68	8	·69	9	·73	10	·69	12·5	·68
·75	10·5	·82	12	·83	13·5	·83	15	·84	18·8	·84
1	14	·94	16	·91	18	·95	20	·95	25	·96
1·25	17·5	1·03	20	1·04	22·5	1·05	25	1·05	31·3	1·07
1·5	21	1·12	24	1·12	27	1·13	30	1·14	37·5	1·16
1·75	21·5	1·18	28	1·2	31·5	1·21	35	1·22	43·8	1·24
2	28	1·25	32	1·27	36	1·28	40	1·29	50	1·31
2·25	31·5	1·3	36	1·33	40·5	1·34	45	1·36	56·3	1·38
2·5	35	1·36	40	1·38	45	1·4	50	1·41	62·5	1·44
2·75	38·5	1·4	44	1·43	49·5	1·45	55	1·47	68·8	1·5
3	42	1·45	48	1·48	51	1·5	60	1·52	75	1·56
3·25	45·5	1·49	52	1·52	58·5	1·55	65	1·57	81·3	1·61
3·5	49	1·53	56	1·56	63	1·59	70	1·61	87·5	1·65
3·75	52·5	1·56	60	1·6	67·5	1·63	75	1·65	93·8	1·7
4	56	1·6	64	1·63	72	1·66	80	1·69	100	1·75
4·25	59·5	1·63	68	1·67	76·5	1·7	85	1·73	106·3	1·78
4·5	63	1·66	72	1·7	81	1·73	90	1·76	112·5	1·82
4·75	66·5	1·68	76	1·73	85·5	1·76	95	1·79	118·8	1·82
5	70	1·71	80	1·76	90	1·79	100	1·83	125	1·89
5·25	73·5	1·73	84	1·78	94·5	1·82	105	1·86	131·3	1·92
5·5	77	1·76	88	1·8	99	1·85	110	1·89	137·5	1·95
5·75	80·5	1·78	92	1·83	103·5	1·87	115	1·91	143·8	1·99
6	84	1·8	96	1·85	108	1·9	120	1·94	150	2·02
6·25	87·5	1·82	100	1·87	112·5	1·92	125	1·96	156·3	2·04
6·5	91	1·84	104	1·89	117	1·94	130	1·98	162·5	2·07
6·75	94·5	1·85	108	1·91	121·5	1·96	135	2·01	168·8	2·09
7	98	1·87	112	1·93	126	1·98	140	2·03	175	2·11
7·25	101·5	1·89	116	1·95	130·5	2	145	2·05	181·3	2·14
7·5	105	1·9	120	1·97	135	2·02	150	2·07	187·5	2·17
7·75	108·5	1·92	124	1·98	139·5	2·04	155	2·09	193·3	2·19
8	112	1·93	128	2	144	2·06	160	2·11	200	2·21
8·25					148·5	2·07	165	2·13	206·3	2·23
8·5					153	2·09	170	2·14	212·5	2·25
8·75					157·5	2·11	175	2·16	218·8	2·27
9					162	2·12	180	2·18	225	2·29
9·25					166·5	2·14	185	2·19	231·3	2·31
9·5					171	2·15	190	2·21	237·5	2·32
9·75					175·5	2·16	195	2·22	243·8	2·34
10					180	2·18	200	2·24	250	2·36

TABLE XLIII.—*Continued.* (*Rectangular.*)

Depth of Water	Bed 30 feet		Bed 35 feet		Bed 40 feet		Bed 50 feet		Bed 60 feet	
	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$
Feet										
1	30	.97	35	.97	40	.98	50	.98	60	.98
1.5	35	1.17	52.5	1.18	60	1.18	75	1.19	90	1.2
2	60	1.33	70	1.34	80	1.35	100	1.36	120	1.37
2.25	67.5	1.39	78.8	1.41	90	1.42	112.5	1.44	135	1.45
2.5	75	1.40	87.5	1.48	100	1.40	125	1.51	150	1.52
2.75	82.5	1.53	96.3	1.54	110	1.56	137.5	1.57	165	1.59
3	90	1.58	105	1.6	120	1.62	150	1.64	180	1.65
3.25	97.5	1.63	113.8	1.66	130	1.67	162.5	1.7	195	1.71
3.5	105	1.68	122.5	1.71	140	1.73	175	1.75	210	1.77
3.75	112.5	1.73	131.3	1.75	150	1.76	187.5	1.81	225	1.83
4	120	1.78	140	1.78	160	1.83	200	1.86	240	1.88
4.25	127.5	1.82	148.8	1.85	170	1.87	212.5	1.91	255	1.93
4.5	135	1.86	157.5	1.89	180	1.92	225	1.95	270	1.08
4.75	142.5	1.9	166.3	1.93	190	1.96	237.5	2	285	2.03
5	150	1.94	175	1.99	200	2	250	2.04	300	2.07
5.25	157.5	1.97	183.8	2.01	210	2.04	262.5	2.08	315	2.11
5.5	165	2.01	192.5	2.04	220	2.08	275	2.12	330	2.16
5.75	172.5	2.04	201.3	2.08	230	2.11	287.5	2.16	345	2.2
6	180	2.07	210	2.11	240	2.15	300	2.2	360	2.24
8.25	187.5	2.1	218.8	2.15	250	2.18	312.5	2.24	375	2.27
6.5	195	2.13	227.5	2.18	260	2.22	325	2.27	390	2.31
6.75	202.5	2.10	236.3	2.21	270	2.25	337.5	2.31	405	2.35
7	210	2.18	245	2.24	280	2.28	350	2.34	420	2.38
7.25	217.5	2.21	253.8	2.26	290	2.31	362.5	2.37	435	2.42
7.5	225	2.24	262.5	2.29	300	2.34	375	2.4	470	2.45
7.75	232.5	2.26	271.3	2.32	310	2.37	387.5	2.43	465	2.48
8	240	2.28	280	2.34	320	2.39	400	2.46	480	2.51
8.25	247.5	2.31	288.8	2.37	330	2.42	412.5	2.49	495	2.54
8.5	255	2.33	297.5	2.39	340	2.44	425	2.52	510	2.57
8.75	262.5	2.35	306.3	2.42	350	2.47	437.5	2.55	525	2.6
9	270	2.37	315	2.44	360	2.49	450	2.57	540	2.63
9.25	277.5	2.39	323.8	2.46	370	2.52	462.5	2.6	555	2.66
9.5	285	2.41	332.5	2.48	380	2.54	475	2.62	570	2.69
9.75	292.5	2.43	341.3	2.5	390	2.56	487.5	2.65	585	2.71
10	300	2.45	350	2.52	400	2.58	500	2.67	600	2.74
10.5							525	2.72	630	2.79
11							550	2.76	660	2.81
11.5							575	2.81	690	2.88
12							600	2.85	720	2.93

TABLE XLIII.—*Continued* (*Rectangular*)

Depth of Water	Bed 70 feet.		Bed 80 feet		Bed 90 feet		Bed 100 feet		Bed 120 feet	
	A	√R	A	√R	A	√R	A	√R	A	√R
Feet										
1	70		80		90		100		120	
1 5	105		120		135		150		180	
2	140		160		180		200		240	
2 25	157 5		180		202 5		225		270	
2 5	175		200		225		250		300	
2 75	192 5		220		247 5		275		330	
3	210		240		270		300		360	
3 25	227 5		260		292 5		325		390	
3 5	245		280		315		350		420	
3 75	262 5		300		337 5		375		450	
4	280		320		360		400		480	
4 25	297 5		340		382 5		425		510	
4 5	315		360		405		450		540	
4 75	332 5		380		427 5		475		570	
5	350		400		450		500		600	
5 25	367 5		420		472 5		525		630	
5 5	385		440		495		550		660	
5 75	402 5		400		517 5		575		690	
6	420		480		540		600		720	
6 25	437 5		500		562 5		625		750	
6 5	455		520		585		650		780	
6 75	472 5		540		607 5		675		810	
7	490		560		630		700		840	
7 25	507 5		580		652 5		725		870	
7 5	525		600		675		750		900	
7 75	542 5		620		697 5		775		930	
8	560		640		720		800		960	
8·25	577 5		660		742 5		825		990	
8 5	595		680		765		850		1020	
8 75	612 5		700		787 5		875		1050	
9	630		720		810		900		1080	
9·25	647 5		740		832 5		925		1110	
9 5	665		760		855		950		1140	
9 75	682 5		780		877 5		975		1170	
10	700		800		900		1000		1200	
10 5	735		840		945		1050		1260	
11	770		880		990		1100		1320	
11 5	805		920		1035		1150		1380	
12	840		960		1080		1200		1440	

TABLE XLIV.—SECTIONAL DATA FOR OPEN CHANNELS

Trapezoidal Sections—Side slopes ½ to 1

Depth of Water	Bed 1 foot		Bed 2 feet		Bed 3 feet		Bed 4 feet		Bed 5 feet	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet .5	.63	.54	1.13	.60	1.63	.63	2.13	.64	2.63	.65
.75	1.03	.62	1.78	.69	2.53	.73	3.28	.76	4.03	.77
1	1.5	.68	2.5	.77	3.5	.82	4.5	.85	5.5	.87
1.25	2.03	.73	3.28	.83	4.53	.88	5.78	.92	7.03	.95
1.5	2.63	.78	4.13	.88	5.63	.94	7.13	.98	8.63	1.02
1.75	3.28	.82	5.03	.92	6.78	.99	8.53	1.04	10.28	1.08
2	4	.86	6	.96	8	1.03	10	1.09	12	1.13
2.25	4.78	.89	7.03	1	9.28	1.07	11.53	1.13	13.78	1.17
2.5	5.63	.92	8.13	1.03	10.63	1.11	13.13	1.17	15.63	1.21
2.75	6.53	.95	9.28	1.07	12.03	1.15	14.78	1.21	17.53	1.25
3	7.5	.99	10.5	1.1	13.5	1.18	16.5	1.24	19.5	1.29
3.25			11.78	1.13	15.03	1.21	18.28	1.27	21.53	1.33
3.5			13.13	1.16	16.63	1.24	20.13	1.30	23.63	1.36
3.75			14.53	1.18	18.28	1.27	22.03	1.33	25.78	1.39
4			16	1.21	20	1.29	24	1.36	28	1.42
4.25					21.78	1.32	26.03	1.39	30.28	1.4[illegible]
4.5					23.63	1.35	28.13	1.41	32.63	1.47
4.75					25.53	1.37	30.28	1.44	35.03	1.5
5					27.5	1.39	32.5	1.46	37.5	1.[illegible]2

TABLE XLIV.—*Continued* (½ to 1.)

Depth of Water	Bed 6 feet.		Bed 7 feet.		Bed 8 feet.		Bed 9 feet.		Bed 10 feet.	
	A	√r	A	√r	A	√r	A	√r	A	√r
Feet										
5	3 13	66	3 63	67	4 13	67	4 63	68	5 13	68
75	4 78	78	5 33	79	6 28	8	7 03	81	7 78	81
1	6 5	89	7 5	9	8 5	91	9 5	92	10 5	93
1 25	8 28	97	9 53	99	10 78	1	12 03	1 01	13·28	1 02
1 5	10 13	1 04	11 63	1 06	13 13	1·07	14 63	1 09	16 13	1 1
1 75	12 03	1 1	13 78	1 12	15 53	1 14	17·28	1 16	19 03	1 17
2	14	1 16	16	1 18	18	1 2	20	1 22	22	1 23
2·25	16·03	1·21	18 28	1 23	20 53	1 25	22 78	1 28	25 03	1 29
2 5	18 13	1 25	20 63	1 28	23 13	1 3	25 63	1 33	28 13	1 34
2 75	20·28	1 29	23·03	1 32	25 78	1 35	28 53	1 38	31 28	1 39
3	22 5	1 33	25 5	1 36	28 5	1 39	31 5	1 42	34 5	1 44
3 25	24 78	1 37	28 03	1 4	31·28	1 43	34 53	1 46	37 78	1 48
3 5	27 13	1 4	30 63	1 44	34 13	1 47	37 63	1 5	41 13	1 52
3 75	29·23	1 43	33·28	1 47	37 03	1 5	40 78	1 54	44 53	1 56
4	32	1 46	36	1 5	40	1 54	44	1 57	48	1 59
4 25	34 53	1 49	38 78	1 53	43 03	1 57	47·28	1 6	51 53	1 63
4 5	37 13	1 52	41 63	1 56	46 13	1 6	50 03	1 63	55 13	1 66
4 75	39 78	1 55	44 53	1 59	49 28	1 63	54 03	1 66	58 78	1 69
5	42 5	1 57	47 5	1 62	52 5	1 65	57 5	1 69	62 5	1 72
5·25	45 28	1 6	50 53	1 65	55 78	1 68	61 03	1 72	65 28	1 75
5 5	48 13	1 62	53 63	1 67	59 13	1 71	64 63	1 74	70 13	1 77
5 75	51 03	1 65	56 78	1 69	62 53	1 73	68 28	1 77	74 03	1 8
6	54	1 67	60	1 71	66	1 76	72	1 79	78	1 82
6 25		..							82 03	1 85
6 5		..							86 12	1 87
6 75									90 28	1 9
7									94 5	1 92
7 25									98 78	1 94
7 5									103 1	1 96
7 75									107 53	1 98
8									112	2

TABLE XLIV.—*Continued* (½ to 1)

Depth of Water	Bed 12 feet		Bed 14 feet		Bed 16 feet		Bed 18 feet		Bed 20 feet	
	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$
Feet										
·5	6·1	·68	7·1	·69	8·1	·69	9·1	·69	10·13	·69
·75	9·3	·82	10·8	·83	12·3	·83	13·8	·84	15·25	·84
1	12·5	·94	14·5	·94	16·5	·95	18·5	·96	20·5	·96
1·25	15·8	1·03	18·3	1·05	20·8	1·05	23·3	1·06	25·8	1·06
1·5	19·1	1·12	22·1	1·13	25·1	1·14	28·1	1·15	31·1	1·15
1·75	22·5	1·19	26	1·2	29·5	1·22	33·3	1·23	36·5	1·23
2	26	1·26	30	1·27	34	1·29	38	1·3	42	1·31
2·25	29·5	1·32	34	1·33	38·5	1·35	43	1·37	47·5	1·38
2·5	33·1	1·37	38·1	1·39	43·1	1·41	48·1	1·43	53·1	1·44
2·75	36·8	1·42	42·3	1·45	47·8	1·47	53·3	1·49	58·8	1·5
3	40·5	1·47	46·5	1·5	52·5	1·52	58·5	1·54	64·5	1·55
3·25	44·3	1·52	50·8	1·55	57·3	1·57	63·8	1·59	70·3	1·6
3·5	48·1	1·56	55·1	1·59	62·1	1·61	69·1	1·64	76·1	1·65
3·75	52	1·6	59·5	1·63	67	1·66	74·5	1·68	82	1·7
4	56	1·64	64	1·67	72	1·7	80	1·72	88	1·74
4·25	60	1·67	68·5	1·71	76	1·74	84·5	1·76	94	1·79
4·5	64·1	1·7	73·1	1·74	82·1	1·78	91·1	1·8	100·1	1·83
4·75	68·3	1·74	77·8	1·78	87·3	1·81	96·8	1·84	106·3	1·86
5	72·5	1·77	82·5	1·81	92·5	1·84	102·5	1·87	112·5	1·9
5·25	76·8	1·8	87·3	1·84	97·8	1·88	108·3	1·91	118·8	1·94
5·5	81·1	1·83	92·1	1·87	103·1	1·91	114·1	1·94	125·1	1·97
5·75	85·5	1·86	97	1·9	108·5	1·94	120	1·97	131·5	2
6	90	1·88	102	1·93	114	1·97	126	2	138	2·03
6·25	94·5	1·91	107	1·96	119·5	2	132	2·03	144·5	2·06
6·5	99·1	1·93	112·1	1·98	125·1	2·02	138·1	2·06	151·1	2·09
6·75	103·8	1·96	117·3	2·01	130·8	2·05	144·3	2·09	157·8	2·12
7	108·5	1·98	122·5	2·03	136·5	2·08	150·5	2·11	164·5	2·15
7·25	113·3	2·01	127·8	2·06	142·3	2·11	156·8	2·14	171·3	2·18
7·5	118·1	2·03	133·1	2·08	148·1	2·13	163·1	2·17	178·1	2·2
7·75	123	2·05	138·5	2·1	154	2·15	169·5	2·19	185	2·23
8	128	2·07	144	2·12	160	2·17	176	2·21	192	2·25
8·25							182·5	2·24	199	2·28
8·5							191·2	2·26	208·2	2·3
8·75							195·8	2·28	213·3	2·32
9							202·5	2·3	220·5	2·34
9·25							209·3	2·33	227·8	2·37
9·5							216·1	2·35	235·1	2·39
9·75							223	2·37	242·5	2·41
10							230	2·39	250	2·43

TABLE XLIV.—*Continued* ($\frac{1}{2}$ to 1)

Depth of Water	Bed 25 feet.		Bed 30 feet.		Bed 35 feet.		Bed 40 feet.		Bed 45 feet.	
	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{R}$
Feet.										
1	25.5	·97	30.5	·97	35.5	·98	40.5	·98	45.5	·98
1.5	38.6	1.17	46.1	1.18	53.6	1.18	61	1.19	68.6	1.19
2	52	1.33	62	1.34	72	1.35	82	1.36	92	1.36
2.25	58.8	1.4	70	1.41	81.3	1.42	92.5	1.43	103.8	1.44
2.5	65.6	1.46	78.1	1.48	90.6	1.49	103.2	1.5	115.6	1.51
2.75	72.5	1.52	86.3	1.54	100	1.56	113.8	1.57	127.5	1.58
3	79.5	1.58	94.5	1.6	109.5	1.62	124.5	1.63	139.5	1.64
3.25	86.5	1.64	102.8	1.66	119	1.68	135.3	1.69	151.5	1.7
3.5	93.6	1.69	111.1	1.71	128.6	1.73	146.1	1.75	163.6	1.76
3.75	100.8	1.74	119.5	1.76	138.3	1.79	157	1.8	175.8	1.82
4	108	1.78	128	1.81	148	1.84	168	1.85	188	1.87
4.25	115.3	1.83	136.5	1.86	157.8	1.89	179	1.9	200.3	1.92
4.5	122.6	1.87	145.1	1.9	167.6	1.93	190.1	1.95	212.6	1.96
4.75	130	1.91	153.8	1.95	177.5	1.97	201.3	2	225	2.01
5	137.5	1.95	162.5	1.99	187.5	2.01	212.5	2.04	237.5	2.06
5.25	145	1.99	171.3	2.03	197.5	2.05	223.8	2.08	250	2.1
5.5	152.6	2.02	180.1	2.06	207.6	2.1	235.1	2.12	262.6	2.14
5.75	160.3	2.06	189	2.1	217.8	2.14	246.5	2.16	275.3	2.18
6	168	2.09	198	2.14	228	2.17	258	2.2	288	2.22
6.25	175.8	2.12	207	2.17	238.3	2.21	269.5	2.24	300.8	2.26
6.5	183.6	2.15	216.1	2.2	248.6	2.24	281.1	2.27	313.6	2.3
6.75	191.6	2.19	225.3	2.24	259.1	2.28	292.8	2.31	326.6	2.34
7	199.5	2.22	234.5	2.27	269.5	2.31	304.5	2.34	339.5	2.37
7.25	207.5	2.25	243.8	2.3	280	2.34	316.3	2.37	352.5	2.4
7.5	215.6	2.27	253.1	2.33	290.6	2.37	328.1	2.4	365.6	2.43
7.75	223.8	2.3	262.5	2.36	301.3	2.4	340	2.44	378.8	2.47
8	232	2.33	272	2.38	312	2.43	352	2.47	392	2.5
8.25	240.3	2.36	281.5	2.41	322.8	2.46	364	2.5	405.3	2.53
								2.52	418.6	2.56
								2.55	432.1	2.59
								2.58	445.5	2.62
								2.61	459.1	2.64
								2.63	472.6	2.67
								2.66	486.3	2.7
								2.69	500	2.72
								2.74	527.6	2.78
11							500.5	2.78	555.5	2.83
11.5							526.1	2.83	583.6	2.87
12							552	2.87	612	2.92

TABLE XLIV.—*Continued* (½ to 1)

Depth of Water	Bed 50 feet		Bed 60 feet		Bed 70 feet		Bed 80 feet		Bed 90 feet	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet										
1	51.5	.98	60.5	.99	70.5	.99	80.5	.99	90.5	.99
1.5	76.1	1.19	91.1	1.2	106.1	1.21	121.1	1.21	136.1	1.21
2	102	1.37	122	1.38	142	1.38	162	1.38	182	1.39
2.25	115	1.45	137.5	1.46	160	1.46	182.5	1.47	205	1.47
2.5	128.1	1.52	153.1	1.53	178.1	1.54	203.1	1.54	228.1	1.54
2.75	141.3	1.59	168.8	1.6	196.3	1.61	224.8	1.61	252.3	1.62
3	154.5	1.65	184.5	1.66	214.5	1.67	244.5	1.68	274.5	1.68
3.25	167.8	1.71	200.3	1.73	232.8	1.74	265.3	1.74	297.8	1.75
3.5	181.1	1.77	216.1	1.78	251.1	1.8	286.1	1.8	321.1	1.81
3.75	194.5	1.83	232	1.84	269.5	1.86	307	1.86	344.5	1.87
4	208	1.89	248	1.9	288	1.91	328	1.92	368	1.93
4.25	221.5	1.93	264	1.96	306.5	1.96	349	1.98	391.5	1.99
4.5	235.1	1.98	280.1	2	325.1	2.02	376.1	2.03	415.1	2.04
4.75	248.8	2.03	296.3	2.05	343.8	2.07	391.3	2.08	438.8	2.09
5	262.5	2.07	312.5	2.1	362.5	2.11	412.5	2.13	462.5	2.14
5.25	276.3	2.12	328.8	2.15	381.3	2.16	433.8	2.18	486.3	2.19
5.5	290.1	2.16	345.1	2.18	400.1	2.2	455.1	2.22	510.1	2.23
5.75	304	2.2	361.5	2.23	419	2.25	476.5	2.26	534	2.28
6	318	2.24	378	2.27	438	2.29	498	2.31	558	2.32
6.25	332	2.28	394.5	2.31	457	2.33	519.5	2.35	582	2.37
6.5	346.1	2.32	411.1	2.35	476.1	2.37	641.1	2.39	606.1	2.41
6.75	360.3	2.36	427.8	2.39	495.3	2.41	562.8	2.43	630.3	2.45
7	374.5	2.39	444.5	2.42	514.5	2.45	584.5	2.47	654.5	2.49
7.25	388.8	2.43	461.3	2.46	533.8	2.49	606.3	2.51	678.8	2.53
7.5	403.1	2.46	478.1	2.5	553.1	2.52	628.1	2.55	703.1	2.57
7.75	417.5	2.49	495	2.53	572.5	2.56	650	2.59	727.5	2.6
8	432	2.52	512	2.56	592	2.6	672	2.62	752	2.64
8.25	446.5	2.55	529	2.59	611.5	2.63	694	2.66	776.5	2.68
8.5	461.1	2.58	546.1	2.63	631.1	2.66	716.1	2.69	801.1	2.71
8.75	475.8	2.61	563.3	2.66	650.8	2.7	738.3	2.73	825.8	2.75
9	490.5	2.64	580.5	2.69	670.5	2.73	760.5	2.76	850.5	2.78
9.25	505.3	2.67	597.8	2.72	690.3	2.76	782.8	2.79	875.3	2.81
9.5	520.1	2.7	615.1	2.75	710.1	2.79	805.1	2.82	900.1	2.84
9.75	535	2.73	632.5	2.78	730	2.82	827.5	2.85	927	2.88
10	550	2.76	650	2.81	750	2.85	850	2.88	950	2.91
10.5	580.1	2.81	685.1	2.86	790.1	2.91	895.1	2.94	1000	2.97
11	610.5	2.86	720.5	2.92	830.5	2.96	940.5	3	1050	3.03
11.5	641.1	2.91	756.1	2.97	871.1	3.02	986.1	3.05	1101	3.08
12	672	2.96	792	3.02	912	3.07	1032	3.11	1152	3.14

TABLE XLIV.—*Continued* (½ to 1)

Depth of Water	Bed 100 feet.		Bed 120 feet.		Bed 140 feet.		Bed 160 feet.	
	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$
Feet.								
1	100·5	·99	120·5	·99	140·5	·99	160·5	·99
1·5	151·1	1·21	181·1	1·21	211·1		241·1	1·4
2	202	1·39	242	1·39	282	1·4	322	1·56
2·25	227·5	1·47	272·5	1·47				
2·5	253·1	1·55	303·1	1·55	353·1	1·56	403·1	
2·75	278·8	1·62	333·8	1·62	388·8	1·63	443·8	1·63
3	304·5	1·69	364·5	1·69	424·5	1·7	484·5	1·7
3·25	330·3	1·76	395·3	1·76	460·3	1·77	525·3	1·77
3·5	356·1	1·82	426·1	1·82	496·1	1·83	566·1	1·83
3·75	382	1·88	457	1·88	532	1·89	607	1·9
4	408	1·94	488	1·94	568	1·95	648	1·96
4·25	434	1·99	519	2	604	2·01	689	2·02
4·5	460·1	2·01	550·1	2·05	640·1	2·06	730·1	2·07
4·75	486·3	2·1	581·3	2·11	676·3	2·12	771·3	2·13
5	512·5	2·15	612·5	2·16	712·5	2·17	812·5	2·18
5·25	538·8	2·2	643·8	2·21	748·8	2·22	853·8	2·23
5·5	565·1	2·24	675·1	2·25	785·1	2·26	895·1	2·28
5·75	591·5	2·29	706·5	2·3	821·5	2·31	936·5	2·32
6	618	2·33	738	2·35	858	2·36	978	2·37
6·25	644·5	2·38	769·5	2·4	894·5	2·41	1020	2·42
6·5	671·1	2·42	801·1	2·44	931·1	2·45	1061	2·46
6·75	697·8	2·46	832·8	2·48	967·8	2·5	1103	2·51
7	724·5	2·5	864·5	2·52	1005	2·54	1145	2·55
7·25	751·3	2·54	896·3	2·56	1041	2·58	1186	2·59
7·5	778·1	2·58	928·1	2·6	1078	2·62	1228	2·63
7·75	805	2·62	960	2·64	1115	2·66	1270	2·68
8	832	2·66	992	2·68	1152	2·7	1312	2·72
8·25	859	2·69	1024	2·72	1189	2·74	1354	2·76
8·5	886·1	2·73	1056	2·75	1226	2·78	1396	2·79
8·75	913·3	2·76	1088	2·79	1263	2·82	1438	2·83
9	940·5	2·8	1121	2·83	1301	2·85	1481	2·86
9·25	967·8	2·83	1153	2·86	1338	2·89	1523	2·9
9·5	995·1	2·86	1185	2·89	1375	2·92	1565	2·93
9·75	1023	2·9	1118	2·93	1413	2·96	1608	2·97
10	1050	2·93	1250	2·96	1450	2·99	1650	3
10·5	1105	2·99	1315	3·03	1525	3·05	1735	3·07
11	1161	3·05	1381	3·09	1601	3·12	1821	3·13
11·5	1216	3·11	1446	3·15	1676	3·18	1906	3·2
12	1272	3·17	1512	3·21	1752	3·24	1992	3·27

Table XLV.—Sectional Data for Open Channels

Trapezoidal Sections—Side-slopes 1 to 1.

Depth of Water	Bed 1 foot		Bed 2 feet		Bed 3 feet		Bed 4 feet		Bed 5 feet.	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet										
5	75	577	1·25	605	1 75	629	2 25	645	2 75	655
75	1 31	652	2 06	707	2 81	741	3 56	763	4 31	779
1	2	723	3	788	4	828	5	856	6	875
1 25	2 81	787	4 06	856	5 31	901	6 56	933	7 81	956
1 5	3 75	846	5 25	917	6 75	965	8 25	1	9 75	1 03
1 75	4 81	899	6 56	971	8 31	1 02	10 06	1 06	11 81	1 09
2	6	95	8	1 02	10	1 08	12	1 12	14	1 15
2·25	7 31	996	9 56	1 07	11 81	1 12	14 06	1 17	16 31	1 2
2 5	8 75	1 04	11 25	1 11	13 75	1 17	16 25	1 21	18 75	1 25
2 75	10 32	1 08	13 06	1 16	15 81	1 21	18 56	1 26	21 31	1 29
3	12	1 13	15	1 2	18	1 25	21	1 3	24	1 33
3 25			17 06	1 24	20 31	1 29	23 56	1 34	26 81	1 37
3 5			19 25	1 27	22 75	1 33	26 25	1 38	29 75	1 41
3 75			21 56	1 31	25 31	1 36	29 06	1 41	32 81	1 45
4			24	1 34	28	1 4	32	1 45	36	1 49
4·25					30 81	1 43	35 06	1 48	39 31	1 52
4 5					33 75	1 47	38 25	1 51	42 75	1 55
4 75					36 81	1 5	41 56	1 54	46 32	1 59
5					40	1 53	45	1 58	50	1 62

TABLE XLV.—*Continued* (1 to 1)

Depth of Water	Bed 6 feet		Bed 7 feet		Bed 8 feet		Bed 9 feet		Bed 10 feet	
	A	√r	A	√r	A	√r	A	√r	A	√r
Feet										
·5	3·25	·662	3·75	·667	4·25	·672	4·63	·667	3·25	·678
·75	5·06	·781	5·81	·798	6·56	·805	7·03	·793	8·06	·815
1	7	·891	8	·902	9	·911	10	·919	11	·926
1·25	9·06	·975	10·31	·989	11·56	1	12·81	1·01	14·06	1·02
1·5	11·25	1·03	12·75	1·07	14·25	1·08	15·75	1·09	17·25	1·1
1·75	13·56	1·11	15·31	1·13	17·06	1·15	18·81	1·16	26·56	1·17
2	16	1·17	18	1·19	20	1·21	22	1·23	24	1·24
2·25	18·56	1·23	20·81	1·25	23·06	1·27	25·31	1·28	27·56	1·29
2·5	21·23	1·28	23·75	1·3	26·25	1·32	28·75	1·33	31·25	1·35
2·75	24·06	1·32	26·81	1·35	29·56	1·37	32·31	1·39	35·06	1·40
3	27	1·37	30	1·39	33	1·42	36	1·44	39	1·45
3·25	36·06	1·41	33·31	1·43	35·56	1·44	39·81	1·48	43·06	1·5
3·5	33·25	1·45	36·75	1·47	40·25	1·51	43·75	1·52	47·25	1·54
3·75	36·56	1·48	40·31	1·51	44·06	1·54	47·81	1·56	51·56	1·58
4	40	1·52	44	1·55	48	1·58	52	1·6	56	1·62
4·25	43·56	1·56	47·81	1·59	52·06	1·61	56·31	1·64	60·56	1·66
4·5	47·25	1·59	51·75	1·62	56·25	1·65	60·75	1·67	65·25	1·69
4·75	51·06	1·62	55·81	1·65	60·56	1·68	63·31	1·71	70·06	1·73
5	55	1·65	60	1·68	65	1·71	70	1·74	75	1·76
5·25	59·06	1·68	64·31	1·72	69·56	1·75	74·81	1·77	80·06	1·8
5·5	63·25	1·71	68·75	1·75	74·25	1·78	79·75	1·8	85·25	1·83
5·75	67·57	1·74	73·32	1·78	79·07	1·81	84·82	1·83	90·57	1·86
6	77	1·77	78	1·8	81	1·83	90	1·86	96	1·89
6·25							95·31	1·89	101·56	1·92
6·5							100·7	1·92	107·2	1·94
6·75							106·3	1·94	113·05	1·97
7							112	1·97	119	2
7·25							117·8	2	125·06	2·03
7·5							123·8	2·02	131·3	2·06
7·75							129·8	2·05	137·56	2·08
8							136	2·07	144	2·11

TABLE XLV.—*Continued* (1 to 1)

Depth of Water	Bed 12 feet		Bed 14 feet		Bed 16 feet		Bed 18 feet.		Bed 20 feet.	
	A	$\sqrt{R}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$
Feet										
·5	0 25	682	7 37	683	8 37	686	9 25	477	10 25	692
75	9 56	823	11 34	824	12 84	825	14 06	694	15 56	839
1	13	936	15	914	17	95	19	[illegible]	21	959
1 25	16 56	1 03	19 06	1 04	21 56	1 07	24 06	1 06	26 56	1 08
1 5	20 25	1 12	23 25	1 13	26 25	1 14	29 25	1 15	32 25	1 15
1 75	24 06	1 10	27 56	1 21	31 06	1 22	34 56	1 23	38 06	1 24
2	28	1 20	32	1 28	36	1 29	40	1 3	44	1 31
2 25	32 06	1 32	36 56	1 34	41 06	1 35	45 56	1 37	50 06	1 38
2 5	36 25	1 38	41 25	1 4	46 25	1 42	51 25	1 43	56 25	1 44
2 75	40 50	1 43	46 06	1 45	51 56	1 47	57 06	1 49	62 56	1 5
3	45	1 48	51	1 51	57	1 53	63	1 54	69	1 56
3 25	49 56	1 53	56 06	1 56	62 56	1 58	69 06	1 59	75 56	1 61
3 5	54 25	1 57	61 25	1 6	68 25	1 62	75 25	1 64	82 25	1 66
3 75	59 06	1 62	66 56	1 65	74 06	1 67	81 56	1 69	89 06	1 71
4	64	1 66	72	1 69	80	1 71	88	1 73	96	1 75
4 25	69 06	1 7	77 56	1 73	86 06	1 75	94 56	1 77	103 1	1 79
4 5	74 25	1 73	83 25	1 77	92 25	1 79	101 3	1 81	110 3	1 84
4 75	79 56	1 77	89 06	1 8	98 56	1 83	108 1	1 85	117 6	1 88
5	85	1 8	95	1 84	105	1 87	115	1 89	125	1 91
5 25	90 56	1 84	101 1	1 87	111 6	1 9	122 1	1 93	132 6	1 95
5 5	96 25	1 87	107 3	1 91	118 3	1 91	129 3	1 96	140 3	1 99
5 75	102 1	1 9	113 6	1 94	125 1	1 97	136 6	2	148 1	2 02
6	108	1 93	120	1 97	132	2	144	2 03	156	2 05
6 25	114 1	1 96	126 6	2	139 1	2 03	151 6	2 06	164 1	2 09
6 5	120 2	1 99	133 3	2 03	146 3	2 06	159 3	2 09	172 3	2 12
6 75	126 6	2 02	140 1	2 06	153 6	2 09	167 1	2 12	180 6	2 15
7	133	2 05	147	2 09	161	2 12	175	2 15	189	2 18
7 25	139 6	2 07	151 1	2 11	168 6	2 15	183 1	2 18	197 6	2 21
7 5	146 3	2 1	161 3	2 14	176 3	2 18	191 3	2 21	206 3	2 24
7 75	153 1	2 13	168 6	2 17	184 1	2 21	199 6	2 24	215 1	2 25
8	160	2 15	176	2 19	192	2 23	208	2 26	224	2 29
8 25					200 1	2 26	216 6	2 29	233 1	2 32
8 5					208 3	2 28	225 3	2 32	242 3	2 35
8 75					216 6	2 31	233 1	2 31	251 6	2 37
9					225	2 33	243	2 37	261	2 4
9 25					233 6	2 35	252 1	2 39	270 6	2 42
9 5					242 3	2 38	261 3	2 41	280 3	2 45
9 75					251 1	2 4	270 6	2 44	290 1	2 47
10					260	2 42	280	2 46	300	2 49

TABLE XLV —*Continued* (1 to 1)

Depth of Water	Bed 25 feet.		Bed 30 feet		Bed 35 feet.		Bed 40 feet		Bed 45 feet	
	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$
Feet										
1	26	·966	31	·970	36	·976	41	·978	46	·981
1·5	39·75	1·17	47·25	1·18	54·75	1·18	62·25	1·19	69·75	1·19
2	54	1·33	64	1·31	74	1·35	84	1·36	94	1·36
2·25	61·31	1·4	72·56	1·41	83·81	1·42	95·06	1·43	106·3	1·44
2·5	68·75	1·46	81·25	1·47	93·75	1·49	106·3	1·5	118·8	1·51
2·75	76·31	1·53	90·06	1·51	103·8	1·56	117·6	1·57	131·3	1·58
3	84	1·58	99	1·6	114	1·62	129	1·63	144	1·64
3·25	91·81	1·64	108·1	1·66	124·3	1·68	140·6	1·69	156·8	1·7
3·5	99·75	1·69	117·3	1·72	134·8	1·73	152·3	1·75	169·8	1·76
3·75	107·8	1·74	126·6	1·77	145·3	1·79	164·1	1·8	182·8	1·81
4	116	1·79	136	1·81	156	1·84	176	1·85	196	1·87
4·25	124·3	1·83	145·6	1·86	166·8	1·88	188·1	1·9	209·3	1·92
4·5	132·8	1·88	155·3	1·91	177·8	1·93	200·3	1·95	222·8	1·96
4·75	141·3	1·92	165·1	1·95	188·8	1·97	212·6	1·99	236·3	2·01
5	150	1·96	175	1·99	200	2·02	225	2·04	250	2·06
5·25	158·8	1·97	185·1	2·03	211·3	2·06	237·6	2·08	263·8	2·1
5·5	167·8	2·03	195·3	2·07	222·8	2·1	250·3	2·12	277·8	2·14
5·75	176·8	2·07	205·6	2·11	234·3	2·14	263·1	2·16	291·8	2·18
6	186	2·11	216	2·15	246	2·18	276	2·2	306	2·22
6·25	195·3	2·14	226·6	2·18	257·8	2·21	289·1	2·24	320·3	2·26
6·5	204·8	2·17	237·3	2·21	269·8	2·25	302·3	2·28	334·8	2·3
6·75	214·3	2·2	248·1	2·25	281·8	2·28	315·6	2·31	349·3	2·34
7	224	2·24	259	2·28	294	2·32	329	2·34	364	2·37
7·25	233·8	2·27	270·1	2·31	306·3	2·35	342·6	2·37	378·8	2·41
7·5	243·8	2·3	281·3	2·34	318·8	2·38	356·3	2·41	393·8	2·44
7·75	253·8	2·33	292·6	2·37	331·3	2·41	370·1	2·45	408·8	2·47
8	264	2·35	304	2·4	344	2·44	384	2·48	424	2·5
8·25	274·4	2·38	315·6	2·43	356·9	2·47	398·1	2·51	439·4	2·54
8·5	284·8	2·41	327·3	2·46	369·8	2·5	412·3	2·54	454·8	2·57
8·75	295·4	2·44	339·1	2·49	382·9	2·53	426·6	2·57	470·4	2·6
9	306	2·46	351	2·52	396	2·56	441	2·6	486	2·62
9·25	316·9	2·49	363·1	2·54	409·4	2·59	455·6	2·62	501·9	2·66
9·5	327·8	2·51	375·3	2·57	422·8	2·61	470·3	2·65	517·8	2·68
9·75	338·9	2·54	387·6	2·6	436·4	2·64	485·1	2·68	533·9	2·71
10	350	2·56	400	2·62	450	2·67	500	2·71	550	2·74
10·5							530·3	2·76	582·8	2·79
11							561	2·81	616	2·85
11·5							593·3	2·86	650·8	2·9
12							624	2·91	684	2·94

TABLE XLV—*Continued* (1 to 1)

Depth of Water	Bed 50 feet		Bed 60 feet		Bed 70 feet		Bed 80 feet		Bed 90 feet	
	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$
Feet.										
1	51	.982	61	.985	71	.987	81	.989	91	.99
1.5	77.25	1.19	92.25	1.2	107.3	1.2				
2	104	1.37	124	1.39	144	1.35	164	1.38	184	1.39
2.25	117.6	1.44	140.1	1.45	162.6	1.46	185.1	1.46	207.6	1.47
2.5	131.3	1.52	156.3	1.53	181.3	1.53	206.3	1.54	231.3	1.54
2.75	145.1	1.58	172.6	1.6	200.1	1.6	227.6	1.61	255.1	1.61
3	159	1.65	189	1.66	219	1.67	249	1.68	279	1.68
3.25	173.1	1.71	205.6	1.72	238.1	1.73	270.6	1.74	303.1	1.75
3.5	187.3	1.77	222.3	1.78	257.3	1.79	292.3	1.8	327.3	1.81
3.75	201.6	1.82	239.1	1.84	276.6	1.85	314.1	1.86	351.6	1.87
4	216	1.88	256	1.9	296	1.91	336	1.92	376	1.93
4.25	230.6	1.93	273.1	1.95	315.6	1.96	358.1	1.97	400.6	1.98
4.5	245.3	1.98	290.3	2	335.3	2.01	380.3	2.03	425.3	2.03
4.75	260.1	2.03	307.6	2.05	355.1	2.06	402.6	2.08	450.1	2.10
5	275	2.07	325	2.1	375	2.11	425	2.13	475	2.14
5.25	290.1	2.12	342.6	2.14	395.1	2.16	447.6	2.17	500.1	2.18
5.5	305.3	2.16	360.3	2.18	415.3	2.2	470.3	2.22	525.3	2.23
5.75	320.6	2.2	378.1	2.23	435.6	2.25	493.1	2.26	550.6	2.28
6	336	2.24	396	2.27	456	2.29	516	2.31	576	2.32
6.25	351.6	2.28	414.1	2.31	476.6	2.33	539.1	2.35	601.6	2.36
6.5	367.3	2.32	433.3	2.35	497.3	2.37	562.3	2.39	627.3	2.41
6.75	383.1	2.35	450.6	2.39	518.1	2.41	585.6	2.43	653.1	2.45
7	399	2.39	469	2.42	539	2.45	609	2.47	679	2.49
7.25	415.1	2.43	487.6	2.46	560.1	2.49	632.6	2.51	705.1	2.53
7.5	431.3	2.46	506.3	2.5	581.3	2.52	656.3	2.55	731.3	2.56
7.75	447.6	2.49	525.1	2.53	602.6	2.56	680.1	2.58	757.6	2.6
8	464	2.53	544	2.57	624	2.6	704	2.62	784	2.64
8.25	480.6	2.56	563.1	2.6	645.6	2.63	728.1	2.65	810.6	2.67
8.5	497.3	2.59	582.3	2.63	667.3	2.66	752.3	2.69	837.3	2.71
8.75	514.1	2.62	601.6	2.66	689.1	2.7	776.6	2.72	864.1	2.74
9	531	2.65	621	2.7	711	2.73	801	2.76	891	2.78
9.25	548.1	2.68	640.6	2.73	733.1	2.76	825.6	2.79	918.1	2.81
9.5	565.3	2.71	660.3	2.76	755.3	2.79	850.3	2.82	945.3	2.84
9.75	582.6	2.74	680.1	2.79	777.6	2.82	875.1	2.85	972.6	2.88
10	600	2.77	700	2.82	800	2.85	900	2.88	1000	2.91
10.5	635.3	2.82	740.3	2.87	845.3	2.91	950.3	2.94	1055.3	2.97
11	671	2.87	781	2.93	891	2.97	1001	3	1111	3.03
11.5	708.3	2.93	823.3	2.98	938.3	3.02	1052.3	3.06	1167.3	3.09
12	744	2.98	864	3.03	984	3.08	1104	3.11	1224	3.14

TABLE XLV—*Continued* (1 to 1)

Depth of Water	Bed 100 feet.		Bed 120 feet.		Bed 140 feet.		Bed 160 feet.	
	A	√r	A	√r	A	√r	A	√r
Feet								
1	101	·991	121	·992	141	·993	161	·994
2	204	1·39	244	1·39	284	1·4	324	1·4
2·25	230·1	1·47	275·1	1·48	320·1	1·47	360·1	1·48
2·5	256·3	1·55	306·3	1·55	356·3	1·56	406·3	1·56
2·75	282·6	1·62	337·6	1·63	392·6	1·63	447·6	1·64
3	309	1·69	369	1·7	429	1·7	489	1·7
3·25	335·6	1·75	400·6	1·76	465·6	1·77	530·6	1·77
3·5	362·3	1·82	432·3	1·82	502·3	1·83	572·3	1·84
3·75	389·1	1·88	464·1	1·89	539·1	1·89	614·1	1·9
4	416	1·93	496	1·94	576	1·95	656	1·96
4·25	443·1	1·97	528·1	2	613·1	2·01	698·1	2·01
4·5	470·3	2·01	560·3	2·05	650·3	2·06	740·3	2·07
4·75	497·6	2·07	592·6	2·11	687·6	2·12	782·6	2·12
5	525	2·13	625	2·16	725	2·17	825	2·18
5·25	552·6	2·19	657·6	2·21	762·6	2·22	867·6	2·23
5·5	580·3	2·24	690·3	2·26	800·3	2·27	910·3	2·28
5·75	608·1	2·29	723·1	2·3	838·1	2·32	953·1	2·33
6	636	2·33	756	2·35	876	2·36	996	2·37
6·25	664·1	2·38	789·1	2·39	914·1	2·41	1039	2·41
6·5	692·3	2·42	822·3	2·44	952·3	2·45	1082	2·46
6·75	720·6	2·46	855·6	2·48	990·6	2·5	1126	2·51
7	749	2·5	889	2·52	1029	2·54	1169	2·55
7·25	777·6	2·54	922·6	2·56	1068	2·58	1213	2·59
7·5	806·3	2·58	956·3	2·6	1106	2·62	1256	2·63
7·75	835·1	2·62	990·1	2·64	1145	2·66	1300	2·67
8	864	2·65	1024	2·68	1184	2·7	1344	2·71
8·25	893·1	2·69	1058	2·72	1223	2·74	1386	2·75
8·5	922·3	2·73	1092	2·75	1262	2·77	1432	2·79
8·75	951·6	2·76	1127	2·79	1302	2·81	1477	2·83
9	981	2·8	1161	2·83	1341	2·85	1521	2·86
9·25	1011	2·83	1196	2·86	1381	2·88	1566	2·9
9·5	1040	2·86	1230	2·89	1420	2·92	1610	2·94
9·75	1070	2·9	1265	2·93	1460	2·95	1655	2·97
10	1100	2·93	1300	2·96	1500	2·99	1700	3·01
10·5	1160	2·99	1370	3·03	1580	3·05	1790	3·07
11	1221	3·05	1441	3·09	1661	3·12	1881	3·14
11·5	1282	3·11	1512	3·15	1642	3·18	1972	3·2
12	1344	3·17	1584	3·21	1824	3·25	2064	3·26

Table XLVI.—Sectional Data for Open Channels.

Trapezoidal Sections—Side slopes 1½ to 1.

Depth of Water	Bed 1 foot		Bed 2 feet		Bed 3 feet		Bed 4 feet		Bed 5 feet	
	A	√R	A	√R	A	√R	A	√R	A	√R
Feet										
.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.64	[illegible]	.64
.75	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	.74	[illegible]	.79	4.5	[illegible]		[illegible]	6.5	[illegible]
1.25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1.5	[illegible]	[illegible]	6.37	[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	[illegible]
1.75	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	[illegible]	[illegible]	10	[illegible]	12	[illegible]	14	1.12	16	1.15
2.25	[illegible]	[illegible]	[illegible]	[illegible]	14.34	1.14	[illegible]	1.17	[illegible]	1.2
2.5	[illegible]	[illegible]	[illegible]	1.14	[illegible]	1.19	[illegible]	1.22	[illegible]	[illegible]
2.75	[illegible]	1.14	[illegible]	1.19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3.25	—	—	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3.5	—	—	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.4	[illegible]	[illegible]
3.75		—	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.44	[illegible]	[illegible]
4	—	—	[illegible]	[illegible]	[illegible]	1.44	[illegible]	[illegible]	44	[illegible]
4.25	—	—	—	—	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4.5	—	—	—	—	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4.75	—	—	—	—	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	—	—	—	—	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE XLVI.—*Continued* (1½ to 1.)

Depth of Water	Bed 6 feet		Bed 7 feet.		Bed 8 feet.		Bed 9 feet.		Bed 10 feet	
	A	√r	A	√r	A	√r	A	√r	A	√r
Feet.										
·5	3·37	·66	3·87	·67	4·37	·67	4·88	·68	5·38	·68
·75	5·34	·78	6·09	·79	6·84	·8	7·59	·81	8·34	·81
1	7·5	·89	8·5	·89	9·5	·9	10·5	·91	11·5	·92
1·25	9·84	·97	11·09	·98	12·34	·99	13·59	1	14·84	1·01
1·5	12·37	1·04	13·87	1·06	15·37	1·07	16·88	1·08	18·38	1·09
1·75	15·09	1·11	16·84	1·12	18·59	1·14	20·34	1·15	22·09	1·16
2	18	1·17	20	1·18	22	1·2	24	1·22	26	1·23
2·25	21·09	1·23	23·34	1·24	25·59	1·26	27·84	1·28	30·09	1·29
2·5	24·37	1·28	26·87	1·3	29·37	1·31	31·88	1·33	34·38	1·34
2·75	27·84	1·33	30·59	1·35	33·34	1·36	36·09	1·38	38·84	1·39
3	31·5	1·37	34·5	1·39	37·5	1·41	40·5	1·43	43·5	1·44
3·25	35·34	1·41	38·59	1·44	41·84	1·46	45·09	1·48	48·34	1·49
3·5	39·37	1·45	42·87	1·48	46·37	1·5	49·88	1·52	53·38	1·54
3·75	43·59	1·49	47·34	1·52	51·09	1·54	54·84	1·56	58·59	1·58
4	48	1·53	52	1·56	56	1·58	60	1·6	64	1·62
4·25	52·59	1·57	56·54	1·59	61·09	1·62	65·34	1·64	69·59	1·66
4·5	57·37	1·6	61·87	1·63	66·37	1·65	70·88	1·68	75·38	1·7
4·75	62·34	1·64	67·09	1·66	71·84	1·69	76·59	1·71	81·34	1·74
5	67·5	1·67	72·5	1·7	77·5	1·72	82·5	1·75	87·5	1·77
5·25	72·84	1·71	78·09	1·73	83·34	1·76	88·59	1·78	93·84	1·8
5·5	78·37	1·74	83·87	1·77	89·37	1·79	94·87	1·81	100·4	1·83
5·75	84·09	1·77	89·84	1·8	95·59	1·83	101·34	1·85	107·1	1·87
6	90	1·81	96	1·83	102	1·85	108	1·88	114	1·9
6·25							114·8	1·91	121·1	1·93
6·5							121·9	1·94	128·4	1·96
6·75							129·1	1·97	135·9	1·99
7							136·5	2	143·5	2·02
7·25							144·1	2·03	151·4	2·05
7·5							151·9	2·05	159·4	2·07
7·75							159·8	2·08	167·6	2·1
8							168	2·11	176	2·13

TABLE XLVI.—SECTIONAL DATA FOR OPEN CHANNELS

Trapezoidal Sections—Side slopes $1\frac{1}{2}$ to 1

Depth of Water	Bed 1 foot		Bed 2 feet		Bed 3 feet		Bed 4 feet		Bed 5 feet	
	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$
Feet										
.5	.87	.56	1.38	.6	1.88	.63	2.38	.64	2.875	.64
.75	1.59	.65	2.34	.71	3.09	.73	3.84	.76	4.59	.77
1	2.5	.74	3.5	.79	4.5	.83	5.5	.85	6.5	.87
1.25	3.59	.81	4.84	.86	6.09	.9	7.34	.93	8.59	.95
1.5	4.48	.87	6.37	.93	7.87	.97	9.37	1	10.87	1.02
1.75	6.34	.93	8.09	.99	9.84	1.03	11.59	1.06	13.34	1.09
2	8	.99	10	1.04	12	1.08	14	1.12	16	1.15
2.25	9.84	1.04	12.09	1.09	14.34	1.14	16.59	1.17	18.84	1.2
2.5	11.87	1.09	14.37	1.14	16.87	1.19	19.37	1.22	21.87	1.25
2.75	14.09	1.14	16.84	1.19	19.59	1.23	22.34	1.27	25.09	1.3
3	16.5	1.18	19.50	1.23	22.5	1.28	25.5	1.31	28.5	1.34
3.25			22.34	1.28	25.6	1.32	28.84	1.36	32.09	1.39
3.5			25.37	1.32	28.87	1.36	32.37	1.4	35.87	1.43
3.75			28.6	1.36	32.34	1.4	36.09	1.44	39.84	1.47
4			32	1.39	36	1.44	40	1.47	44	[illegible]
4.25					39.84	1.48	44.09	1.51	48.34	[illegible]
4.5					43.87	1.51	48.37	1.55	[illegible]	[illegible]
4.75					48.09	[illegible]	52.84	1.58	[illegible]	1.61
5					[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE XLVI—*Continued* (1½ to 1)

Depth of Water	Bed 6 feet.		Bed 7 feet.		Bed 8 feet.		Bed 9 feet.		Bed 10 feet.	
	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$	A	$\sqrt{P}$
Feet.										
·5	3·37	·66	3·87	·67	4·37	·67	4·88	·68	5·38	·68
·75	5·34	·78	6·09	·79	6·81	·8	7·59	·81	8·34	·81
1	7·5	·89	8·5	·89	9·5	·9	10·5	·91	11·5	·92
1·25	9·81	·97	11·09	·98	12·31	·99	13·59	1	14·84	1·01
1·5	12·37	1·04	13·87	1·06	15·37	1·07	16·88	1·08	18·38	1·09
1·75	15·09	1·11	16·84	1·12	18·59	1·14	20·31	1·15	22·09	1·16
2	18	1·17	20	1·18	22	1·2	24	1·22	26	1·23
2·25	21·09	1·23	23·34	1·21	25·59	1·26	27·84	1·28	30·09	1·29
2·5	24·37	1·28	26·87	1·3	29·37	1·31	31·88	1·33	34·38	1·34
2·75	27·84	1·33	30·59	1·35	33·34	1·36	36·09	1·38	38·84	1·39
3	31·5	1·37	34·5	1·39	37·5	1·41	40·5	1·43	43·5	1·44
3·25	35·31	1·41	38·59	1·44	41·84	1·46	45·09	1·48	48·34	1·49
3·5	39·37	1·45	42·87	1·48	46·37	1·5	49·88	1·52	53·38	1·54
3·75	43·59	1·49	47·31	1·52	51·09	1·54	54·84	1·56	58·59	1·58
4	48	1·53	52	1·56	56	1·58	60	1·6	64	1·62
4·25	52·59	1·57	56·54	1·59	61·09	1·62	65·34	1·64	69·59	1·66
4·5	57·37	1·6	61·87	1·63	66·37	1·63	70·88	1·68	75·38	1·7
4·75	62·34	1·64	67·09	1·66	71·84	1·69	76·59	1·71	81·34	1·74
5	67·5	1·67	72·5	1·7	77·5	1·72	82·5	1·75	87·5	1·77
5·25	72·84	1·71	78·09	1·73	83·34	1·76	88·59	1·78	93·84	1·8
5·5	78·37	1·74	83·87	1·77	89·37	1·79	94·87	1·81	100·4	1·83
5·75	84·09	1·77	89·84	1·8	95·59	1·83	101·34	1·85	107·1	1·87
6	90	1·81	96	1·83	102	1·85	108	1·88	114	1·9
6·25							114·8	1·91	121·1	1·93
6·5							121·9	1·94	128·4	1·96
6·75							129·1	1·97	135·9	1·99
7							136·5	2	143·5	2·02
7·25							144·1	2·03	151·4	2·05
7·5							151·9	2·05	159·4	2·07
7·75							159·8	2·08	167·6	2·1
8							168	2·11	176	2·13

TABLE XLVI.—*Continued* (1½ to 1)

Depth of Water	Bed 12 feet		Bed 14 feet		Bed 16 feet		Bed 18 feet		Bed 20 feet	
	A	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$	*A*	$\sqrt{R}$
Feet.										
.5	6.37	.68	7.37	.68	8.37	.69				
.75	9.84	.82	11.34	.82	12.84	.83	14.34	.83	15.8	.84
1	13.5	.93	15.5	.93	17.5	.94	19.5	.95	21.5	.95
1.25	17.34	1.02	19.84	1.04	22.34	1.04	24.84	1.05	27.34	1.05
1.5	21.38	1.11	24.37	1.12	27.37	1.13	30.37	1.14	33.37	1.15
1.75	25.59	1.18	29.09	1.2	32.59	1.21	36.09	1.22	39.59	1.23
2	30	1.25	34	1.26	38	1.28	42	1.29	46	1.3
2.25	34.59	1.31	39.09	1.33	43.59	1.34	48.09	1.36	52.59	1.37
2.5	39.38	1.37	44.37	1.39	49.37	1.4	54.37	1.42	59.37	1.43
2.75	44.34	1.42	49.84	1.44	55.34	1.46	60.84	1.48	66.34	1.49
3	49.5	1.47	55.5	1.5	61.5	1.51	67.5	1.53	73.5	1.54
3.25	54.84	1.52	61.34	1.55	67.84	1.56	74.34	1.58	80.84	1.6
3.5	60.38	1.57	67.37	1.59	74.37	1.61	81.37	1.63	88.37	1.65
3.75	66.09	1.61	73.59	1.64	81.09	1.66	88.59	1.68	96.09	1.69
4	72	1.65	80	1.68	88	1.7	96	1.72	104	1.73
4.25	78.09	1.69	86.59	1.72	95.09	1.74	103.6	1.76	112.1	1.78
4.5	84.38	1.73	93.37	1.76	102.4	1.78	111.4	1.8	120.4	1.82
4.75	90.84	1.76	100.3	1.79	109.8	1.82	119.3	1.84	128.8	1.86
5	97.5	1.8	107.5	1.83	117.5	1.86	127.5	1.88	137.5	1.9
5.25	104.3	1.83	114.8	1.86	125.3	1.89	135.8	1.91	146.3	1.91
5.5	111.4	1.87	122.4	1.9	133.4	1.93	114.4	1.95	155.4	1.97
5.75	118.6	1.9	130.1	1.93	141.6	1.96	153.1	1.98	164.6	2.01
6	126	1.94	138	1.97	150	2	162	2.02	174	2.04
6.25	133.6	1.96	146.1	2	158.6	2.03	171.1	2.05	183.6	2.08
6.5	141.4	2	154.4	2.03	167.4	2.06	180.4	2.09	193.4	2.11
6.75	149.4	2.02	162.9	2.06	176.4	2.09	189.9	2.12	203.4	2.14
7	157.5	2.05	171.5	2.09	185.5	2.12	199.5	2.15	213.5	2.17
7.25	165.9	2.08	180.4	2.12	194.9	2.15	209.4	2.18	223.9	2.2
7.5	174.4	2.11	189.5	2.15	204.4	2.18	219.4	2.21	234.4	2.23
7.75	183.1	2.14	198.6	2.17	214.1	2.21	229.6	2.24	245.1	2.26
8	192	2.17	208	2.2	224	2.21	240	2.27	256	2.28
8.25							250.6	2.3	267.1	2.31
8.5							261.4	2.32	278.4	2.34
8.75							272.5	2.34	289.8	2.37
9							283.5	2.37	301.5	2.4
9.25							294.8	2.4	313.3	2.42
9.5							306.4	2.42	325.4	2.45
9.75							318.1	2.45	337.6	2.47
10							[illegible]	2.47	350	2.5

TABLE XLVI—*Continued* (1½ to 1)

Depth of Water	Bed 25 feet.		Bed 30 feet.		Bed 35 feet.		Bed 40 feet.		Bed 45 feet.	
	A	√r	A	√r	A	√r	A	√r	A	√r
Feet.										
1	26·5	·96	31·5	·97	36·5	·97	41·5	·98	46·5	·98
1·5	40·88	1·16	48·38	1·18	55·88	1·18	63·38	1·18	70·88	1·19
2	56	1·32	66	1·33	76	1·34	86	1·35	96	1·36
2·25	63·84	1·39	75·09	1·4	86·34	1·41	97·59	1·42	108·8	1·43
2·5	71·88	1·45	84·38	1·47	96·88	1·49	109·4	1·49	121·9	1·5
2·75	80·09	1·51	93·84	1·53	107·6	1·55	121·3	1·56	135·1	1·57
3	88·5	1·57	103·5	1·59	118·5	1·61	133·5	1·62	148·5	1·63
3·25	97·09	1·63	113·3	1·65	129·6	1·67	145·8	1·68	162·1	1·69
3·5	105·9	1·68	123·4	1·7	140·9	1·72	158·4	1·73	175·9	1·75
3·75	114·8	1·73	133·6	1·75	152·3	1·77	171·1	1·79	189·8	1·8
4	124	1·78	144	1·8	164	1·82	184	1·84	204	1·85
4·25	133·3	1·82	154·6	1·85	175·8	1·87	197·1	1·89	218·3	1·9
4·5	142·9	1·86	165·4	1·89	187·9	1·91	210·4	1·93	232·9	1·95
4·75	152·6	1·9	176·3	1·93	200·1	1·96	223·8	1·98	247·6	2
5	162·5	1·94	187·5	1·97	212·5	2	237·5	2·03	262·5	2·04
5·25	172·6	1·98	198·8	2·01	225·1	2·04	251·3	2·07	277·6	2·08
5·5	182·9	2·02	210·4	2·05	237·9	2·08	265·4	2·11	292·9	2·13
5·75	193·3	2·06	222	2·09	250·8	2·12	279·6	2·15	308·3	2·16
6	204	2·09	234	2·13	264	2·16	294	2·18	324	2·2
6·25	214·8	2·13	246·1	2·16	277·3	2·2	308·6	2·22	339·8	2·24
6·5	225·9	2·16	258·4	2·2	290·9	2·23	323·4	2·26	356	2·28
6·75	237·1	2·19	270·9	2·23	304·6	2·27	338·4	2·29	372·1	2·32
7	248·5	2·22	283·5	2·27	318·5	2·3	353·5	2·33	388·5	2·35
7·25	260·1	2·25	296·4	2·3	332·6	2·33	368·9	2·36	405·1	2·38
7·5	271·9	2·29	309·4	2·33	346·9	2·36	384·4	2·39	421·9	2·42
7·75	283·8	2·31	322·6	2·36	361·3	2·39	400·1	2·43	438·8	2·45
8	296	2·34	336	2·39	376	2·42	416	2·46	456	2·48
8·25	308·4	2·37	349·6	2·42	390·9	2·45	432·1	2·49	473·4	2·51
8·5	320·9	2·4	363·4	2·45	405·9	2·48	448·4	2·52	490·9	2·55
8·75	333·6	2·43	377·3	2·48	421·1	2·51	464·8	2·55	508·6	2·58
9	346·5	2·46	391·5	2·5	436·5	2·54	481·5	2·58	526·5	2·61
9·25	359·6	2·48	405·8	2·53	452·1	2·57	498·3	2·61	544·6	2·64
9·5	372·9	2·51	420·4	2·56	467·9	2·6	515·4	2·64	562·9	2·66
9·75	386·4	2·53	435·1	2·58	483·9	2·63	532·5	2·66	581·3	2·69
10	400	2·56	450	2·61	500	2·65	550	2·69	600	2·72
10·5							585·4	2·74	637·9	2·77
11							621·5	2·79	676·5	2·83
11·5							658·4	2·84	715·9	2·83
12							696	2·89	756	2·93

TABLE XLVI —*Continued* (1½ to 1)

Depth of Water	Bed 50 feet		Bed 60 feet		Bed 70 feet		Bed 80 feet		Bed 90 feet	
	A	$\sqrt{P}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{R}$	A	$\sqrt{P}$
Feet										
1	51 5	98	61 5	98	71 5	98	81 5		91 5	
1 5	78 38	1 19	91 13	1 18	108 4	1 19	123 4		138 4	
2	106	1 36	126	1 37	146	1 37	166		186	
2 25	120 1	1 44	142 6	1 45	165 1	1 45	187 6		210 1	
2 5	134 4	1 51	159 4	1 52	184 4	1 53	209 4		234 4	
2 75	148 8	1 58	176 3	1 59	203 8	1 6	231 3		258 8	
3	163 5	1 64	193 5	1 65	223 5	1 66	253 5		283 5	
3 25	178 3	1 7	210 8	1 71	243 3	1 73	275 8		308 3	
3 5	193 4	1 76	228 4	1 77	263 4	1 79	298 4		333 4	
3 75	208 6	1 81	246 1	1 83	283 6	1 84	321 1		358 6	
4	224	1 86	264	1 88	304	1 9	344		384	
4 25	239 6	1 92	282 1	1 94	324 6	1 93	367 1		409 6	
4 5	255 4	1 96	300 4	1 99	345 4	2	390 4		435 4	
4 75	271 3	2 01	318 8	2 03	366 3	2 05	413 8		461 3	
5	287 5	2 05	337 5	2 08	387 5	2 1	437 5		487 5	
5 25	303 8	2 1	356 3	2 12	408 8	2 14	461 3		513 8	
5 5	320 4	2 14	375 4	2 17	430 4	2 19	485 4		540 4	
5 75	337 1	2 18	394 6	2 21	452 1	2 23	509 6		567 1	
6	354	2 22	414	2 25	474	2 27	534		594	
6 25	371 1	2 26	433 6	2 29	496 1	2 32	558 6		621 1	
6 5	388 4	2 3	453 4	2 33	518 4	2 36	583 4		648 4	
6 75	405 9		473 4		540 9		608 4		675 9	
7	423 5	2 37	493 5	2 4	563 5	2 43	633 5		703 5	
7 25	441 4		513 9		586 4		658 9		731 4	
7 5	459 4	2 44	534 4	2 47	609 4	2 51	684 4		759 4	
7 75	477 6	2 47	555 1		632 6	2 54	710 1		787 6	
8	496	2 5	576	2 54	656	2 57	736		816	
8 25	514 6		597 1		679 6		762 1		844 6	
8 5	533 4	2 57	618 4	2 61	703 4	2 64	788 4		873 4	
8 75	552 3	2 6	639 8	2 64	727 3		814 8		902 3	
9	571 5	2 63	661 5	2 67	757 5	2 71	841 5		931 5	
9 25	590 8		683 3	2 7	775 8	2 74	868 3		960 8	
9 5	610 4	2 69	705 4	2 73	800 4	2 77	895 4		990 4	
9 75	630	2 72	727 5	2 76	825	2 8	922 5		1020	
10	650	2 75	750	2 79	850	2 83	950		1050	
10 5	690 4	2 8	795 4	2 85	900 4	2 89	1006		1110	
11	731 5	2 86	841 5	2 9	951 5	2 94	1062		1173	
11 5	773 4	2 91	888 4	2 96	1003	3	1118		1233	
12	816	2 96	936	3 01	10 6	3 05	1176		1296	

TABLE XLVII.—SECTIONAL DATA FOR OVAL SEWERS (Art 3)

Metropolitan Oval

Dimensions	Full.		Two-thirds full		One third full.	
	A	$\sqrt{R}$	A	$\sqrt{r}$	A	$\sqrt{r}$
1 0" × 1 6"	1·15	·51	·76	·56	·28	·45
1 2" × 1 9"	1·16	·58	1·03	·61	·39	·49
1 4" × 2 0"	2·04	·62	1·31	·65	·51	·[illegible]3
1 6" × 2 3"	2·[illegible]8	·66	1·7	·69	·64	·[illegible]6
1 8" × 2 6"	3·19	·69	2·1	·73	·79	·59
1 10" × 2 9"	3·86	·73	2·54	·76	·99	·62
2 0" × 3 0"	4·[illegible]9	·76	3·02	·79	1·14	·64
2 2" × 3 3"	5·30	·79	3·55	·83	1·33	·67
2 4" × 3 6"	6·25	·82	4·12	·86	1·55	·69
2 6" × 3 9"	7·18	[illegible]	4·72	·88	1·78	·72
2 8" × 4 0"	8·17	·88	5·38	·92	2·02	·74
2 10" × 4 3"	9·22	·91	6·07	·95	2·28	·79
3 0" × 4 6"	10·31	·93	6·8	·97	[illegible] 6	·[illegible]9
3 2" × 4 9"	11·[illegible]2	·96	7·58	1	2·85	·81
3 4" × 5 0"	12·76	·98	8·4	1·03	3·16	·83
3 6" × 5 3"	14·07	1·01	9·26	1·05	3·48	·85
3 8" × 5 6"	15·41	1·03	10·16	1·08	3·8[illegible]	·87
3 10" × 5 9"	16·88	1·06	11·11	1·1	4·17	·89
4 0" × 6 0"	18·38	1·08	12·09	1·12	4·54	·91
4 2" × 6 3"	19·94	1·1	13·12	1·15	4·93	·93
4 4" × 6 6"	21·57	1·12	14·19	1·17	5·33	·95
4 6" × 6 9"	23·26	1·14	15·31	1·19	5·75	·96
4 8" × 7 0"	25·01	1·16	16·46	1·21	6·19	·98
4 10" × 7 3"	26·83	1·18	17·66	1·23	6·64	1
5 0" × 7 6"	28·71	1·2	18·9	1·26	7·1	1·02
5 2" × 7 9"	30·67	1·22	20·18	1·28	7·58	1·03
5 4" × 8 0"	32·67	1·[illegible]1	21·5	1·3	8·08	1·05
5 6" × 8 3"	34·[illegible]4	1·26	22·86	1·32	8·59	1·07
5 8" × 8 6"	36·88	1·28	24·[illegible]7	1·34	9·12	1·09
5 10" × 8 9"	39·08	1·3	25·[illegible]	1·36	9·66	1·1
6 0" × 9 0"	41·35	1·32	27·21	1·38	10·[illegible]2	1·11

TABLE XLVIII.—SECTIONAL DATA FOR OVAL SEWERS (Art 3)

Hawksley's Oval

Transverse Diameter	Full		Two thirds full		One third full	
	A	√P	A	√P	A	√R
1 0″	1	53	67	50	26	44
1 2″	1 36	57	91	6	35	48
1 4	1 77	61	1 19	64	46	51
1 6	2 24	64	1 51	68	58	54
1 8″	2 77	68	1 87	72	71	57
1 10″	2 35	71	2 25	75	86	6
2 0	3 98	74	2 69	79	1 03	63
2 2″	4 67	77	3 14	82	1·21	66
2 4	5 42	8	3 66	85	1 4	68
2 6	6 22	83	4 2	88	1 61	7
2 8	7 08	86	4 77	91	1 83	72
2 10″	7 89	89	5 38	94	2 06	74
3 0	8 97	91	6 04	96	2 31	77
3 2	9 98	94	6 73	99	2 58	79
3 4	11 06	96	7 46	1 02	2 85	81
3 6	12 2	98	8 22	1 04	3 15	83
3 8	13 38	1 01	9	1 07	3 45	85
3 10″	14 63	1 03	9 87	1 09	3 78	87
4 0	15 03	1 05	10 74	1 11	4 11	89
4 2	17 28	1 07	11 66	1 14	4 46	91
4 4″	18 69	1 09	12 57	1 16	4 82	93
4 6″	20 18	1 12	13 6	1 18	5 20	94
4 8	21 68	1 14	14 62	1 2	5 59	96
4 10″	23 25	1 16	15 68	1 22	6	98
5 0″	24 89	1 18	16 79	1 24	6 42	1
5 2″	26 57	1 2	17 92	1 27	6 86	1 01
5 4″	28 32	1·21	19 1	1 29	7 31	1 03
5 6″	30 11	1·23	20 26	1 31	7 76	1 04
5 8	31 56	1 25	51 5	1 33	8 24	1 06
5 10″	33 87	1 27	22 84	1 34	8 74	1 07
6 0″	35 84	1 29	24 17	1 36	9 25	1 09

TABLE XLIX.—SECTIONAL DATA FOR OVAL SEWERS (Art 3)

Jackson's Peg top Section

Dimensions	Full.		Two thirds full		One third full	
	A	$\sqrt{r}$	A	$\sqrt{r}$	A	$\sqrt{r}$
1' 0" × 1' 6"	1 039	·52	·646	·53	·242	·44
1' 2" × 1' 9"	1 414	·56	·88	·57	·33	·47
1' 4" × 2' 0"	1 846	·6	1 148	·61	·431	·5
1' 6" × 2' 3"	2 337	·63	1 473	·65	·545	·53
1' 8" × 2' 6"	2 885	·67	1 793	·68	·63	·56
1' 10" × 2' 9"	3 491	·7	2 115	·72	·813	·59
2' 0" × 3' 0"	4 154	·73	2 583	·75	·969	·62
2' 2" × 3' 3"	4 874	·76	3 032	·78	1 136	·64
2' 4" × 3' 6"	5 654	·79	3 516	·81	1 319	·67
2' 6" × 3' 9"	6 491	·82	4 034	·84	1 513	·69
2' 8" × 4' 0"	7 385	·84	4 593	·86	1 722	·71
2' 10" × 4' 3"	8 337	·87	5 184	·89	1 943	·73
3' 0" × 4' 6"	9 347	·89	5 813	·92	2 179	·76
3' 2" × 4' 9"	10 41	·92	6 478	·94	2 427	·78
3' 4" × 5' 0"	11 54	·94	7 172	·97	2 692	·8
3' 6" × 5' 3"	12 72	·97	7 912	·99	2 967	·82
3' 8" × 5' 6"	13 96	·99	8 401	1 01	3 254	·84
3' 10" × 5' 9"	15·26	1 01	9 492	1·03	3 556	·85
4' 0" × 6' 0"	16 62	1 03	10 33	1 06	3 874	·87
4' 2" × 6' 3"	18 03	1 06	11 22	1 08	4·201	·89
4' 4" × 6' 6"	19 5	1 08	12 13	1 1	4 542	·91
4' 6" × 6' 9"	21 03	1 1	13 08	1 12	4 903	·93
4' 8" × 7' 0"	22 62	1 12	14 07	1 14	5 274	·94
4' 10" × 7' 3"	24·26	1 14	15 09	1 16	5 663	·96
5' 0" × 7' 6"	25 96	1 16	16 14	1 18	6·064	·98
5' 2" × 7' 9"	27 72	1 18	17·24	1 2	6 46	·99
5' 4" × 8' 0"	29 54	1 19	18 37	1 22	6 844	1 01
5' 6" × 8' 3"	31 42	1 21	19 54	1 24	7 321	1 02
5' 8" × 8' 6"	33 35	1 23	20 74	1 26	7 77	1 04
5' 10" × 8' 9"	35 34	1 25	21·98	1 28	8·234	1·06
6' 0" × 9' 0"	37 39	1 27	23 25	1 3	8 718	1·07

Table L —Ratios of Combined Length of Two Side slopes to Depth of Water

Side slope	=½ to 1	¾ to 1	1 to 1	1⅓ to 1	1½ to 1	2 to 1	2½ to 1	3 to 1
Ratio	=2 236	2 5	2 828	3 333	3 606	4 472	5 385	6 325

These ratios can be used for calculating R for channels outside the range of tables xliii xlvi

Table La —Circular Channels partly full (Art 6)

The Diameter of the Channel is supposed to be 1.

Depth of Water	Angle subtended by Wet Portion of Border	Relative Values of A	Relative Values of $\sqrt{R}$
Feet 25	120°	196	707
5	180°	5	1
75	240°	804	1 1
1	360°	1	1

For actual values of A and $\sqrt{R}$ see table xxiii , page 142.

CHAPTER VII

OPEN CHANNELS—VARIABLE FLOW

[For preliminary information see chapter II articles 10 to 14 and 17 to 21]

SECTION I—BENDS AND ABRUPT CHANGES

1 Bends —The loss of head at a change in direction in an open stream is, as in the case of a pipe, greater for an elbow than for a bend The formula for loss of head at a bend arrived at by observations on the Mississippi is $H=\frac{V^2 \sin^2\theta}{134}$ where θ is the angle subtended by the bend This takes no account of the radius In a bend of 90° the loss of head by this formula is $48\frac{V^2}{2g}$ Generally a single bend with ordinary velocities causes little heading up, but if a stream has a long succession of bends their cumulative effect may be considerable It is practically the same as that of an increase of roughness, and may be allowed for by taking a lower value of the co-efficient C How far the loss of head at a bend depends on the radius of the bend is not known (Cf chap v art 4)

At a bend there is a 'set of the stream towards the concave bank, the greatest velocity being near that bank, and there is a raising of the water level there, so that the surface has a transverse slope (Fig 117) There is also a deepening near the concave bank and a shoaling at the opposite one, but this is not all due to the direct action of centrifugal force The high water level at the concave bank, due to centrifugal force, gives a greater pressure and tends to cause a transverse current from the concave towards the convex bank This tendency is, in the greater part of the cross section, resisted by the centrifugal force But the water near the bed and sides has a low velocity, the centrifugal

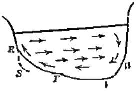

FIG. 117

force is therefore smaller, and transverse flow occurs Solid material is thus rolled towards the convex bank, and it accumulates there because the velocity is low To compensate for the low level current towards the convex bank there are high level currents towards the concave bank The directions of the currents are shown by the arrows on Fig 117 In Fig 118 the dotted line shows the direction of the strongest surface current and the arrows the currents near the bed This explanation is due to Thomson, and has been confirmed by him experimentally When the channel is of masonry or even very hard soil the deepening *TVW* cannot occur, but the bank *RST* may still be formed, the material for it being brought down by the stream

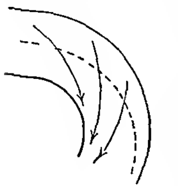

Fig 118

As the transverse current and transverse surface slope cannot commence or end abruptly there is a certain length in which they vary In this length the radius of curvature of the bend and the form of the cross section also tend to vary This can often be seen in plans of river bends the curvature being less sharp towards the ends This principle has been adopted in constructing river training walls and it appears to be sound as tending against any abruptness in the change of section For training walls to remove bars at the mouth of the Mississippi it has been proposed to construct instead of two walls only one wall having a curve concave to the stream The success of this plan would appear to depend on whether the curve is sharp enough to ensure the stream keeping close to the wall and not going off in another direction

The sectional area of a stream is often less at a bend than in straight reaches, especially when the channel is hard, so that the stream cannot excavate a hollow to compensate for the silt bank, but the surface width is often greatest at bends, and in constructing training walls the width between the walls is sometimes increased at bends In the silt clearances of some tortuous canals in India it was once the custom to remove the silt *RST*, the dotted line showing the section of the cleared channel in the straight reaches No allowance was made for the hollow *TVW* A silt-bank so removed quickly forms again Its removal is equivalent to the digging of a hole or recess in the bed

When once a stream has assumed a curved form, be it ever so slight, the tendency is for the bend to increase The greater velocity and greater depth near the concave bank react on each other, each inducing the other The concave bank is worn away, or becoming vertical by erosion near the bed, cracks, falls in, and is washed away The bend may go on increasing as indicated by the dotted lines in Fig 119, a deposit of silt occurring at the convex bank, so that the width of the stream remains tolerably constant Some of the large Indian rivers flowing through alluvial soil sometimes cut away, at bends, hundreds of acres of land, together with the trees, crops, and villages standing thereon Works to check the erosion would cost many times as much as the value of the property to be saved When a bend has formed in a channel previously straight, the stream at the lower end of the bend, by setting against the bank, tends to cause another bend of the opposite kind to the first Thus the tendency is for the stream to become tortuous, and while the tortuosity is slight the length, and therefore the slope and velocity, are little affected, but the action may continue until the increase in the length of the stream materially flattens the slope, and the consequent reduction in velocity causes erosion to cease Or the stream during a flood may find, along the chord of a bend, a direct route, with of course a steeper slope Scouring a channel along this route it straightens itself, and its action then commences afresh

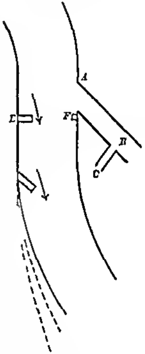

Fig. 119

2 Changes of Section —An "obstruction" is anything causing an abrupt decrease of area in a part of the cross section of a stream such as a pier or spur There may or may not be a decrease in the sectional area of the stream as a whole There is a tendency to scour alongside an obstruction owing to the increased velocity, and downstream of it owing to the eddies When a spur is constructed for the purpose of deflecting a stream or checking erosion of the bank, the scour near the end of the spur may be very severe, even though there may be very little contraction of the stream as a whole If the bed is soft the spur may be undermined A continuous lining of the bank with

protective material is not open to such an attack Similarly a hole may be formed alongside of and downstream of a bridge pier The hole may work back to the upstream side of the obstruction though there is little original tendency to scour there

When an obstruction reaches up to the surface, or nearly up to it, there is a heaping up of the water on its upstream side due to the checking of the velocity In the eddy downstream of an obstruction the water level is depressed The changes of water level and velocity are local, that is, they do not necessarily extend across the stream, and they are independent of the effects of any general change—supposing such to occur—in the sectional area of the stream Their amounts cannot be calculated, but they often have to be recognised They should be avoided in observing water levels where accuracy is required, as for instance when finding the surface slope The discharge of a branch will be increased by a spur or obstruction just below it, and decreased by one just above it On some irrigation canals in India, where the velocity is high and the channel of boulders, the cultivators sometimes run out small spurs below their water course heads in order to obtain more water

An obstruction causes a 'set of the stream,' that is a strong current, as shown by the arrows in Fig 119, but the distance to which such a current extends depends entirely on its impetus, and is not usually great If a spur is merely intended to cause slack water or silt deposit on its own side of the stream several short spurs will do as well as one long one, but when the object is to cause a stream to set against the opposite bank the spurs may have to be very long

In a short deep recess in the bed or bank of a stream or down stream of an obstruction if it is large enough to cause dead water, there is generally a rapid deposit of silt, but not where strong eddies occur

When an obstruction causes material reduction of the section of the stream the velocity past it is increased, and the scour may be excessive, both from the high velocity past it and (if there is a subsequent expansion of the stream) the eddies downstream of it Thus a partly formed dam *EF* (Fig 119) is, unless the gap is quickly closed, liable to be destroyed by the stream, and so is any structure which reduces the water way In order to lessen scour of the banks downstream of contracted water ways the channel is sometimes widened out so as to form a basin in which the eddies exhaust themselves

3 **Bifurcations and Junctions**—The general effects of these have been stated in chapter II (art 20) In an irrigation distributary constructed in India the velocity was exceptionally high, and it was found that the discharges of some narrow masonry outlets, taking off from the distributary at right angles, were so small that it became necessary to rebuild them at a smaller angle On the other hand, it was once the custom to build the heads of the distributaries themselves at an angle of 45° with the canal, but they are now built at right angles The velocity in the canal is 2 or 3 feet per second, and that in the distributary less A slight fall into the distributary is not objectionable A skew head is suitable in cases where loss of head is not permissible

When there is a bend in the main stream importance is sometimes attached to the set of the stream as affecting the supply in a branch taking off on the concave bank The velocity in the branch is that due to its slope and to the depth of water in it The advantage possessed by the branch as compared with one on the opposite bank is the greater depth of water, owing to velocity of approach This advantage is small except in the case of a sharp bend and a high velocity A river about 20 feet deep was eroding the concave bank at a bend An attempt was made to divert it by a straight cut, about a mile long across the bend Owing to the high level of the sub-soil water, the cut could only be dug down to about 2 feet below the water level of the river The slope of the cut was about one and a half times that of the river, but owing to the small depth of water the velocity was low, and the cut or at least its upper part, rapidly silted up The reason given for its failure was that its head was not so placed as to catch the set of the stream at the bend next above This set might have given an inch or two more water and the cut might have taken a few days longer to silt up

Sometimes the deposit of silt in a branch channel is attributed to some peculiarity in its offtake, such as the angle of offtake or the arrangement of the head gates If these matters have any importance it can only be when they affect the stratum of water drawn upon (thus altering the silt charge or the amount of rolled material brought in), or when they directly affect the gauge reading or depth of water which, in a given channel is the only factor governing the velocity, and, so far as is known, the silt-supporting power

In river diversion works spurs are sometimes used to 'drive the river' down a branch channel A spur may make the current set against the branch head (art 1), but unless the spur is so long

as to greatly contract the water way, the rise of water level will not be great except in cases of very high velocities, and the river will continue to distribute itself according to the discharging capacities of the two branches It is only by closing or thoroughly obstructing one branch or enlarging the other that the stream can be forced to alter its distribution of discharge

At a junction of one stream with another there are the usual eddies and inequalities in the water level, all depending as before on the sharpness of the angle and on the velocity When the main stream is not much larger than the tributary, the latter may cause a set of the current against the opposite bank and erode it

4 Relative Velocities in Cross section —In every case of abrupt contraction in a stream there are (chap ii art 21) eddies which extend back to the point where the fall in the surface begins Upstream of these eddies the distribution of the velocities in the cross section is not affected In the case of a pier, even a wide one, in the middle of a straight uniform stream, the maximum velocity remains in mid stream till just before the pier is reached If a plank or gate obstructs the upper portion of a stream from side to side, the surface velocities are affected for only a short distance upstream A spur or sudden decrease of width causes slack water for only a short distance In all these cases the state of the flow further upstream, as far as regards the distribution of the velocities is precisely the same as if no obstruction existed In the case of a weir visual evidence is wanting, but by analogy the same law holds good

SECTION II —VARIABLE FLOW IN A UNIFORM CHANNEL

(General Description)

5 Breaks in Uniformity —Variable flow may be caused by a change in slope (Figs 16 and 17, pp 24 and 25) or in roughness (Figs 120 and 121) by a debouchure into a pond or river (Figs 122 and 123), by a weir (Figs 124 and 125) by a change in width (Figs 126 and 127) or in bed level (Figs 1[illegible] and 1[illegible]9) Heading up may be caused by a local contraction or submerged weir

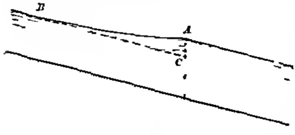

Fig 1[illegible]a

(Fig 130), but the analogous case of a local enlargement has no effect A change of hydraulic radius seldom occurs without a change of sectional area, and it need not therefore be considered as a separate case A bend generally causes some degree of heading up In each case the line BC is the 'natural water surface' of the upper reach, that is, the surface as it would have been if no change had occurred The profiles of the water-surface touch the natural surface at points far upstream Above

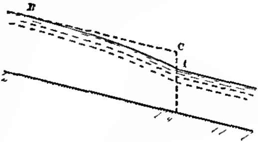

Fig 121

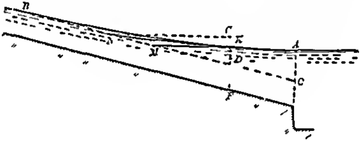

Fig 122

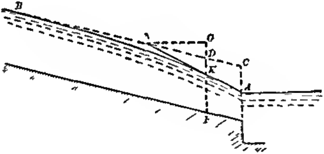

Fig 123

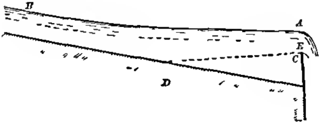

Fig 124

as to greatly contract the water way, the rise of water level will not be great except in cases of very high velocities, and the river will continue to distribute itself according to the discharging capacities of the two branches It is only by closing or thoroughly obstructing one branch or enlarging the other that the stream can be forced to alter its distribution of discharge

At a junction of one stream with another there are the usual eddies and inequalities in the water level, all depending as before on the sharpness of the angle and on the velocity When the main stream is not much larger than the tributary, the latter may cause a set of the current against the opposite bank and erode it

4 **Relative Velocities in Cross section** —In every case of abrupt contraction in a stream there are (chap ii art 21) eddies which extend back to the point where the fall in the surface begins Upstream of these eddies the distribution of the velocities in the cross section is not affected In the case of a pier, even a wide one, in the middle of a straight uniform stream, the maximum velocity remains in mid stream till just before the pier is reached If a plank or gate obstructs the upper portion of a stream from side to side, the surface velocities are affected for only a short distance upstream A spur or sudden decrease of width causes slack water for only a short distance In all these cases the state of the flow further upstream, as far as regards the distribution of the velocities, is precisely the same as if no obstruction existed In the case of a weir visual evidence is wanting, but by analogy the same law holds good

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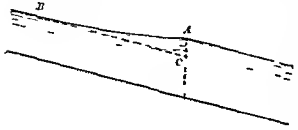

Fig 1[illegible]a.

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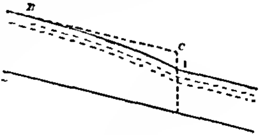

Fig 121

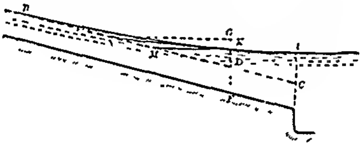

Fig 122

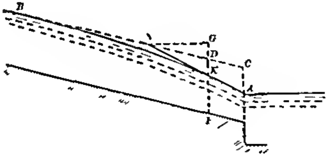

Fig 123

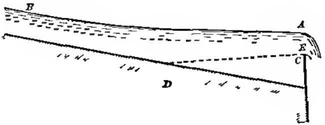

Fig 124

as to greatly contract the water way, the rise of water level will not be great except in cases of very high velocities, and the river will continue to distribute itself according to the discharging capacities of the two branches It is only by closing or thoroughly obstructing one branch or enlarging the other that the stream can be forced to alter its distribution of discharge

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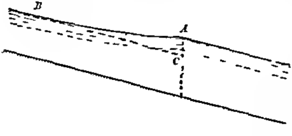


Fig 1[illegible]a

(Fig. 130), but the analogous case of a local enlargement has no effect. A change of hydraulic radius seldom occurs without a change of sectional area, and it need not therefore be considered as a separate case. A bend generally causes some degree of heading up. In each case the line *BC* is the 'natural water surface' of the upper reach, that is, the surface as it would have been if no change had occurred. The profiles of the water-surface touch the natural surface at points far upstream. Above

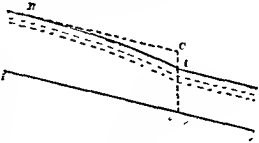


FIG. 121

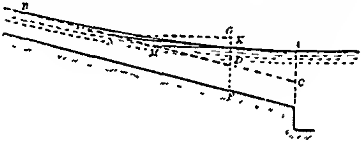


FIG. 122

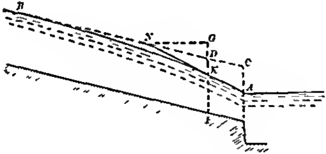


FIG. 123

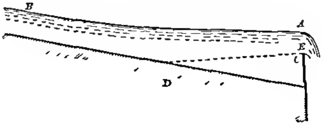


FIG. 124

these points the flow is uniform if the reach extends far enough. In heading up there is a tendency to silt and in drawing down to scour.

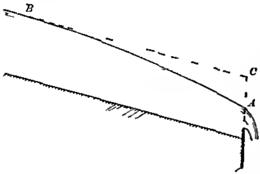


Fig 1°

In the cases shown in Figs 126 to 130 there are abrupt changes in the sectional area, falls in the surface when the area decreases, and perhaps rises where it increases (chap ii arts 18 and 19). In Figs 124 and 125 the weir formula gives the discharge having reference to the surface above the local fall which therefore need not be considered. In the other cases there are no abrupt changes in section and therefore no local changes in level.

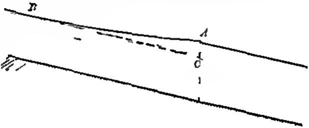


Fig 1

A change of one kind may be combined with another so that the change of water level is altered or suppressed. For instance the changes of roughness may be accompanied by changes in slope so that the water level in the lower reach is at C and the flow is uniform but any local falls or rises due to abrupt change of section (Figs 126 to 130) will remain. The rises are generally however negligible, and the falls are much reduced if the changes are not actually sudden (chap ii art 17).

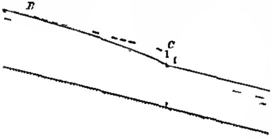


Fig 1

In all cases whatever the upstream level has to accommodate itself to the downstream level. The water level in the lower reach or pond or on the crest of the fall is known or can be ascertained. The local fall or rise, if any, must be found and there will be heading up or drawing down or neither in the reach above according as

the level found is above or below or equal to the natural level in that reach.

When the variable flow extends upstream to a point where there is another break in uniformity the flow in the reach is said to be "variable throughout." If the bed of the reach is level or slopes upward (Figs. 135 and 136 p. 240) the flow must be variable throughout however long the reach may be and the surface convex upward.

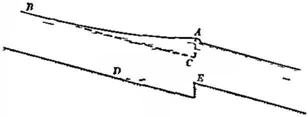


Fig. 128.

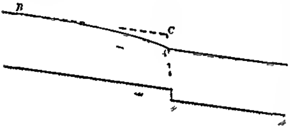


Fig. 129.

In a uniform channel let CD (Fig. 131) be a "flume" of the same section as the rest of the channel but of smoother material. If the flume extended upstream far enough the water surface would be CGH. Actually it will be CGL, GL being a curve of drawing down. The height DG will generally be very small and no appreciable change in the velocity will be caused, but if surface slope observations are made a serious error may occur if the upstream point of observation falls at M. The slope required is $ECDL$; that actually observed is EM. Often a flume has vertical sides and is of a different section to the rest of the channel. If the change is made gradually there may possibly be no interference with the straight line of the water surface; the smaller

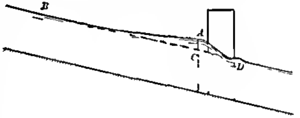


Fig. 130.

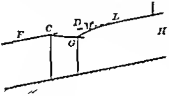


Fig. 131.

sectional area and hydraulic radius of the flume compensating for its smoother material. But this is not likely to be the case exactly. If the change of section is abrupt there will be a change in the water level at the entrance of the flume. In the Roorkee Hydraulic Experiments observations were made in a masonry aqueduct 900 feet long in the Ganges Canal. The surface slope, instead of being observed within the aqueduct, was obtained from points lying far outside it in the earthen channel, and the results of the experiments, so far as concerns the relation between slope and velocity in masonry channels, were vitiated.[1]

6. Bifurcations and Junctions.—A bifurcation or junction may cause variable flow upstream of it. At a junction let Q_1 and Q_2 be the discharges of the two tributaries. The flow in the main stream is uniform, and its water level is that corresponding to the discharge Q_1+Q_2. If the conditions of the debouchure of either tributary are such as to cause any local fall or rise, the amount of this must be estimated, and the water level in the tributary just above the junction is then known. There will be heading up or drawing-down or neither in the tributary, according as its natural water level is below or above or equal to that so found. There may be heading up in one tributary and drawing down in the other.

At a bifurcation let Q be the discharge of the main stream. The flow in the branches is uniform. Assume discharges Q_1 and Q_2 for them—Q_1+Q_2 being equal to Q—and find their water levels. Allow for any local fall or rise, and if the water levels upstream of them are equal the assumed discharges Q_1 and Q_2 are correct, and the water level found is that of the main stream. If they are not equal it is necessary to alter the quantities Q_1 and Q_2 and make a second trial. In the main stream there will be heading up or drawing down or neither, according as the water level found is higher or lower than, or equal to, its natural water level. If a stream flows out of a reservoir the flow will be uniform down stream of the fall in the surface (chap. iv, art. 15) which occurs at the head. If more than two streams meet or separate at one place the discharges Q_1, Q_2, Q_3, etc., must be considered, and the above processes adopted. The variable flow caused by a junction or bifurcation may be counteracted wholly or partly by any other cause, just as in the other instances of variable flow.

In a paper[2] on the designing of trapezoidal notches at canal falls it has been observed that a distributary usually takes off a short

[1] *Transactions Society of Engineers* 18[illegible]

[2] *Punjab Irrigation Branch Paper No.* [illegible]

distance above a fall, and that though the notch must obviously be able to pass the whole discharge when the distributary is closed, it has to be settled in each case whether the design of the notch should be such as to cause draw when the distributary is open or heading up when it is closed The question must occur with every distributary, and not only with those taking off above falls If the canal is designed so as to give uniform flow with the distributary closed, then there must be draw when it is open If there is uniform flow when the distributary is open, there must be heading up when it is closed The best arrangement depends on engineering considerations which need not be discussed here

The opening of an escape or branch may cause scouring upstream of it. One method of freeing the upper reach of a canal from silt is to make an escape from a point some distance below its head leading back to the river If there is a weir across the river the slope of the escape may be great By opening the escape scour is caused in the canal, but this may cause some deposit in the canal downstream of the escape, unless it can be shut off when the escape is opened

There were once to be seen in a large canal two gauges, one just above and the other just below the offtake of an escape channel It was stated that the two gauges had been erected in order that, by noting the difference of their readings, the quantity of water passing down the escape could be estimated Both gauges were carefully read, and copies of the readings sent to various officials But when the escape was opened the water level on the upper gauge fell practically as much as that on the lower one Both gauges always read the same The assistant in charge put up a temporary gauge half a mile upstream This also fell when the escape was opened The proper arrangement in such a case is to have one gauge in the canal below the escape and one in the escape Again, some irrigators who wanted a new water course were anxious that its offtake should be placed just above and not just below the offtake of an existing water course Practically it made no difference whether it was above or below There was no sudden fall in the water level of the canal If a branch whose discharge is to be q is to be supplied from a channel whose discharge is Q, it is necessary first to find what the water level in the channel will be when its discharge is $Q-q$ and then to design the branch so that it will obtain a discharge q with the water level thus found

7 **Effect of Change in the Discharge** —An increase or decrease of

the discharge is always accompanied by a rise or fall of the water level throughout every reach except at the points A (Figs 122 and 123), where the stream enters or leaves a river or pond whose water level is not affected by the alteration of discharge It is clear, however, that for a given change of discharge the changes in the water levels at two distant points may be very different from one another In changes of slope, roughness, width, or bed level, a change in the discharge causes no change in the character of the flow, that is, there is always heading up or draw, whichever there was at first In a local contraction there is always heading up and also with a drowned weir if there is no fall in the bed In the other cases (change of bed level, weirs debouchures) there will be heading up if the supply falls low enough, and drawing down if it rises high enough (See also chap IV arts 12, 15, and 17)

At a bifurcation, if the branches are such that the flow in the main stream is uniform with the average discharge, and if the beds of all three channels are at one level, the flow in the main stream will probably be nearly uniform with all discharges At a junction a similar rule obtains only if the discharges of the *tributaries vary in the same proportion*

Above a weir or a rise in the bed the water approaches the line DF (Figs 124 and 128) as the discharge is reduced, the tendency to silt increases, supposing the water to be silt laden, and deposit will doubtless occur if the discharge falls low enough A fall in the bed (*Fig* 129) is converted into a clear 'fall' (*Fig* 79, p 99) at low supply, and in that case there will probably be scour or 'cutting back' owing to the high velocity

8 **Effects of Alterations in a Channel** —When a natural or artificial change occurs in a channel, such as deepening, widening, silting, the erection or removal of a structure, or the manipulation of a gate or sluice, the consequent change of water level may exte

the

the depth at the head of the channel remains constant, [illegible] surface slope alters, and with it the discharge, or a change in the channel may cause an alteration in the quantity of water lost by evaporation, percolation, or flooding, and so affect the discharge But if the discharge of the channel is unaltered, the effect on the water level and velocity caused by any change in the channel is wholly upstream The building for instance, of a weir in a stream ordinarily causes little difference to persons further down the stream as long as water is not permanently diverted

In a discussion [1] on some oblique weirs erected in the Severn it is implied that the weirs caused a lowering of the flood level and a deepening upstream. Above the weirs basins had been made by widening the channel, and the widening might, by itself, have caused some slight reduction in the flood level, but not when a weir was added. It was not contended that the flood discharge at the weir was reduced. The water level at D (Fig. 130) would therefore be the same as it was originally, and since there must always be some fall from A to D, the flood level at A must have been raised. No deepening due to the weir could occur except close alongside a very oblique weir. (See also chap. iv. art. 18.)

Upstream of a place where changes occur a gauge reading affords no proper indication of the discharge, and a discharge table, if it can be made at all, must be one of double entry, showing the discharge as depending not only on the gauge reading, but on other conditions. If gates or shutters are worked there may be any number of water surfaces corresponding to one discharge. An instance of this has already been given in the case of the flow upstream of an escape. Gauges are sometimes fixed in canals near their heads, and tables are made showing the discharges as depending on the gauge reading. The deposit of silt in the heads alters the discharges, vitiates the tables, and destroys the utility of statistics based on the discharges obtained from them. Gauges ought to be placed below the reaches in which the deposits occur. The deposit of silt changes both the section and the slope, and it is next to impossible to allow for it by merely observing the depth at the gauge.

Sometimes shoals or masses of silt travel down a stream. On the Western Jumna Canal there is a gauge at Jhind and another about twenty miles upstream. When the upper gauge is kept steady that at Jhind sometimes slowly fluctuates in a curious manner, although no water is drawn off in the intervening reaches. This has been ascribed to travelling masses of silt, and no other explanation presents itself.

If a channel AB (Fig. 119) is drawn from a source whose water level is not affected, and if, near the head of the channel, a branch BC is taken off, the discharge of the channel below B may be very little affected. A very slight lowering of the water level at B increases the slope AB, and causes more water to be drawn in. The water level in the channel may rise slightly at L (chap. ii. art. 20). A case occurred in which an engineer, wishing to

[1] *Minutes of Proceedings, Institution of Civil Engineers*, vol. lx.

reduce the supply in an overcharged canal, caused a breach to be made in the bank a short distance below its off take from the river He was surprised to find, that although a large volume of water passed out of the breach, there was no appreciable diminution of the canal discharge below the breach In the case of an irrigation distributary which takes out of a canal and has itself a number of water courses taking out of it not far from its head, the discharge of the distributary may partly depend on whether the water courses are open or not (Cf case of branched water main, chap v art. 3)

Let a straight cut be made across a bend in a uniform stream The slope in the cut is increased and the longitudinal section is as in Fig 132 If the discharge is unaltered the water level at B is as before, and there is tendency to scour at A and to silt at B The bed and water surface tend to assume the positions shown by the dotted lines, and the probability of this occurring must be considered in making a cut If it is desired to keep the water level at A the same as before, the cut AB must be made smaller than the original channel, but the velocity in it will be greater, and there will therefore be a still greater tendency to scour If the abandoned loop is left open the velocity in it will be greatly reduced, owing to the lower water level at A, and at B will be further reduced by heading up It generally silts up

FIG 132

To increase the discharge of a channel ABC (Fig 130, p 210), supposed to be of shallow section, without enlarging it throughout, the plan involving least work is to alter the bed to DB As D recedes from A the discharge increases, but so does the tendency to silt (Cf chap vi art 2)

9 **Effect of a Weir or Raised Bed**—*The tendency to silting* common to all cases of heading up, may be somewhat enhanced in the case of a rise in the bed or a weir extending across a channel, because of the obstruction offered to rolling material This however does not seem to be very great The silt may *form a long* slope against the weir, and material may be rolled up the slope Usually even this slope is not formed Probably the eddies stir up the silt, and it is carried over

The deposit occurring upstream of a rise or a weir *has caused it* to be supposed that there is a layer of still water upstream of and

below the level of the crest This idea is absolutely untenable The general velocity undoubtedly decreases as the rise or weir is approached This is due to the increasing section of the stream If the water below DF (Figs 124 and 128) were still the section would be decreasing The same amount of heading up might be caused by obstructions of other forms, but it has been shown, (art 4) not only that the water upstream of them is moving, but that upstream of the eddies not even the distribution of the velocities is affected The same is no doubt true of a rise or weir If in a silt-bearing stream the water near the bed were still, there would be a rapid deposit of silt as there is in a short hollow or recess But the contrary often happens In some of the large canals in India the bed upstream of bridges has been scoured for miles, to a depth of perhaps two feet below the masonry floors of the bridges which are left standing up, and forming, in fact, submerged weirs This alone shows the preposterous nature of the still water theory

The idea might have been supposed to be exploded, but for a somewhat recent case In a paper on the Irrawaddy[1] it is stated that, if the discharges for the water levels A, C, etc (Fig 133), are plotted, the discharge seems to become zero at E, which is level with a sand bar four miles down stream, although the depth EG was 34 feet, and that 'this dead area of cross section lying below the level of the bar regulating the discharge, exists on almost all rivers' It is natural that the discharge should become zero at E As the water level falls the effect of the obstruction at F increases (art 7), and the surface slope becomes flatter If the water level ever fell to E the surface would be horizontal and the discharge zero But the reduction of the discharge to zero is due to the flattening of the slope, and not to a portion of the section of the stream being still If it were still it could never have been scoured out, or being in existence it would quickly silt up

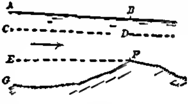


FIG 133

'Profile walls' are sometimes built across a channel at intervals They are useful for showing the correct form of the cross section, but will not prevent scour, unless built extremely close together A single wall built at a point where the bed slope becomes steeper will not prevent scour If scour does occur, walls or weirs will of course stop it eventually

[1] *Minutes of Proceedings, Institution of Civil Engineers*, vol. cxiii.

In clearing the silt from a canal it is often convenient to make the level of the cleared bed coincide with the level of a masonry bridge floor, but it is not a fact that any deeper clearance is useless. The deeper bed gives an increased discharge for the same water level, and there is not necessarily a deposit of silt upstream of the raised floor. Similarly, there is no particular harm in omitting the clearance in any reach where, the depth of the deposit being small, say half a foot, it is troublesome to clear it.

SECTION III.—VARIABLE FLOW IN A UNIFORM CHANNEL

(Formulæ and Analysis)

10. Formulæ.—To find the length L between two points where the depths are D_1 and D_2 (Fig. 134) let S be the bed slope. Then

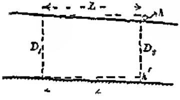


FIG. 134

$$h = D_1 - D_2 + LS$$

And from equation 17, p. 22,

$$L = \frac{C^2R}{V^2}(D_1 - D_2 + LS + h_v)$$

or $V^2L - C^2RS\,L = C^2R(D_1 - D_2 + h_v)$

Therefore
$$L = \frac{C^2R(D_1 - D_2 + h_v)}{V^2 - C^2RS} \quad . \quad (74),$$

where C, R, and V have values suited to the mean section between the two points. The quantity h_v is nearly always small compared to $(D_1 - D_2)$. In heading up $(D_1 - D_2)$ and $(V^2 - C^2RS)$ are negative, so that in equation 74 both numerator and denominator are negative. In drawing down the above quantities are positive.

To find the surface slope S at any point, consider a point midway between the two sections, and suppose them very near together, so that the changes are very small. Let $V_1 - V_2 = v$, then $V_1^2 - V_2^2 = \left(V + \frac{v}{2}\right)^2 - \left(V - \frac{v}{2}\right)^2 = 2Vv$ and equation 17 becomes

$$h = \frac{V^2L}{C^2R} - \frac{Vv}{g} \qquad (75)$$

Let A be the sectional area and B the surface width at the midway point. Let a be the difference in area in the length l. Then
$$Q = VA = \left(V + \frac{v}{2}\right)\left(A - \frac{a}{2}\right) = VA + \frac{vA}{2} - \frac{Va}{2} - \frac{va}{4},$$

neglecting the very small last term, $vA = Va$ or $v = \frac{aV}{A}$

Therefore from equation 75, $h=\frac{V^2L}{C^2R}-\frac{V^2a}{gA}$ But $a=L(D_2-D_1)$ and if d is the mean depth in the cross section, $A=l\,d$

Therefore $h=\frac{V^2L}{C^2R}-\frac{V^2}{g}\cdot\frac{D_2-D_1}{d}=\frac{V^2L}{C^2R}-\frac{V^2}{gl}(LS-h)$

or
$$h\left(1-\frac{V^2}{gd}\right)=L\left(\frac{V^2}{C^2R}-\frac{V^2S}{gl}\right)$$

Therefore
$$S=\frac{h}{L}=\frac{V^2}{C^2R}\;\frac{1-\frac{C^2RS}{gl}}{1-\frac{V^2}{gd}} \qquad (76)$$

The difference between the bed slope and the surface slope is

$$S-S=\frac{S\left(1-\frac{V^2}{gl}\right)-V^2\left(\frac{1}{C^2R}-\frac{S}{gl}\right)}{1-\frac{V^2}{gl}}=\frac{S-\frac{V^2}{C^2R}}{1-\frac{V^2}{gl}} \qquad (77)$$

The fraction by which $\frac{V^2}{C^2R}$ is multiplied in equation 76 is the ratio of the surface slope to what it would be in a uniform stream with the same velocity and hydraulic radius This fraction may be written $\frac{1-\frac{V'^2}{gd}}{1-\frac{V^2}{gl}}$ where V' is the velocity in a uniform stream with the same values of C and R but with a slope equal to the bed slope For ordinary depths and velocities the numerator is not much less than unity In cases of heading up the denominator is still nearer unity, but in drawing down less so In a stream of shallow section R is nearly as d and V is as $\frac{1}{d}$, so that, neglecting the above fraction S is for moderate changes in depth roughly as $\frac{1}{d}$ In order that the slope obtained by observing the water levels at the ends of a reach may agree with the local slope at the centre of the reach, the sectional areas of the stream at the two ends of the reach must not differ, in ordinary cases, by more than 10 or 12 per cent

Equation 76 establishes a direct connection between the depth at any cross section and the surface slope at that section, but not the connection between the depth or slope at any section and the position of the section To find this, the profile must be worked

out in short reaches (restricted as above as to length) by equation 74, or by a method which will be given below

To find the length of a tangent from any point K (Figs 122 and 123, p 229) to N, where it meets the line of natural water surface. Let D be the depth at K and D' the natural depth. Let $GN=x$, $GD=y$. Then $y=xS$ and $y+D-D'=xS'$. Therefore $D-D'=x(S'-S)$

and $$x=\frac{D-D'}{S'-S}=(D-D')\frac{1-\frac{V^2}{gd}}{S-\frac{V^2}{C^2R}} \qquad (78)$$

When the bed is level or slopes upward (Figs 135 and 136)

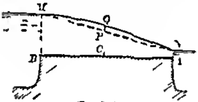


Fig 135

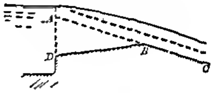


Fig 136

S' in equations 74 and 76 is zero or negative. In the former case

$$L=\frac{C^2R(D_1-D_2+h_v)}{V^2} \qquad (79)$$

and $$S-\frac{V^2}{C^2R}\quad\frac{1}{1-\frac{V^2}{gd}} \qquad (80)$$

11 Standing Wave.—If a stream has a high velocity relatively to the depth of water in it V^2 may be greater than gd. Let heading up occur in such a stream, so that V^2 becomes less than gd. Then the curve of heading up does not extend back till it touches the natural water surface, but ends abruptly at a point A (Fig 137). At this point $V^2=gd$, the denominator in equation 76 is zero, and the slope therefore infinite, that is, the water surface is vertical, or a standing wave occurs. In order that the velocity may be sufficiently high, relatively to the depth, to produce a standing wave, the slope must be steep or the channel smooth. It is not necessary that there should be any variable flow except at the wave. The flow in both the upstream and downstream reaches may be uniform. Instances may be seen

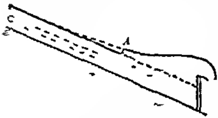


Fig 137

where a steep wooden trough tails into a pond or downstream of a sloping weir or contracted water way. One occurs where the Amazon suddenly changes its slope. The quantity $\frac{V_1^2-V_2^2}{2g}$ in equation 17 is greater than, and of opposite sign to the quantity, $\frac{V^2L}{c^2R}$. In order that V^2 or C^2RS may be greater than gd, S must be greater than $\frac{g}{C^2}$ assuming R and d to be equal If C is 100, S must be more than 0032.

At the foot of a rapid forming the left flank of the weir across the river Ravi at the head of the Bari Doab Canal the standing wave, when floods are passing, is 6 or 8 feet high, not counting the masses of broken water on the crest of the wave Logs 6 feet in diameter brought down by the flood disappear into the wave.

The following statement shows some results observed by Bidone —

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
D_1	D_2	V_1	V_2	$\frac{V_1^2-V_2^2}{2g}$	D_2-D_1	Difference of Columns 5 and 6	$\frac{(V_1-V_2)^2}{2g}$
Feet. 149	Feet ·423	4 59	1 62	287	Feet 274	013	137
246	739	6·28	2 09	545	493	052	273

Column 7 shows (chap II art 1) the head lost This is small and is nothing like $\frac{(V_1-V_2)^2}{2g}$ (chap II art 18), but it is much greater for the second case than for the first It appears that for very small streams where perhaps there is no foam or broken water, the loss of head is slight, and the height of the wave may be calculated, but it does not seem possible to do this accurately in large streams, the loss of head being probably great

Let AB (Fig 138) be a stream, and let it be desired to lower

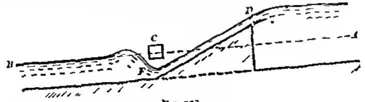


Fig 138

the water level at E, say in order that floating logs or rafts may clear a structure C, or in order to allow of a drainage outfall into

the stream The object can be to some extent attained by heading up the stream and introducing a rapid *DE* It is conceivable that some practical application of this principle might occur (Cf case of constricted pipe, chap x art 7)

12 **The Surface curve** —In any given channel with a given discharge there is only one curve of heading up and one of drawing down, whatever the cause of the variable flow may be If the cause operating at *A* (Figs 122 and 123) be removed and another cause introduced say at *K*, making the water level at *K* as before, the curve *BK* is the same as before The water in the reach *BK* is only concerned with accommodating itself to the water level at *K*, and not with the question how that water level has been caused If the surface curve is once found, it will not have to be found again for any lesser change of water level, but only a part of the same curve used Theoretically the curve extends to an infinite distance upstream, approaching indefinitely near to the line *BC*, which is an asymptote of the curve Practically the curve extends to a limited distance beyond which no change in the natural water surface is perceptible The less the ratio of *AD* to *AF* the greater is the relative length of the curve *BA* If the discharge of the channel is altered, the curve is entirely changed, and no part of it is the same as any part of the original curve If the natural water level is higher than before, a change of the same amount as before will cause a smaller ratio of *AD* to *AF*, and therefore a longer curve The greater the relative area of that part of the cross section of a stream which lies over the side slopes of the channel, the more rapidly does the section change with change of water level the more, therefore, does the surface slope at *A* differ from the natural slope and the less the length of the curve The length of the curve is of course less the steeper the bed slope

13 **Method of finding Surface curve** —To obviate the tedious process of working out length by length, and obtain a direct approximation to the surface curve, one or two methods have been used An old rule, given by Neville for cases of heading up is that the total length of the curve *BK* (Fig 122 p 229) is 1 5 to 1 9 times the length of the horizontal line *AM* This is only an approximation, or rather guess of the very roughest kind and it gives no idea of the form of the curve, that is of the depths at intermediate points For an imaginary case in which the bed width is infinite, the sides vertical, and the coefficient *C* constant for all depths, an equation to the curve can be found by integration

It is far too complicated for practical use, but certain tables have been based on it. Such tables, owing to the wholly imaginary condi

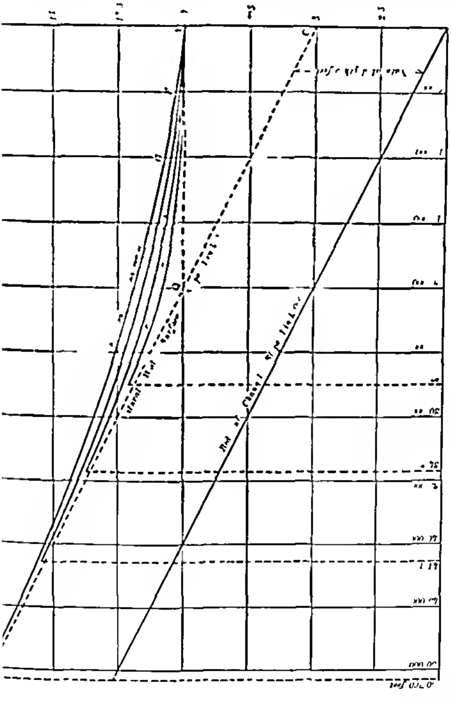


FIG. 139

tions of the case, are of very limited use. For channels with vertical sides they are not accurate, for others not even fairly accurate.

Fig. 139 shows four curves worked out length by length by equation 74 (p. 238) for streams 5 feet deep with a slope of 1 in 4000, the co-efficient C being about 60 when the depth is 5 feet. For other depths the co-efficient is suitably increased. The curves all tend to become straight lines as the depth increases. This is owing to the minuteness of the surface slope at great depths. The fall in GF has a great relative difference to the fall in FA, but both are so small that the divergence of the curve from a straight line is sometimes imperceptible. The curves are drawn up to a depth of 10 feet in one direction and 5.125 feet in the other. Below this depth the curve again tends to become straight. The three uppermost curves are for channels of rectangular section. The uppermost curve represents the extreme limit possible, the bed being assumed of width zero, or, what is the same thing, assumed to be quite smooth, the sides being only taken into account in calculating R, which is therefore constant. In the second curve l increases from 2.50 feet to 3.33 feet. The third curve is for a channel of infinite width, but it is not the imaginary curve mentioned above, because the co-efficient C has been increased as D increases, instead of being constant. As D increases from 5 to 10 feet l also increases from 5 to 10 feet. In channels with sloping sides increase of depth is accompanied by a rapid increase of section and of R and C. The profiles curve more rapidly, and the points where the curves become straight are sooner reached. The lowest curve is for a triangular section (bed width zero) and represents the extreme limit possible. For greater bed widths the effect of the side slopes becomes less and vanishes when the bed width is infinite. The third curve therefore represents the other limit in this case. The surface slopes at A are, for the four curves [illegible] and [illegible], *the last being only* [illegible]*th of the slope at* J.

From equation 74 (p. 238)

$$\frac{l}{L}=\frac{l^2}{C^2 I(D_1-D_2+h_v)}-\frac{S}{D_1-D_2+h_v} \qquad (81)$$

Let $x=\frac{D_1-D_2}{S}$, then x is the length in which the bed level changes by (D_1-D_2) feet and l is the length in which the depth changes by (D_1-D_2) feet. If the ratio $\frac{x}{l}$ is known L can be easily found.

This ratio, for each of the above curves (… the … which is not needed) and for some ot… approximately in table … for a ra…

$2D$, the value of $(D_1 - D_2)$ being usually $\frac{D'}{10}$, which gives reaches sufficiently short to enable equation 74 or 81 to apply without any considerable error. The approximate ratios $\frac{x}{L}$ are easily found by disregarding h_r. Then, putting $C'^2 R S' = V'^2$, from equation 81,

$$\frac{x'}{L} = \frac{D_1 - D_2}{S L} = \frac{V'^2}{V^2} - 1 \qquad (82)$$

This quantity, since $D_2 > D_1$, is negative, and in table II the quantity $1 - \frac{V'^2}{V^2}$ is shown instead.

Now the ratios $\frac{x}{L}$ in table II apply, not only to the cases from which they were deduced, but with certain corrections to most other cases. Let the size, roughness, or bed slope of the stream alter in any manner, the proportions of the stream being maintained, and the proportionate change in C with change of R being also maintained, and let $\frac{D_1 - D_2}{D'}$ be as before, then $\frac{V'^2}{V^2}$ and $\frac{x}{L}$ are as before. Thus the ratios in table II can be used, with suitable interpolations, for any channel whose section is rectangular or trapezoidal. For a curvilinear or irregular section the section most resembling it can be adopted. For most cases the above approximation will be sufficient, but greater exactness can be obtained as follows:—

Denoting by C_1 the value of C for the natural depth D, and C_2 the value for the headed up depth $2D$, column 14 of table II shows the ratios $\frac{C_2}{C_1}$ or M, which actually occurred in the cases worked out. These ratios are fair averages, being such as occur with streams 5 feet to 10 feet deep with N about ·0275, but for other cases the ratio may be different. For a very smooth deep stream it will be less and for a rough shallow stream more. For values of R (in the reach of natural flow) ranging from 2 feet to 8 feet, and N ranging from ·017 to ·030 the value of M (Kutter and Bazin) may possibly vary as shown in columns 15 and 16. For any given stream it will be difficult to say what the value is and the extreme values shown are not likely to occur. Suppose that, for the second case shown in table II, it is believed that M' is 1·16. Then $\frac{M'}{M} = \frac{1\cdot16}{1\cdot10} = 1\cdot055$ and $\frac{M'^2}{M^2} = 1\cdot11$ nearly. Corrections can be applied as follows:—

	Column of table II	3	4		11	12	13
Corrections	In $C'^2 R'$ or V'^2 (+) say,	½	1	2,	9	10	11 per cent.
	In $V'^2 - V^2$ (−) say,	½	1	2	8,	8½	9 per cent.
	In $\frac{x}{L}$ (+) say,	4½	4,	4,	2½	2	2 per cent.

The correction to be applied to $\frac{x}{L}$ is + or − according as $\frac{M'}{M}$ is > 1·0 or < 1·0.

tions of the case, are of very limited use For channels with vertical sides they are not accurate, for others not even fairly accurate

Fig 139 shows four curves worked out length by length by equation 74 (p 238), for streams 5 feet deep with a slope of 1 in 4000, the co efficient C being about 60 when the depth is 5 feet For other depths the co efficient is suitably increased The curves all tend to become straight lines as the depth increases This is owing to the minuteness of the surface slope at great depths The fall in GF has a great relative difference to the fall in FA but both are so small that the divergence of the curve from a straight line is sometimes imperceptible The curves are drawn up to a depth of 10 feet in one direction and 5 125 feet in the other Below this depth the curve again tends to become straight The three uppermost curves are for channels of rectangular section The uppermost curve represents the extreme limit possible, the bed being assumed of width zero, or, what is the same thing, assumed to be quite smooth, the sides being only taken into account in calculating R which is therefore constant In the second curve R increases from 2 50 feet to 3 33 feet The third curve is for a channel of infinite width, but it is not the imaginary curve mentioned above, because the co efficient C has been increased as D increases, instead of being constant As D increases from 5 to 10 feet R also increases from 5 to 10 feet In channels with sloping sides increase of depth is accompanied by a rapid increase of section and of R and C The profiles curve more rapidly, and the points where the curves become straight are sooner reached The lowest curve is for a triangular section (bed width zero), and represents the extreme limit possible For greater bed widths the effect of the side slopes becomes less and vanishes when the bed width is infinite The third curve, therefore, represents the other limit in this case The surface slopes at A are, for the four curves, $\frac{1}{16\,085}$, $\frac{1}{24\,491}$, $\frac{1}{43\,233}$ and $\frac{1}{182\,559}$ the last being only $\frac{1}{40}$th of the slope at L

From equation 74 (p 238),

$$\frac{1}{L}=\frac{V^2}{C^2R(D_1-D_2+h_v)}-\frac{S}{D_1-D_2+h_v} \qquad (81)$$

Let $x=\frac{D_1-D_2}{S}$, then x is the length in which the bed level changes by (D_1-D_2) feet, and L is the length in which the depth changes by (D_1-D_2) feet If the ratio $\frac{x}{L}$ is known L can be easily found

This ratio, for each of the above curves (except the uppermost, which is not needed) and for some intermediate cases, is given approximately in table li for a range of depth extending up to

$2D'$, the value of $(D_1 - D_2)$ being usually $\frac{D'}{10}$, which gives reaches sufficiently short to enable equation 74 or 81 to apply without any considerable error. The approximate ratios $\frac{x}{L}$ are easily found by disregarding I_r. Then, putting $C^2 I S = V'^2$, from equation 81,

$$\frac{x}{L} = \frac{D_1 - D_2}{S L} = \frac{V^2}{V'^2} - 1 \qquad (82)$$

This quantity, since $D_2 > D_1$, is negative, and in table li the quantity $1 - \frac{V^2}{V'^2}$ is shown instead.

Now the ratios $\frac{x}{L}$ in table li apply, not only to the cases from which they were deduced, but with certain corrections to most other cases. Let the size, roughness, or bed slope of the stream alter in any manner, the proportions of the stream being maintained, and the proportionate change in C with change of R being also maintained, and let $\frac{D_1 - D_2}{D'}$ be as before, then $\frac{V^2}{V'^2}$, and $\frac{x}{L}$ are as before. Thus the ratios in table li can be used, with suitable interpolations, for any channel whose section is rectangular or trapezoidal. For a curvilinear or irregular section the section most resembling it can be adopted. For most cases the above approximation will be sufficient, but greater exactness can be obtained as follows:—

Denoting by C_1 the value of C for the natural depth D', and C_2 the value for the headed up depth $2D'$, column 14 of table li shows the ratios $\frac{C_2}{C_1}$ or M, which actually occurred in the cases worked out. These ratios are fair averages being such as occur with streams 5 feet to 10 feet deep with N about ·0275, but for other cases the ratio may be different. For a very smooth deep stream it will be less, and for a rough shallow stream more. For values of R (in the reach of natural flow) ranging from 2 feet to 8 feet, and N ranging from ·017 to ·030 the value of M (Kutter and Bazin) may possibly vary as shown in columns 15 and 16. For any given stream it will be difficult to say what the value is, and the extreme values shown are not likely to occur. Suppose that, for the second case shown in table li, it is believed that M' is 1·16. Then $\frac{M'}{M} = \frac{1·16}{1·10} = 1·055$ and $\frac{M''}{M} = 1·11$ nearly. Corrections can be applied as follows:—

	Column of table li	3	4	5	11	12	13
Corrections	In $C^2 I$ or V'^2 (+) say	½	1	2,	9	10	11 per cent
	In $V^2 - V'^2$ (−) say,	½	1,	2	8,	8½	9 per cent
	In $\frac{x}{L}$ (+) say,	4½	4,	4,	2½,	2	2 per cent

The correction to be applied to $\frac{x}{L}$ is + or − according as $\frac{M'}{M}$ is > 1·0 or < 1·0.

For trapezoidal channels table li gives the ratio $\frac{A_b}{A_s}$, but the channels concerned had side slopes of 4 to 3 For other side slopes the increase of I even with the same value of $\frac{A_b}{A_s}$, may differ somewhat, but the difference is likely to be considerable only for a deep narrow channel In any case a correction can be made, as above, by considering the change in $\frac{C_2^2 R_2}{C_1^2 I_1}$ instead of in $\frac{C^2}{C_1^2}$ The actual values of R_1 and R_2 were as follows —

Section ratio	—	Infinity	3	75	0 0
R_1	=	5 0	3 64	2 69	2 0
R_2	=	10 0	6 25	4 78	4 0
$\frac{R_2}{I_1}$	=	2·0	1 72	1 78	2 0

Regarding the hitherto neglected quantity h, the following table shows such values of it as have been worked out for the above cases Except with

Values of h_v

Section Ratio (see table 1)		Velocity when depth is 5 feet	Depths of Water: 5 125 to 5 25	5 25 to 5 5	5 5 to 6	6 to 6 5	6 5 to 7	7 to 7 5	to 8	8 to 8 5	8 5 to 9	9 to 9 5	9 5 to 10
			Values of D_1 D: 125	25	5	5	5	5	5	5	5	5	5
Rectangl	2	0 0	025	046	074	058	040	036	030	020	021	018	015
	4	1 73			006	005			0025			0014	0013
	Infinity	2 12	*003*	*000*	*009*	*007*		*0046*					00?
Trapezoidal	3	1 81			008	006				0023			
	75	1 6			007					0015		001	
	0 0	2 68		013	023	015	018	007	005	004			0016

high velocities h_v is small compared to $(D_1 - D_2)$ For a smaller channel $(D_1 - D)$ will be less, but probably V and h_v will also be less By interpolating and noting that h_v is as V^2 the values of h_v for any case can be approximately obtained and $\frac{x}{L}$ corrected by multiplying it by $\frac{D_1 - D_2}{D_1 - D_2 + h_v}$, which since $D_2 > D_1$ is greater than unity, so that the correction increases $\frac{x}{L}$

Ordinarily the corrections have little effect, because D changes less rapidly than $\frac{x}{L}$ Suppose the ratio $\frac{x}{L}$ used is wrong by 4 per

cent, then instead of giving the point where D is, say, 1 30, it gives the point where D is 1 28 or 1 32

The profile can be easily extended with accuracy to a point where the depth is greater than $2D$ by simply calculating the surface slopes at the two ends of the extension and drawing two straight lines or even one

Table lii shows some co efficients $\frac{x''}{L}$ for cases of drawing down extending to half the natural depth As with the curves of heading up the greatest change of slope and the shortest curve occurs with a channel of triangular section Fig 140 shows one of the

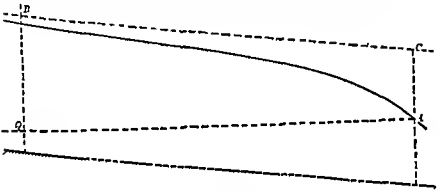


FIG 140.

curves The channels are the same as before, but the natural depth D is now 10 feet, so that column 1 is not as before, and $D_1 - D_2$ is $\frac{D'}{20}$

C_1 now refers to the depth D and C_2 to the depth $\frac{D}{2}$ The correction to be applied to $\frac{x''}{L}$ for change in M is, as before, + or − according as $\frac{M}{M'}$ is > 1 0 or < 1·0, but it is greater than before in relative amount The values of $\frac{R_1}{R_2}$ for the trapezoidal channels are the same as the values of $\frac{P_2}{P_1}$ given above The correction for h_v is the same as before and as before has the effect of increasing $\frac{x''}{L}$

Where D is not much less than D the surface curve is very similar to that of heading up, with similar proportionate depths, but as D decreases the resemblance ceases, and the curvature increases rapidly, a tangent to the curve tending to eventually become vertical instead of horizontal, as in heading up

The ratios in tables li and lii have been arranged in the form

For trapezoidal channels table h gives the ratio $\frac{A_b}{A_s}$, but the channels concerned had side slopes of 4 to 3. For other side slopes the increase of R even with the same value of $\frac{A_b}{A_s}$, may differ somewhat, but the difference is likely to be considerable only for a deep narrow channel. In any case a correction can be made, as above, by considering the change in $\frac{C_2^2 R_2}{C_1^2 h_1}$ instead of in $\frac{C_2^2}{C_1}$. The actual values of R_1 and R_2 were as follows —

Section ratio	=	Infinity	3	75	0 0
R_1	=	5 0	3 64	2 69	2 0
R_2	=	10 0	6 25	4 78	4 0
$\frac{R}{R_1}$	=	2·0	1 72	1 78	2 0

Regarding the hitherto neglected quantity h_e, the following table shows such values of it as have been worked out for the above cases. Except with

VALUES OF h_e

Section Ratio (see table 1).		Velocity when depth is 5 feet	Depths of Water										
			5 15 to 5 25	5 25 to 5 5	5 5 to 6	6 to 6 5	6 5 to 7	7 to 7 5	7 5 to 8	8 to 8 5	8 5 to 9	9 to 9 5	9 5 to 10
			Values of D_1 D										
			125	25	5	5	5	5	5	5	5	5	5
Rectangular	2	6 0	025	046	074	058	046	036	030	026	021	·018	015
	4	1 73			006	005			0025			0014	0013
	Infinity	2 12	003	006	009	007		0046					002
Trapezoidal	3	1 81			008	006				0023			
	75	1 56			007					0015		001	
	0 0	2 68		013	023	015	018	007	005	004			0016

high velocities h_e is small compared to $(D_1 - D_2)$. For a smaller channel $(D_1 - D)$ will be less, but probably V and h_e will also be less. By interpolating and noting that h_e is as V^2 the values of h_e for any case can be approximately obtained and $\frac{x}{L}$ corrected by multiplying it by $\frac{D_1 - D_2}{D_1 - D + h_e}$, which, since $D_2 > D_1$ is greater than unity, so that the correction increases $\frac{x}{L}$.

Ordinarily the corrections have little effect, because D changes less rapidly than $\frac{x}{L}$. Suppose the ratio $\frac{x}{L}$ used is wrong by 4 per

cent, then instead of giving the point where D is, say, 1 30, it gives the point where D is 1 28 or 1 32

The profile can be easily extended with accuracy to a point where the depth is greater than $2D'$ by simply calculating the surface slopes at the two ends of the extension and drawing two straight lines or even one

Table lii shows some co-efficients $\frac{x''}{L}$ for cases of drawing down extending to half the natural depth As with the curves of heading up the greatest change of slope and the shortest curve occurs with a channel of triangular section Fig 140 shows one of the

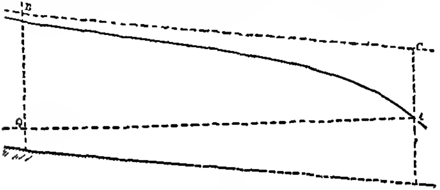


FIG 140.

curves The channels are the same as before, but the natural depth D' is now 10 feet, so that column 1 is not as before, and $D_1 - D_2$ is $\frac{D'}{20}$

C_1 now refers to the depth D and C_2 to the depth $\frac{D}{2}$ The correction to be applied to $\frac{x''}{L}$ for change in M is, as before, + or − according as $\frac{M}{M'}$ is > 1 0 or < 1·0, but it is greater than before in relative amount The values of $\frac{R_1}{R_2}$ for the trapezoidal channels are the same as the values of $\frac{R_2}{P_1}$ given above The correction for k_2 is the same as before, and, as before, has the effect of increasing $\frac{x''}{L}$

Where D is not much less than D' the surface curve is very similar to that of heading up, with similar proportionate depths, but as D decreases the resemblance ceases, and the curvature increases rapidly, a tangent to the curve tending to eventually become vertical instead of horizontal, as in heading up

The ratios in tables li and lii have been arranged in the form

For trapezoidal channels table II gives the ratio $\frac{A_b}{A_e}$, but the channels concerned had side slopes of 4 to 3. For other side slopes the increase of R even with the same value of $\frac{A_b}{A_e}$, may differ somewhat, but the difference is likely to be considerable only for a deep narrow channel. In any case a correction can be made, as above, by considering the change in $\frac{C_2^2 P}{C_1^2 P_1}$ instead of in $\frac{C^2}{C_1}$. The actual values of R_1 and R_2 were as follows —

Section ratio	= Infinity	3	75	0 0
R_1	= 5 0	3 64	2 69	2 0
R_2	= 10 0	6 25	4 78	4 0
$\frac{R}{R_1}$	= 2 0	1 72	1 78	2 0

Regarding the hitherto neglected quantity h, the following table shows such values of it as have been worked out for the above cases. Except with

VALUES OF h

Section Ratio (see table II)		Velocity when depth is 5 feet	Depths of Water: 5 125 to 5 25	5 25 to 5 5	5 5 to 6	6 to 6 5	6 5 to 7	7 to 7 5	7 5 to 8	8 to 8 5	8 5 to 9	9 to 9 5	9 5 to 10
			Values of $D_1 - D$: 125	25	5	5	5	5	5	5	5	5	5
Rectangular	2	6 0	025	046	074	058	040	036	030	026	021	·018	017
	4	1 73			006	005			0025			0014	0013
	Infinity	2 12	003	006	009	007		0046					002
Trapezoidal	3	1 81			008	006				0023			
	75	1 56			007					0015		001	
	0 0	2 68		013	023	01[illegible]	018	007	00[illegible]	004			0016

high velocities h_v is small compared to $(D_1 - D)$. For a smaller channel $(D_1 - D)$ will be less but probably V and h_v will also be less. By interpolating and noting that h_v is as V^2 the values of h_v for any case can be approximately obtained and $\frac{x}{L}$ corrected by multiplying it by $\frac{D_1 - D_2}{D_1 - D_2 + h_v}$, which since $D_2 > D_1$, is greater than unity, so that the correction increases $\frac{x}{L}$.

Ordinarily the corrections have little effect, because D changes less rapidly than $\frac{x}{L}$. Suppose the ratio $\frac{x}{L}$ used is wrong by 1 per

discharge will be a maximum, BM being given, let $BM=D$ and $NA=y$. The section CQ is nearly as $\frac{D+y}{2}$, $\sqrt{R}$ as $\sqrt{\frac{D+y}{2}}$, and $\sqrt{S}$ as $\sqrt{\frac{D-y}{L}}$. Then assuming C constant, Q is nearly as $(D+y)(D^2-y^2)^{\frac{1}{2}}$,

$$\frac{dQ}{dy}=\text{constant}\times\{(D^2-y)^{\frac{1}{2}}-y(D+y)(D^2-y^2)^{\frac{1}{2}}\},$$
$$=\text{constant}\times(D^2-y^2-Dy-y^2)$$

When the expression in brackets is zero $y+\frac{D}{4}=\pm\frac{3D}{4}$.

The discharge is a maximum when $y=\frac{D}{2}$ and a minimum when $y=D$. The discharge, however, varies little for a considerable variation in y. In the case just referred to, when D was 8 feet, the discharges found were, C being constant,

$y=1$ ft	2 ft	3 ft	4 ft	5 ft	6 ft
$Q=249$	253	255	259	240	229

Similar interesting problems occur on Inundation Canals, though, owing to the temporary nature of the conditions, approximate solutions are sufficient. When the head reach of a canal is silted and the time is approaching when the canal, owing to the falling of the river, will go dry, a reserve head channel is often opened. Sometimes the first one is also left open. Whether it should be left open or not depends on what extra supply it will give (when the water level at the junction is raised by the opening of the reserve head) and on whether the slope in it will be so flat as to cause it to silt excessively. If only one head is to be open it is sometimes better to keep the reserve head closed, as the slope along it may be flat owing to the conditions in the shifting river.

On the Choa branch of the Sirhind Canal the water four miles from the head, was headed up in order to work a mill, and the variable flow extended up to the head, thus vitiating the discharge table which depended on the reading of the head gauge. The use of the table was abandoned, but it would be possible to correct it on the above principles, a gauge above the mill being also read. The case of a silted canal head (art. 8) is different because the bed is constantly changing.

Section IV.—Variable Flow in General

15. Flow in a Variable Channel.—Sections ii and iii of this chapter treat of uniform channels but though the propositions

given so as to admit of corrections being applied, or at least to show how the corrections affect them Otherwise it would be more convenient to show $\frac{L}{x}$ instead of $\frac{x}{L}$ It is, however, easy to convert the figures It is clear that the total length of a curve (say of heading up and starting from the point where D_1 is $1\ 025D$) is relatively very great when the heading up is small, and that co efficients showing its total length would require a table as large as table h, and similarly with drawing down

14 **Calculations of Discharges and Water levels**—When the flow in a reach is not variable throughout, the discharge can be found from the depth—or *vice versa*—in its upper portion, and thus V is known Then, the depth at the lower end, or at any point in the variable length, being also known, the surface curve can be found by the method of the preceding article

When the flow is variable throughout a reach, such as AK (Figs 122 and 123, p 229), supposing a breach in uniformity to occur at K, an approximate discharge can be found by the formula for uniform flow, the slope being KA and the depth being greater or less than the mean of the two depths at K and A, according as draw or heading up exists The reach can then be divided into a few lengths, or left undivided (according as the relative difference in the two depths at K and A is great or small) and a nearer approximation made by using equation 74 If the depths at K and A are very different the channel can be assumed to extend up to B and table h or hi used In any case the correct discharge is obtained when, the water level at one end being assumed, that at the other end comes out correct

Whether or not the flow is variable throughout the reach, if the discharge is so great as to affect the original water level at the head of the reach, allowance must be made for this in assuming the water level at B or K

A case occurred[1] in which a cut, BA, with a level bed (Fig 135, p 240) connected two rivers It was desired to ascertain how much water would flow along the cut The writer of the article worked out the discharge from first principles by the aid of the calculus, the working occupying several pages This case, as well as that shown in Fig 136, can be dealt with as above, except that, D being infinite, tables h and hi cannot be used, and that for the level bed equation 79 (which is simpler) is to be used instead of 74

To find approximately the depth AN (Fig 135) for which the

[1] *Minutes of Proceedings Institution of Civil Engineers* vol lui

discharge will be a maximum, LM being given, let $BM = D$ and $NA = y$. The section A is nearly as $\frac{D+y}{2}$, $\sqrt{R}$ as $\sqrt{\frac{D+y}{2}}$, and $\sqrt{S}$ as $\sqrt{\frac{D-y}{L}}$. Then assuming C constant, Q is nearly as $(D+y)(D^2-y^2)^{\frac{1}{2}}$,

$$\frac{dQ}{dy} = \text{constant} \times \{(D^2-y^2)^{\frac{1}{2}} - y(D+y)(D^2-y^2)^{-\frac{1}{2}}\},$$

$$= \text{constant} \times (D^2 - y^2 - Dy - y^2).$$

When the expression in brackets is zero $y + \frac{D}{4} = \pm\frac{3D}{4}$.

The discharge is a maximum when $y = \frac{D}{2}$ and a minimum when $y = D$. The discharge, however, varies little for a considerable variation in y. In the case just referred to, when D was 8 feet, the discharges found were, C being constant,

$y=$ 1 ft	2 ft	3 ft	4 ft	5 ft	6 ft
$Q=$ 249	253	255	259	240	229

Similar interesting problems occur on Inundation Canals, though, owing to the temporary nature of the conditions, approximate solutions are sufficient. When the head reach of a canal is silted and the time is approaching when the canal, owing to the falling of the river, will go dry, a reserve head channel is often opened. Sometimes the first one is also left open. Whether it should be left open or not depends on what extra supply it will give (when the water level at the junction is raised by the opening of the reserve head) and on whether the slope in it will be so flat as to cause it to silt excessively. If only one head is to be open it is sometimes better to keep the reserve head closed, as the slope along it may be flat owing to the conditions in the shifting river.

On the Choa branch of the Sirhind Canal the water, four miles from the head, was headed up in order to work a mill, and the variable flow extended up to the head, thus vitiating the discharge table which depended on the reading of the head gauge. The use of the table was abandoned, but it would be possible to correct it on the above principles a gauge above the mill being also read. The case of a silted canal head (art. 8) is different because the bed is constantly changing.

SECTION IV.—VARIABLE FLOW IN GENERAL

15. Flow in a Variable Channel.—Sections ii and iii of this chapter treat of uniform channels, but though the propositions

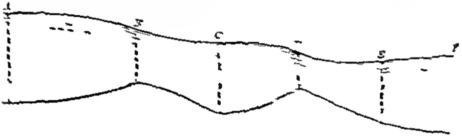
[illegible] they [illegible] them [illegible] natural flow and [illegible] have [illegible] For [illegible] down [illegible] variable [illegible] 5 [illegible] drawing [illegible] the flow natural.

Generally a [illegible] One problem [illegible] will be produced [illegible] channel. [illegible] surface [illegible] length [illegible] varying in one [illegible] It the channel [illegible] and [illegible] level [illegible] will often extend [illegible]

The [illegible] 76 (p. 23 [illegible] $V = C\sqrt{RS}$ where S is [illegible] B be the width of the [illegible] Then [illegible]

$$ [illegible] $$

Since $B = $ [illegible] manner to [illegible] Fig. 141 [illegible] Here d, P



[illegible]

The surface slopes at opposite banks of a stream are not generally equal unless it is quite uniform and straight

17 Rivers —A river, especially at low water, may be a series of separate streams with numerous junctions and bifurcations The water level in a side channel *CAB* (Fig 142) may afford only a very poor indication of the general water level in the river Suppose that with a good supply the water level at *A* is the same as that at *B* If there is silt in the channel *CA*—the silt being deepest at *C*—a moderate decrease of the river discharge may cause a great decrease in the discharge of *CA*, or even a total cessation of discharge This causes great difficulties in the matter of gauge readings in some Indian rivers Suppose a gauge to have been originally at *B* If erosion of the bank sets in the gauge has to be moved, and sometimes it is difficult to find another place (free from practical difficulties in the matter of reading the gauge and despatch of readings), except at such a place as *A* in a side channel In floods, especially when the sandbanks between the channels are submerged, there is a general tendency for the water surface to become level across, but it by no means follows that it becomes so When the deep stream is at one side of the river channel the flood level is nearly always higher on that side than at the opposite side

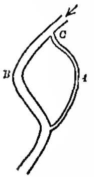


Fig 14

Since a small cross section tends to cause scour and a large one silting, it follows that every stream tends to become uniform in section The remarks made in articles 1, 2, and 8 also show that it tends to destroy obstructions, to assume a constant slope, and to become curved in such a way that its velocity will suit the soil through which it flows If a river always discharged a constant volume its regimen would probably be permanent It is the fluctuations in the discharge that cause disturbance

Examples

Example 1 —In the channel considered in example 3 of chapter vi a heading up of 1 25 ft is caused by a weir What heading up is caused 2000 feet upstream of the weir?

Table xlv shows $I = 102.6$ sq ft Also $A_b = 80 \times 4.75 = 380$ sq ft $A_s = 22.6$ sq ft and $\frac{I_b}{I_s} = 17$ nearly, so that $\frac{y'}{l}$ lies between the values for the first and second cases in the second part of table li, and somewhat nearer to the first than the second

Since $S=\frac{1}{5000}$ and $D_1-D_2=\frac{D}{10}=475$ ft $x'=\frac{D_1-D_2}{S}=475\times5000=2375$ ft

The headed up depth at the weir is 6 ft $=4\ 75\times1\ 264$ From table li $\frac{x}{L}$ is about 550 when D_1 is $1\ 2D$ and D_2 is $1\ 3D'$ Therefore $L=\frac{x'}{550}=\frac{2375}{550}=4318$ ft The distance of the weir downstream from the point where the depth is $1\ 20D'$ is $\frac{1\ 264-1\ 200}{1\ 30-1\ 20}\times4318=2764$ ft The point 2000 ft upstream of the weir is thus 764 ft from the above point, and the change of depth in this length is $(1\ 30-1\ 20)D'\times\frac{764}{4318}=018D$, so that the heading up is $(1\ 218-1\ 00)D'$, or 218×475 ft, or 1 04 ft Corrections if applied to this case might alter the result by 01 ft

Example 2—From the stream considered in the first trial in example 2 of chapter vi a branch is taken off and discharges 120 c ft per second What lowering of the water level is caused 1500 ft upstream of the branch?

Table xliv shows $A=356\ 3$ Also $A_b=40\times7\ 5=300$ sq ft $A_s=56\ 3$ sq ft and $\frac{A_b}{A_s}=5\ 32$, so that $\frac{x''}{L}$ lies between the values in the first two lines of the second part of table lii The discharge below the bifurcation is 967 c ft, and this is given by a depth of 7 ft, so that the lowering is 5 ft

Since $S''=\frac{1}{5000}$ and $D_1-D_2=\frac{D}{20}=375$ ft $x''=\frac{D-D_2}{S''}=375\times5000=1875$ ft The drawn-down depth at the bifurcation is 7 ft $=7\ 5\times93$ ft From table lii $\frac{x''}{L}$ is about 33, when D_1 is $95D'$ and D_2 is $90D$ Therefore $L=\frac{x''}{33}=\frac{1875}{33}=5682$ ft The distance of the bifurcation downstream from the point where the depth is $95D'$ is $\frac{95-93}{95-90}\times5682=1894$ ft The point 1500 ft upstream of the weir is thus 394 ft from the above point, and the change of depth in this length is $(95-90)D'\times\frac{394}{5682}=00347D$, so that the drawing-down is $D'-(95-0035)D'$ or $0535\times7\ 5=401$ ft

TABLE LI.—RATIOS FOR CALCULATING PROFILE OF SURFACE WHEN HEADED UP. (Art. 13.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Section Ratio	Ratios [illegible] and 1 [illegible]	Depth Ratios. Upper figures show $\frac{D_1}{l}$, lower figures $\frac{D}{D}$											Values of M or $\frac{C_1}{C_1}$		
		1·0[illegible] to 1·05	1·05 to 1·10	1·10 to 1·[illegible]0	1·[illegible]0 to 1·30	1·30 to 1·40	1·40 to 1·[illegible]0	1·50 to 1·[illegible]0	1·[illegible]0 to 1·[illegible]0	1·[illegible]0 to 1·[illegible]0	1·80 to 1·[illegible]0	1·90 to 2·0	Actual [illegible] occurred	Extreme Values Maximum	Minimum
Rectangular Sections. Ratio of Width to Depth as in column 1.															
2	1 −1 $\frac{x}{L}$ or 1−(1 −1[illegible])	·003 ·09[illegible]	[illegible] 180	68[illegible] 318	548 45[illegible]	443 55[illegible]	3[illegible]1 629	313 68[illegible]	[illegible] 733	[illegible] 771	[illegible] 801	1[illegible] 8[illegible]5	1·07	1·1[illegible]	1·0[illegible]
4	1 −1 $\frac{x}{L}$ or 1−(1 −1[illegible])	[illegible]9 108	804 196	[illegible]6[illegible] 341	510 484	410 584	33 661	[illegible]0 [illegible]20	[illegible]34 766	[illegible]01 79[illegible]	[illegible] 829	14[illegible] 851	1·10	1·16	1·03
Infinity	1 −1 $\frac{x}{L}$ or 1−(1 −1[illegible])	880 120	[illegible] 223	614 386	458 51[illegible]	3[illegible]1 649	[illegible]74 [illegible]96	[illegible]18 782	1[illegible]5 825	143 8[illegible]7	118 882	09[illegible] 901	1·17	1·28	1·0
Trapezoidal Sections. Ratio $\frac{A_b}{A_s}$ = area of section over bed / area over side slopes, as in column 1.															
Infinity	(The figures are the same as for the preceding case.)												1·17	1·28	1·0
3	1 −1[illegible] $\frac{x}{L}$ or 1−(1 −1)	[illegible]64 136	[illegible]60 240	[illegible]00 [illegible]00	440 560	334 ·666	25 743	198 802	154 916	1[illegible] 878	098 ·902	·080 920	1·13	1·21	1·01
75	1 −1 $\frac{x}{L}$ or 1−(1 −1[illegible])	84[illegible] 153	[illegible]30 270	[illegible]43 457	3[illegible]0 ·630	[illegible] 731	194 806	146 8[illegible]1	109 891	0[illegible] 918	0[illegible]4 936	05[illegible] 918	1·1[illegible]	1·21	1·01
0·0	1[illegible]−1[illegible] $\frac{x}{L}$ or 1−(1[illegible]−1[illegible])	·31 185	·6 328	4[illegible]0 5[illegible]0	9 708	19 80[illegible]	130 870	·0[illegible]0 910	·0[illegible] 930	046 951	·034 [illegible]	0[illegible] ·971	1·18	1·[illegible]8	1·0[illegible]

TABLE LII.—RATIOS FOR CALCULATING PROFILE OF SURFACE WHEN DRAWN DOWN

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Section Ratio	Ratios $\frac{V^2}{V'^2}$ and $\frac{V^2}{V'^2}-1$	Depth Ratios Upper figures show $\frac{D_1}{D}$ lower figures $\frac{D}{D'}$									Values of M or $\frac{C_2}{C_1}$		
		·95 to ·90	·90 to ·85	·85 to ·80	·80 to ·75	·75 to ·70	·70 to ·65	·65 to ·60	·60 to ·55	·55 to ·50	Actual which occurred	Extreme Values Maximum	Extreme Values Minimum
Rectangular Sections Ratio of Width to Depth as in column 1													
1	$V^2 \div V'^2$	1·21	1·39	1·62	1·90	2·25	2·72	3·33	4·14	5·31	·935	·89	·98
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·21	·39	·62	·90	1·25	1·72	2·33	3·14	4·31			
2	$V^2 \div V'^2$	1·24	1·45	1·69	2·02	2·42	3·00	3·72	4·76	6·24	·909	·86	·97
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·24	·45	·69	1·02	1·42	2·00	2·72	3·76	5·24			
Infinity	$V^2 \div V'^2$	1·31	1·59	1·91	2·42	3·04	3·89	5·07	6·80	9·35	·85	·78	·95
	$\frac{x}{L}$ or $V^2 \div V'^2 - 1$	·31	·59	·91	1·42	2·04	2·89	4·07	5·80	8·35			
Trapezoidal Sections Ratio $\frac{A_b}{A_s} = \frac{\text{area of section over bed}}{\text{area over side slopes}}$, as in column 1													
Infinity	(The figures are the same as for the preceding case.)										·85	·78	·9[illegible]
1·5	$V^2 \div V'^2$	1·34	1·67	2·10	2·70	3·50	4·55	6·00	8·18	11·35	·88	·83	[illegible]
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·34	·67	1·10	1·70	2·50	3·55	5·00	7·18	10·35			
·375	$V^2 \div V'^2$	1·35	1·73	2·30	3·08	4·1[illegible]	5·69	[illegible]·84	11·50	17·1[illegible]	·88	·83	[illegible]
	$\frac{x''}{L}$ or $V^2 \div V'^2 - 1$	·35	·73	1·30	2·08	3·12	4·69	6·84	10·50	16·17			
0·0	$V^2 \div V'^2$	1·52	2·06	2·84	3·99	5·[illegible]	8·49	1[illegible]·9[illegible]	18·62	31·3[illegible]	·84	·78	·95
	$\frac{x}{L}$ or $V^2 \div V'^2 - 1$	·52	1·06	1·84	2·99	4·77	7·49	11·94	17·62	30·32			

CHAPTER VIII

HYDRAULIC OBSERVATIONS

[For general remarks on Hydraulic Observations, see chap II art 25]

SECTION I—GENERAL METHODS

1 **Velocities**—When the velocity is observed at one or more points in the cross section of a stream, the process is termed 'Point Measurement' When the mean velocity on a line in the plane of the cross section is found directly, it is known as an 'Integrated Measurement' Velocity measuring instruments are of two classes, namely, 'Floats' and 'Fixed Instruments' Fixed Instruments give the velocities in one cross section of a stream Floats give the average velocity in the 'run' or length over which they are timed, and not that at one cross section Floats are used only in open streams, but fixed instruments sometimes in pipes

With most instruments time observations are necessary The best instrument for this is a chronometer beating half seconds, similar to those used at sea, or a stop-watch which can be read to quarter seconds The next best is a common pendulum swinging in half seconds, and after that an ordinary watch The error in timing with a chronometer is not likely to exceed half a second, with an ordinary watch it may be one or even two seconds Some stop-watches and watches not only do not keep proper time, but are not regular in their speed If any such defect is suspected the instrument should be tested The time over which an observation extends should be such that any error in timing will be relatively small In order to eliminate the 'personal equation' of the observer similar observations at the beginning and end of the time should be performed by the same individual, or if performed by two they should frequently change places

Floats include surface floats, sub-surface floats, and rod floats The first two are used for point measurement, the last for integrated measurements on vertical lines A float travels with the stream, and so interferes little with the natural motion of the

water Its velocity is supposed to be the same as that of the water which it displaces

Fixed Instruments are divided into Current Meters and Pressure Instruments In the former the velocity of the stream is inferred from that of a revolving screw, in the latter from indications caused directly by the pressure of the water[1] Velocities cannot be obtained by Fixed Instruments until they have been 'Rated,' that is, until it has been ascertained that certain indications of the instrument correspond to certain velocities Fixed instruments interfere with the natural motion of the stream, but this need not cause error The disturbance is almost entirely downstream of an obstruction (chap ii art 21), and if those parts of the instrument which are intended to receive the effect of the current are kept well upstream, no difficulty arises, except perhaps in very small streams If a boat is used the bow can be kept pointing upstream and the instrument upstream of the bow, a platform being made to project over the bow Even if the boat or instrument is so large (which is not likely) relatively to the stream as to cause a general heading up, this will not prevent a correct measurement of the discharge, though it may affect the surface slope In order that disturbance may not be caused by moorings the boat should (unless it is a steam launch which can maintain its position) be held by shore lines If attached by its bow to a pulley running on a transverse rope, it can quickly be brought, by using the rudder, to any required point Another transverse rope serves to keep the boat steady and, if divided by marks, shows its position In a wide stream containing shallows the ropes may rest on trestles placed at the shallows Where moorings must be used it is best to moor two boats side by side, as far apart as practicable, and to work from a platform between them, keeping the instrument well upstream

The choice of an instrument for velocity measurement depends on various considerations Floats require a regular stream, but fixed instruments can be used in any stream In comparing the Current-Meter, or Pitot's Tube with Floats, regard must be had to the design and quality of the instruments available, and to the manner in which they were rated Sub surface floats are unsuit

[1] Further information concerning Fixed Instruments is given in Sections IX and X, but the varieties and details are very numerous and cannot all be discussed There are many papers on these instruments in the *Minutes of Proceedings of the Institution of Civil Engineers* and *Transactions of the American Society of Civil Engineers*

able when the stream is rapid or when there are weeds growing in it, fixed instruments unsuitable when the velocity is very low For surface velocities alone surface floats are, in regular streams, the best instruments unless there is considerable wind For integrated measurements the rod float is as good as any instrument, provided the bed is even enough to allow of a rod of the proper length being used

The above considerations refer to accuracy only As regards the time occupied and the number of observers required, fixed instruments generally have the advantage In a discharge measurement of a large river current meter integration measurements can be made while the soundings across the channel are being taken On the other hand, the time occupied in rating the fixed instruments, their initial cost, and their liability to damage or loss, especially in out-of the way places, may be very important factors If a stream is too wide to be reached at all points without a boat, has no suitable bridge, but is still narrow enough for the floats to be thrown in from the sides, and if no soundings are required, float observations may take less time than others

2 **Discharges**—The discharge of an aperture or pipe is not usually found by measuring the velocity but by letting the water pass into a tank and measuring the volume added in a given time[1] Whenever leakage, absorption, or evaporation occur, allowance must be made for them, but some error is likely to result

The discharge of an open stream is usually found by observing the depths and mean velocities on a number of verticals Let *ABC* (Fig 143) be the mean velocity curve, and *ADEFC* a curve

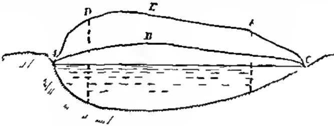


Fig 143

whose ordinates are found by multiplying the depth on each vertical by the corresponding velocity Then *ADEFC* is the dis

[1] The velocities in large pipes may however be observed (art 14) When this is done it is best to divide the section into concentric circles of equal areas

charge curve, and its area is the discharge If floats are used the velocities obtained are the averages in the run, and the depths must also be averages in the run The more numerous the verticals the more accurate the result For ordinary work ten is a fair number, *for very accurate work, twenty* In the segments *AD*, *FC*, near the sides the verticals should be nearer together than elsewhere, because the ordinates change rapidly The equal spacing of the verticals in each segment is not essential, but it simplifies the calculation, as it is only necessary to add together all the ordinates in a segment—deducting half the first and last—and multiply the sum by the distance between the ordinates The discharges of all the segments added together gives that of the stream If the number of equal spaces in a segment is even Simpson's rule can be used, but ordinarily the results of formulæ such as this differ very little from those of the simpler rule

Sometimes the spacing in a segment cannot be equal If there is in the cross section any marked angle, whether salient or re entering, a measurement should be made there Sometimes when floats are used in rivers the velocities must be observed where the floats happen to run In such cases the depths at these exact points need not be measured, but may be inferred from those observed at fixed intervals or found by plotting the section

If the mean velocity on a vertical is obtained by multiplying the observed surface velocity by the co efficient β (chap VI art 9), and if β is the same for all verticals, the discharge may be calculated as if the surface velocities were the means on verticals and the whole discharge multiplied by β

Discharge observations in an open stream are greatly facilitated by the construction of a 'Flume' A short length of the channel is constructed of masonry or timber The sides may be sloping but are preferably vertical In the absence of silt deposit the section of the stream is known from the water level, and if rod floats are used they are *all of one length* Flumes may, however, prevent proper surface slope observations (chap VII art 5) Discharges can be obtained with more or less exactness by the observation of U or U_s and the use of a or δ (chap VI art 10), but a flume is often unsuitable for this (chap II art 21)

When a discharge table has been prepared for any site or aperture, the discharge can be found by simply observing the water level or head or—in the case of a pipe—the hydraulic gradient The discharge of a pipe may be altered by corrosion,

and that of an open channel by changes occurring, not only at the site but downstream of it Hence orifices and weirs in thin walls afford the best means of measuring moderate quantities of water For larger quantities, and for small quantities when no fall is available, measurement in a flume or regular channel is adopted, but the velocities should be observed sometimes, if not in the whole section, then in the centre

3 Soundings —Soundings are generally taken to obtain a cross section of a stream, but longitudinal sections may be required in order to find the most regular site, or in connection with float observations In water not more than about 15 feet deep soundings are best taken with a rod, which may carry a flat shoe to prevent its being driven into the bed In greater depths a weighted line is used

Unless the velocity is very low it is best to observe soundings from a boat drifting downstream The current then exerts little force on the rod or line, which can thus be kept vertical It can be held so as to clear the bed by a small amount, and lowered at the proper moment This plan is particularly suitable for obtaining the mean cross section in the run when floats are used As the boat drifts the bottom is frequently touched with the rod or line, and the readings booked and averaged Any local shallow likely to interfere with the use of rod floats is also thus detected When shore lines can be used the boat can be worked and the widths measured as described in article 1 In wide rivers lines of flags or 'range poles' are used instead of ropes An observer on the boat or on shore can note the moment when the boat crosses the line, and give a signal for the soundings to be taken To determine the distance of the boat from the bank an observer in the boat reads an angle with a sextant, or an observer on shore reads it with a theodolite, following the boat with his instrument and keeping the cross wires on some part of it When the line is reached the motion of the instrument is stopped and the angle read off

4 Miscellaneous —The diameters of pipes, while water was flowing, were measured by Williams, Hubbell, and Fenkell by means of a rod with a hook inserted through a stuffing box For obtaining the mean diameter in a length of pipe one method is to fill it with water, which is afterwards measured or weighed If the joints are not closely filled in some error may be caused, and Smith in some experiments filled each separate piece of pipe before it was laid, and weighed the water it contained

For ascertaining c_c and c_v for orifices special arrangements are required. The velocity of the jet is found by observing its range on a horizontal plane. A ring or movable orifice of nearly the size of the section of the jet may be placed so that the jet passes through it, the flow stopped, and the necessary distances measured. The actual velocity can then be found from equation 29 or 30 (p. 52) and the actual head being measured c_v is easily found.

When observations of any kind are made a suitable form should be prepared and filled in. It should have spaces set apart for recording the date, time, gauge reading and (at least when floats are used) the direction and force of the wind.

SECTION II.—WATER LEVELS

5. Gauges.—For observing the water level of an open stream the simplest kind of gauge is a vertical scale fixed in the stream and graduated to tenths of a foot. It may be of enamelled iron, screwed to a wooden post which is driven into the bed or spiked to a masonry work. The zero may conveniently be at the bed level, so that the reading gives the depth of water. The actual gauge may extend only down to low water level. If a gauge is exposed to the current it may be damaged by floating bodies, and it is difficult to read it accurately owing to the piling up of the water against the upstream face and the formation of a hollow downstream. These irregularities can be greatly reduced by sharpening the upstream and downstream faces of the post or the upstream face only. Greater accuracy can be obtained by placing the gauge in a recess in the bank, but not where it is exposed to the effects of irregularities in the channel (chap. vii, art. 2) and by watching the fluctuations of the water level, noting the highest and lowest readings within a period of about half a minute, and taking their mean; but very great accuracy by direct reading of a fixed gauge is difficult because of the adhesion of the water to the gauge, and the differences in level of the point observed and the eye of the observer.

With floating gauges these difficulties are almost got rid of. The graduated rod is attached at its lower end to a float which rises and falls with the water level. The rod travels vertically between guides and it is read by means of a fixed pointer on a level with the eye of the observer. The float and rod should be of metal, so that they may not alter in weight by absorbing moisture, the float perfectly water tight and its top conical so that it may not

form a resting place for solid matter. The gauge should occasionally be tested by comparison with a fixed gauge or bench mark. For a given weight of float and rod the smaller the horizontal section of the float at the water surface the more sensitive the gauge will be.

To reduce the oscillations of the surface a gauge, whether fixed or floating, may be placed in a masonry well communicating with the stream by a narrow vertical slit. It is not certain that the average water level in the well is exactly the same as in the stream, but the difference can only be minute. The larger the well the better the light and the less the oscillation of the water. The advantage of a slit as compared with a number of holes is that it can always be seen whether the communication is open, but in order to avoid the necessity for frequent inspection the oscillation of the water in the well should not be entirely destroyed. In observations made downstream of the head gates of irrigation distributaries in India the oscillations were very violent—amounting to 60 foot—but they were reduced to 03 foot in the well by slits 005 foot wide.

Where a gauge does not exist the water level can be measured from the edge of a wall or other fixed point, either above or below the surface. Owing to the oscillation of the water the end of the measuring rod cannot be held exactly at the mean water level. It should be held against the fixed point, and the mean reading taken as explained above. A self registering gauge can be made by means of a paper band travelling horizontally and moved by clock work and a pencil moving vertically and actuated by a float. The pencil draws a diagram showing the gauge readings. The water level in a tank may be shown by a graduated glass tube fixed outside the tank and communicating with it.

The level of still water can be observed with extraordinary accuracy by Boyden's Hook Gauge, which consists of a graduated rod with a hook at its lower end. The rod slides in a frame carrying a fixed vernier, and is worked by a slow motion screw. If the hook is submerged, the frame fixed and the rod moved upwards, the point of the hook, before emerging causes a small capillary elevation of the surface. The rod is then depressed till the elevation is just visible. By this means the water level can be read to the thousandth of a foot, and even to one five thousandth in still water, by a skilled observer in certain lights. The hook gauge is not of much use in streams because of the surface oscillation. It is most used in still water upstream of weirs

To destroy oscillation and ripples, a box having holes in it may be placed in the water and the readings taken in the box When observing with a hook gauge in water not perfectly still the point of the hook should be set so as to be visible half the time A pointed plumb bob hung over the water from a closely graduated steel tape is sometimes used, and by it the surface level can be observed to within 005 foot The adjustment of the level of the zero of the gauge above a weir may be effected by a levelling instrument If effected from the level of the water when just beginning to flow over the crest capillary action may cause some error

6 Piezometers —The name 'Piezometer,' used chiefly for the pressure column of a pipe, is also used to include a gauge well and its accompanying arrangements In all such cases the surface, where the opening is, should be parallel to the direction of flow and flush with the general boundary of the stream, and the opening should be at right angles If it is oblique the water level in the piezometer will be raised or depressed according as the opening points upstream or downstream The well or pressure tube can be connected with any convenient point by flexible hose terminating in fixed glass graduated tubes With high pressures the piezometers may be connected with columns of mercury, which may be surrounded by a water jacket to keep the temperature nearly constant Common pressure gauges are not accurate enough

In the piezometers of pipes air is somewhat liable to accumulate and cause erroneous readings When the presence of air is suspected the tubes should be allowed to flow freely for a few minutes If flexible they can be shaken and if stiff rapped with a hammer Very small tubes are liable to obstruction by leaves or deposits and should be avoided, as also should glass gauge tubes small enough to be affected by capillarity The orifices should be drilled and made carefully flush Instead of one orifice there may be four, 90° apart, in one cross section of a pipe, all opening into an annular space from which the piezometer tube opens It is not certain that this gives greater exactness but with a single opening from the top of the pipe the accumulation of air is probably greatest The air probably travels along the pipe at the top

The Venturi meter for pipe observations has been described in chapter x (art 7) It has been patented and can be obtained with automatic recording apparatus

form a resting place for solid matter The gauge should occasionally be tested by comparison with a fixed gauge or bench mark For a given weight of float and rod the smaller the horizontal section of the float at the water surface the more sensitive the gauge will be

To reduce the oscillations of the surface a gauge, whether fixed or floating, may be placed in a masonry well communicating with the stream by a narrow vertical slit It is not certain that the average water level in the well is exactly the same as in the stream, but the difference can only be minute The larger the well the better the light, and the less the oscillation of the water The advantage of a slit as compared with a number of holes is that it can always be seen whether the communication is open but in order to avoid the necessity for frequent inspection the oscillation of the water in the well should not be entirely destroyed In observations made downstream of the head gates of irrigation distributaries in India the oscillations were very violent—amounting to 60 foot—but they were reduced to 03 foot in the well by slits 005 foot wide

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The arrangements at the weirs where the most important observations (chap ix art 1) have been made were as below In all cases the surface containing the orifice was parallel to the axis of the stream

Bazin —An opening near the bed 4 inches square communicating with a well

Francis —A small box[1] with 1 inch holes in the bottom

Fteley and Stearns —For the 19 foot weir there was an opening 04 foot in diameter and 4 feet lower than the crest of the weir From the opening a rubber pipe led to a pail below the weir

For the 5 foot weir there was a board parallel to the side of the channel and 1 5 feet from it The pipe leading to the pail started from an auger hole in the board 9 feet above the bed of the channel

To find the heads on weirs piezometers connected with perforated tubes placed horizontally in the channel have been used in America, but they appear to give unreliable results, even when the holes open vertically In experiments made at Cornell University[2] the 'middle piezometer' was a transverse 1 inch pipe, laid 8 inches above the bed and 10 feet upstream of the weir The 'upper piezometer' was similar, but 15 feet further upstream A 'flush piezometer' was also 'set in the bottom of the flume,' 6 inches upstream of the upper piezometer The readings of these two differed on one occasion by 3 foot The readings of the upper and the middle also differed It is believed that the opening from the rounded surface of the pipe instead of from a plane surface causes error, and that the error is one of defect A 'longitudinal piezometer' was formed by certain perforated pipes With high heads—a little over 3 feet—the longitudinal piezometer read 099 foot higher than the upper piezometer With a head of about 17 foot there was no difference between the two Experiments made by Williams[3] also show that the readings obtained with a transverse pipe with holes opening downwards, do not agree with those obtained by a simple opening in the side of the channel being higher with low supplies and lower with higher supplies It seems clear that all perforated pipe arrangements are to be avoided until their action is better understood

7 Surface-slope —Probably the best method of observing the slope in a short length of open stream is to dig two ditches from the extremities of the slope length, both leading into a well divided into two by a thin partition The difference between the water levels on the two sides of the partition is the local surface fall It can be very accurately measured, especially if the ditches

[1] The box projected somewhat into the stream, and this was not free from objection, as it caused an abrupt change

[2] *Transactions of the American Society of Civil Engineers*, vol xliv

[3] *Ibid* vol xliv

are treated as gauge wells, that is, open into the stream by narrow slits This is perhaps the only way by which the slope in a really short length can be found [1] Slight leakage in the partition is probably of no consequence as long as it gives rise to no perceptible current in the ditch The slope should unless the stream is perfectly uniform and straight, be observed at both banks and the mean taken (chap vii art 16)

For measuring the loss of head in a short length of pipe a differential gauge may be used consisting of two parallel glass tubes with a scale fixed between them Capillarity does not vitiate the results because it is the difference that is taken If the tubes are partly filled with water and the space above the water is occupied by air the difference in the heights of the water columns gives the difference in head To obtain great exactness Williams, Hubbell, and Fenkell used an inverted U tube and substituted kerosene oil for air This causes the difference in the readings in the two legs to be magnified about five times Another differential gauge used was a simple mercury gauge All these gauges had to be 'calibrated' (their constants determined) and corrections were applied for changes in temperature [2]

In whatever way slope is observed the openings of any pair of gauge wells, ditches, or piezometers must be exactly similar, and the observations should be repeated at intervals as long as the velocity observations last

SECTION III — FLOATS

8 **Floats in general** — The size of a float used for point measurement is limited by the consideration that the mean velocity of the stream within the 'direct area' of the float (the area of its projection on a cross section of the stream) must be practically equal to that at the point where the velocity is sought The depth of the submerged part of a surface float may be about one twentieth of the depth of water and the depth of a sub surface float one tenth or, at the point of maximum velocity one twentieth of the depth of water The width of a float of any kind may be about one twentieth of the width of the stream except for use near the bank, when it may be about one tenth of the distance from the bank to the line of the float The length is

[1] This method can be used for a pipe provided the hydraulic gradient is at a convenient level

[2] *Transactions of the American Society of Civil Engineers*, vol xlvii

similarly limited because the float may revolve The exposed part of a surface float should be small compared to the submerged part For deep water a good surface float is made by a bottle submerged all but the neck, or a log deeply submerged, for shallow water by a disc almost totally submerged and carrying a small cylinder or knob With all kinds of floats the exposed part should be of such a colour that it can easily be seen

The 'lines' or boundaries of the run are marked by ropes stretched across the stream at right angles, or, if the width is great, by lines of flags Observers signal each float as it crosses the lines, and another observer notes the times When ropes are used the *float-courses* can be marked by 'pendants' of cloth or brass Usually about three floats are signalled in rapid succession at the first line and then at the second If on reaching the second line they have changed order, this affects the individual times recorded, but not the mean With a stop watch the time observer may also be the float-observer He can start and stop the watch while noting the float But in this case each float must complete its course before another can be timed With a slow current the time observer may also start the floats, and he may even use an ordinary watch In a wide river the course of a float can be observed by an angular instrument (see art 3)

A float required to travel in any course usually deviates from it The deviation increases the distance over which it travels, but this is of no consequence because the object is to obtain the forward velocity (chap i art 3) The deviation is of consequence only when the velocities in adjacent parts of the stream differ much from one another, that is, generally, near the banks In such cases the 'run' of the float can be shortened, the deviation noted, and the mean course adopted When ropes are used the approximate deviation can be seen by the float-starter by means of the pendants, especially when the rope is at a low level

The length over which a float travels, upstream of the run, in order that it may acquire the velocity of the water, is called the 'dead run' The float may be taken out into the stream, or thrown in from the bank, or placed in it from a bridge or boat Throwing in is often practicable with surface floats, and sometimes with rods A low level single span bridge is the most suitable arrangement, but if there are piers or abutments which interfere with the stream they disturb the flow, and a site down stream of them is unsuitable for velocity measurements, at least with floats (chap ii art. 21) Even a boat causes disturbance

downstream Two small boats or pontoons carrying a platform are better than a large boat

The length of run to be adopted depends on the velocity and uniformity of the stream, the accuracy of the timing, and the distance of the float-course from the bank, this last consideration having reference to deviation Ordinarily the length may be so fixed that the probable maximum error in timing will be only a small percentage of the time occupied The length may, however, have to be reduced if the stream is not regular, especially if rods are used Reduction of the length in order to avoid excessive deviation is most likely to be necessary for observations near the bank, especially with surface floats The surface currents near the bank set towards the centre of the stream (chap vi art 7), so that the tendency to deviation is greater, while the admissible deviation is less Most observations are made at a distance from the bank, and the rejections for excessive deviation need not generally be numerous A moderate number of rejections, owing to a long run, does not cause much loss of time, because in order to obtain a particular degree of approximation to the average velocity of the stream the number of floats recorded must be inversely proportional to the length of the run

9 Sub surface Floats —A float used for measuring the velocity at a given depth below the surface is called a 'double float' A submerged 'lower float' somewhat heavier than water, is suspended by a thin 'cord' from a 'buoy' which moves on the surface In the ordinary kind of double float the buoy is made small, and the velocity of the instrument is assumed to be that of the stream at a depth represented by the length of the cord but it is really different because of the current pressures on the buoy and cord, and the 'lift' of the float due to these pressures There is also 'instability' of the lower float, caused chiefly by the eddies which rise from the bed Any lateral deviation of the lower float adds to the lift, but otherwise is not of consequence, except near the banks The resultant effect of all the faults is a distortion of the velocity curves obtained When the maximum velocity is at the surface (Fig 112, p 166) the buoy and cord accelerate the lower float, and the lift brings it into a part of the stream where the velocity exceeds that at the assumed depth Hence the velocity obtained is always too great and the 'observation curve' which is shown dotted, lies outside the true curve When the maximum velocity is below the surface the curve is distorted as in Fig 113

A double float is best suited to a slow current The higher the velocity of the stream the greater the differences among the velocities at different levels and the greater the lift of the lower float the greater also the strength of the eddies and the instability

The defects of the double float cannot be removed, but they can be much reduced by attention to the design In order that the lower float may be as free as possible from instability, its shape should be such as to afford little hold to upward eddies In order that it may be little affected by the current pressures on the buoy and cord, it should afford a good hold to the horizontal current It should therefore consist of vertical plates say of two cutting each other at right angles, with smooth surfaces, and lower edges sharpened The upper edges should not be sharpened Any downward current will then act as a corrective to instability If *the float tilts much its efficiency is reduced* but *tilting can be prevented by avoiding a high ratio of width to height, and by making the upper and lower parts respectively of light and heavy materials say wood and lead* If the thickness of the plates is uniform the resistance to tilting is a maximum when the heights of the heavy and light portions are inversely as the square roots of the specific gravities of the materials *It is an improvement to remove the central portions of the plates and to substitute for them a hollow vertical cylinder, in the middle of which the cord is attached by a swivel* This causes the pull of the cord, however the float revolves on its vertical axis, to be applied at the point where the average horizontal current pressure acts The cord should be of the finest wire, and the buoy of light material say hollow metal smooth and spindle shaped the cord being attached towards one end so as to make the float point in the direction of the resistance

Given the velocity of the stream the force tending to cause instability of the lower float depends on its superficial area Its stability depends on the ratio of its weight to its superficial area that is on the thickness of the plates For all floats of the same shape and materials there is a certain thickness of plate which is the least consistent with stability, and a float should be composed of plates of this thickness in order that the thickness of the cord and volume of buoy may be small This thickness cannot be determined theoretically, but is a matter of judgment and experience Of any two similar double floats that which has the *larger lower float is the more efficient* If the direct areas of the lower floats are as f and 1, their weights and the submerged

volumes of the buoys are as 4 and 1. But the direct areas of the buoys, if their shapes are similar, are as $4^{\frac{2}{3}}$ and 1 or nearly as 2·5 and 1. The thicknesses and direct areas of the cords are also theoretically as 2 and 1 In both cases the larger instrument has greatly the advantage, and practically, if the lower float is small, it is physically impossible to make the cord thin enough The dimensions are limited by the considerations set forth above The larger the stream the greater the admissible size of float

The following statement shows that the double floats which have been actually used in important experiments have been of bad design —

Channel.	Observer	Greatest Depth of Water	Description of Lower Float.	Ratio of Direct Areas at Maximum Depth		
				Lower Float.	Cord.	Buoy
		Feet.				
Mississippi.	Humphreys and Abbott.	110	Keg with top and bottom removed.	1·0	1·75	·03
Irrawaddy	Gordon	70	Block of wood loaded with clay	1·0	73	·06
Ganges Canal.	Cunningham.	11	Ball (3 inches and $1\frac{5}{8}$ inch)	1·0	$\begin{Bmatrix} 15 \\ 72 \end{Bmatrix}$	10

It is obvious that when the lower float was near the bed—or supposed to be near it—the observed velocities must, owing to the very great relative current-actions on the cord, and probably also to instability, have been so much in excess of the truth as to render them mere approximations, the general values found for bed velocities being perhaps about halfway between the real bed velocity and the mean velocity from the surface to the bed The vertical velocity curves obtained with the above instruments often show marked peculiarities in form, the velocity sometimes seeming to remain constant or even increase as the bed is approached

In the 'twin float' the submerged part of the buoy or 'upper float' is of the same size, shape, and roughness as the lower float, and the velocity of the instrument is assumed to be a mean between the stream velocities at the surface and at the level of the lower float The surface velocity is observed separately and eliminated This causes additional trouble The best form and size for the lower float are arrived at in the same manner as in the

ordinary double float The difficulties arising from tilting and instability can be overcome by making the lower float heavy and the upper one light The current pressure on the cord is less than with the ordinary double float, but its inclination greater The instrument has been very little used

Cunningham has proposed a triple float for measuring the mean velocity on a vertical when the depth is too great for rod floats, or the bed too uneven It has a small buoy and two large submerged floats at depths of 21 and 79 respectively of the full depth, the upper of the two being light and the lower heavy The instrument is supposed to give the mean of the velocities at these two depths, and this is nearly equal to the mean on the whole vertical The figures 21 and 79 were arrived at theoretically by Cunningham, and they are the best for general use, the depth of the line of maximum velocity being supposed to be unknown *It would be preferable to use a multiple float with several equi*distant submerged floats, the lower ones heavy and the upper ones light, the distance of the lowest from the bed and of the highest from the surface being half the distance between two adjoining floats All these floats are best suited to slow currents

10 Rod floats—A rod float is a cylinder or prism ballasted so that in still water it floats upright In flowing water it tilts because of the differences in the velocities of the stream By using a rod reaching nearly to the bed the mean velocity on the vertical is obtained Owing to the irregular movements of the water the rod does not move steadily Both its submerged length and tilt vary The clearance below the bottom of the rod must be sufficient to prevent the bed being touched The great advantage of a rod as compared with a multiple float is that there is no uncertainty as regards lift and instability

Rods are usually made of wood or tin and weighted with lead A wooden rod is liable to alter in weight from absorption of water, and it may then become too deeply submerged or sink A cap containing shot fitted to the lower end of the rod gives a ready means of adjustment In a rapid stream a wooden rod may have an excessive tilt, and a tin rod is better It is lighter and can carry more ballast. It is, however, liable to damage and to spring a leak A rod may sometimes sink, owing to its grounding and being turned over by the current In a rapid stream a wooden rod may be turned over even without grounding Wooden rods can be more easily made square than of other sections In any case the *section and degree of roughness must* be uniform throughout

For a rod 1 foot long, 1 inch, and for one 10 feet long, $2\frac{1}{2}$ inches are suitable diameters Rods are often made up in sets, the lengths increasing by half feet, or for small depths by quarter feet, but this does not give sufficient exactitude, and it often leads to the use of rods much too short Owing to the unevenness of the bed a rod of the proper theoretical length is usually too long, and the next length is perhaps 10 or 15 per cent shorter A set of short adjusting pieces to screw on to the tops of the rods should be provided Rods for use in very deep water are sometimes made in lengths screwed together It is convenient to have rods divided into feet, beginning from the bottom If the tilt is likely to be great, allowance can be made for it in selecting the length to be used

It has been said that a rod, owing to its not reaching down to the slowest part of the stream, must move with a velocity greater than the mean on the whole vertical Cunningham has attempted to show theoretically that the length of a rod must be 945, 927, or 930 of the full depth of water according as the point of maximum velocity is at the surface, at one third depth, or at half depth The proof rests on the assumption that the vertical velocity curve is a parabola It has been shown (chap vi art 9) that it is not a parabola, and that the velocity probably decreases very rapidly close to the bed, and for this last reason it is probable that a rod reaching close to the bed would move too slowly The proper length of rod cannot be calculated theoretically in the present state of knowledge

A large number of experiments with rod floats were made by Francis The discharges obtained by rods in a masonry flume of rectangular section with a depth of water of 6 feet to 10 feet were compared with the discharges obtained from a weir in a thin wall, and the following formula was deduced

$$V = V_r\left(1\ 012 - 1\ 116\sqrt{\frac{D-d}{D}}\right)$$

when V is the mean velocity on the vertical, V_r the rod velocity, d the length of the rod, and D the depth of the stream According to this formula the correct length of rod so that V and V_r may be equal, is $93D$ and the errors due to shortness of rod are as follows —

$\frac{d}{D}=$	75	80	85	90	91	[illegible]	93	97	[illegible]	[illegible]
$\frac{V}{V_r}=$	954	961	968	975	991	[illegible]	[illegible]	[illegible]	[illegible]	1 00

The discharges obtained by the weir are believed to be very nearly correct, and the acceptance of the above figures is recommended. Accepting them, the proper length of a rod is 99 of the full depth, and if the length is only 93 of the full depth the velocity found is 2 per cent in excess. In earthen channels a rod of the proper length can hardly ever be used, but allowance can be made for its shortness.

Section IV.—Current Meters

11. **General Description.**—The current meter consists of a screw, resembling that of a ship, and mechanism for recording the number of its revolutions. Frequently this mechanism is on the same frame as the screw, and by means of a cord it can be put in and out of gear. The reading having been noted the meter is placed in the water, the recording apparatus brought into gear, and, after a measured time, put out of gear and a fresh reading taken. The difference in the readings gives the number of revolutions, and this divided by the time gives the number of revolutions per second. This again, by the application of a suitable co-efficient determined when the instrument is rated, can be converted into the velocity of the stream. The co-efficient depends on the 'slip' of the screw, and varies for each instrument and each velocity. With many meters the recording apparatus is above water, and there is electric communication between it and the screw. The meter can then be allowed to run for an indefinite time without raising to read. For each meter there is a minimum velocity below which the screw ceases to revolve. This may be as low as six feet per minute, but is generally much higher.

Sometimes a current-meter is carried on a vertical pivot and provided with a vane. The irregularity of the current causes the instrument to swing about, and so to register the total and not the 'forward' velocity. It is better to keep the instrument fixed with the axis parallel to that of the stream, but if the axis swings through a total angle of 20—10 either way—the velocity registered is only 75 per cent in excess of the forward velocity, and if the total angle is 10°, 3 per cent in excess.

A current meter may be used in a small stream *from the bank* or from a bridge, but generally it is used from a boat. This has already been referred to (art. 1). The rod or chain to which the meter is attached should be graduated. If a rod is used it

may be sharpened or rounded on its upstream face, the down stream face being flat, and resting against a portion of the platform fixed at right angles to the centre line of the boat The rod can be provided with a collar, which can be clamped on to it in such a position, that when it rests on the platform the meter is at the depth required In water 53 feet deep Revy attached the meter to a horizontal iron bar, which was lowered by ropes fastened to its ends, and was kept in position by diagonal ropes In shallow water an iron rod is sometimes fixed, on which the meter slides up and down, but this causes delay

In some experiments the time in quarter seconds, position of the meter, and number of revolutions of the screw have been automatically recorded on a band driven by clockwork With a meter having electric communication with the bank a wire rope has been stretched across a wide stream, the meter carried on a frame slung from the rope and the discharge of the stream thus observed In other cases the observers travel in a cage slung from a wire rope It is quite usual to have several meters working simultaneously at different depths In integration it is not necessary for the descending and ascending velocities to be equal, and two or three up and down movements may be made without raising to read It is a common practice, after taking an observation lasting a few minutes, to check it by a shorter one To facilitate the computation of the meter velocity the times may be whole numbers of hundreds of seconds A stop-watch may be started and stopped by the same movement which puts the instrument in and out of gear

The rate of a current meter is liable, at first, to increase slightly owing to the bearings working smoother by use It should be allowed to run for some time before being rated Oil should not be used, as it is gradually removed by the water, and the rate may then alter Every time a meter is used the screw should be spun round by hand to see that it is working smoothly A gentle breeze should keep it revolving A second instrument should be kept at hand for comparison Occasionally a short test of the rating should be made If tests made at two or three velocities all show small or proportionate changes of one kind similar corrections may be applied to other velocities but if the changes are great or irregular the instrument should be rated afresh

The speed of a current meter is liable to be affected by weed leaves, etc. becoming entangled in the working parts If any are found when the instrument is read the observation can be rejected

but some may become entangled and detached again without being seen The effect must be to reduce the velocity, and any abnormally low result may be rejected The rate of the instrument is also liable to be affected by silt and grit getting into the working parts and increasing the friction The only rubbing surface which has a high velocity is the axis of the screw, and this is probably the part chiefly affected In using a current-meter of the kind illustrated (Fig 144) it was found on one occasion that it rapidly became stiff The meter having been cleaned, the screw ran freely again, but again became stiff The stream was six feet deep and

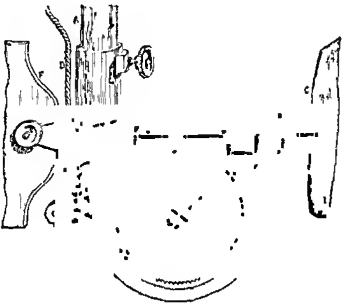


Fig 144

had a velocity of about seven feet per second The water contained silt and probably fine sand, which gradually increased the friction The clogging was most rapid in observations below mid depth and it is probable that there was more sand in that part of the stream

12 Varieties of Current-meters —There are probably twenty kinds of current meter Each kind has its own special advantages or disadvantages Fig 144 shows a meter sold by Elliott Brothers, London The instrument is attached by the clamping screw to a rod A By pulling the cord D the wheel I is geared with the screw A vane F can if desired be attached A meter very similar to the above is made in the Canal workshops at Roorkee,

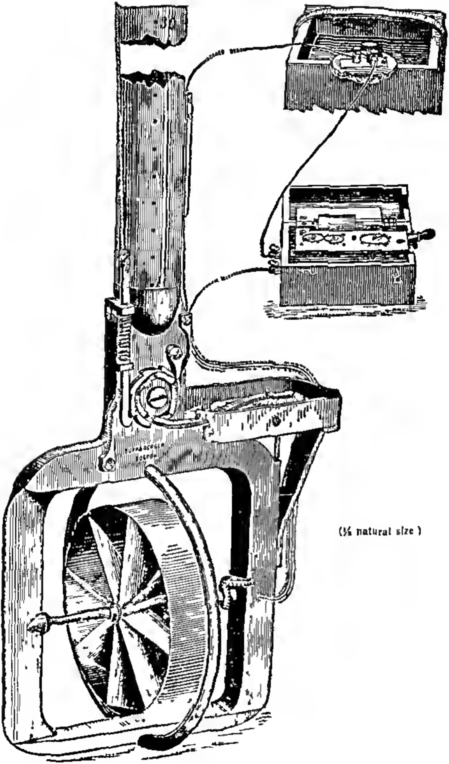
India, but it is pivoted on the tube which carries the screw for clamping it to the rod.

In Revy's current meter friction is reduced by a hollow boss on the axle of the screw of such a size that the weight of the whole is equal to that of the water displaced. The recording mechanism is enclosed in a box covered by a glass plate, filled with clear water, and communicating by a small hole with the water in the stream, so that the glass may not be broken by the pressure at great depths. A horizontal vane is attached under the screw, so that it may revolve freely while the meter rests on the bed.

Moore's current meter consists of a brass cylinder, 10½ inches long, provided with screw blades. In front of the cylinder is an ogival head which is fixed to the frame. The cylinder, which is water tight, revolves, and the recording apparatus is inside it, the reading being observed through a pane of glass. *The instrument is hung from a cord or chain. This renders it easier* to manipulate. To prevent its being forced far out of position, a weight is suspended to the frame, and it should be sufficient to prevent the instrument being temporarily displaced by the tightening of the gearing cord. The instrument has horizontal and vertical vanes and can swing in any direction.

In Harlacher's current meter there is electric connection between the worm wheel driven by the screw and a box above water. At every hundred revolutions of the screw the worm wheel makes an electrical contact, and an electro-magnet in the box exposes and withdraws a coloured disc. The meter slides on a fixed wooden rod. A tube lying along the rod carries the electric wires, and serves to adjust the meter on the rod. In one variety the axle of the screw carries an eccentric which makes an electric contact every revolution, and thus enables individual revolutions to be noted.

Fig. 115 shows a current meter sold by Buff and Berger, Boston, U.S.A. The object of the band encircling the screw is to protect the blades from accidental changes of form, which would cause a change in the rate of the instrument. A bar underneath the screw and a stout wire running round at a short distance outside it affords additional protection, and enables the instrument to be used close to the bed or *side of a channel*. There are two end bearings and a very light screw and axle, and the screw revolves with one fourth of the velocity required to turn a similar one with the usual sleeve bearing. The friction is so small that the rate is not altered by silt or grit. The meter is fixed to a brass tube, which has a line along it to show the direction of the axis when the meter cannot be seen. The meter is sold with the recording apparatus either on the frame or with electric connection, as in the figure. Stearns used a meter of this type, and provided with two screws, either of which could be used. One



(½ natural size)

F 14[illegible]

had eight vanes and the other ten In the latter half the vanes had one pitch and the other half a different pitch The eight-vane screw began to move with a velocity of 104, and the ten vane screw with a velocity of 094, feet per second

One kind of current meter has no regular recording apparatus, but simply a device for making and breaking circuit and a sounder The revolutions are counted by the clicks A current-meter made by von Wagner gave its indications by sound, but the counting was effected by an arrangement like the seconds hand of a watch At each stroke, or with high velocities at every fourth stroke, the observer pressed a button which caused the hand to move one division

13 **Rating of Current meters** —The usual method of rating is to move the instrument through still water with a uniform velocity, and to repeat the process with other velocities covering a wide range *The instrument may be held at the bow of a boat, or* attached to a car running on rails, or on a suspended wire In case the water should not be quite still the runs should be taken alternately in reverse directions

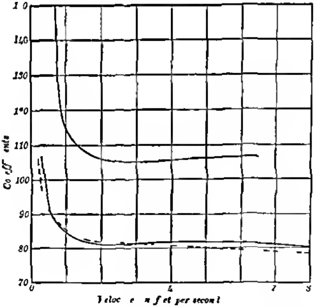
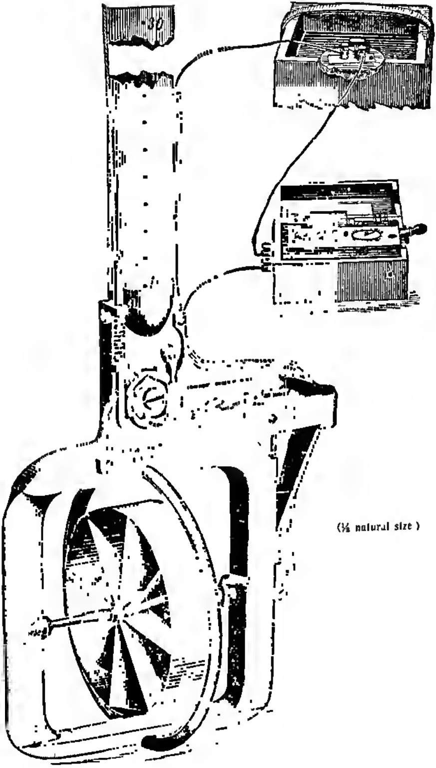


Fig 14

When rating a meter the length of run being a fixed quantity, it is only necessary to record for each observation the time occupied and the difference of the meter readings After several



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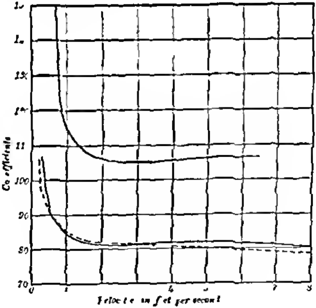


Fig. 14.

When rating a meter, the length of run being a fixed quantity, it is only necessary to record for each observation the time occupied and the difference of the meter readings. After several

Both equations give curves of the same general form, and becoming practically straight lines at high velocities. They can never agree exactly with curves having a sag and as the constants cannot be arrived at until some experimental co efficients have been found the equations are not of much practical value.

It has been shown by Stearns[1] that rating by ordinary towing through still water is not perfect. In a flowing stream the velocity and direction of the water constantly vary, but in rating this is not so. Stearns shows theoretically that the screw turns more rapidly when the velocity varies than when it is constant, that an ordinary screw probably turns more rapidly when the current strikes at an angle than when it is parallel to the axis, but that with his meter (Fig. 145) the band and parts of the frame intercept portions of the oblique currents, and so cause a decrease in the number of revolutions, the net result depending chiefly on the design of the instrument. He also moved the meter with mean velocities ranging up to 3.7 feet per second through still water, first with an irregularly varying velocity and then with its axis inclined to the direction of motion. He found that inclining the axis 8° and 11° had no appreciable effect but that inclinations of 24° and 41° decreased the number of revolutions about 9 per cent,[2] and that with irregular velocities the number of revolutions was increased, the increase varying from zero to 5 per cent, being generally greater for low velocities, and in one case reaching 13 per cent when the mean velocity was only 85 feet per second. This velocity was not a very low one when compared with that for which the screw ceased to revolve.

By measuring with the same current meter the discharges in a masonry conduit, the depths varying from 1.5 to 4.5 feet, and the velocities from 1.7 to 2.9 feet per second, and comparing the results with others known to be practically correct, Stearns found that, with point measurement, the discharge given by the meter observations was practically correct, both in the ordinary condition of the stream and when the water was artificially disturbed, and that with integration the discharge was correct when the rate of integration was 5 per cent of the velocity of the stream, but too small by 9 per cent when the rate was 58 per cent of the velocity. In the above experiments both the eight bladed

[1] *Transactions of the American Society of Civil Engineers*, vol. xii.

[2] Other experiments have shown that inclinations of 25°, 35° and 45° give a decrease in the number of revolutions of 8, 15 and 23 per cent respectively.

and ten bladed screws were used, the results being generally similar

It seems clear that, with the instrument used, the increase in the velocity due to the variations in the velocity of the stream was counter balanced by the decrease due to oblique currents, and that the instrument gave correct results with point measurements even when the water was disturbed, but with an instrument of different design, and especially one without a band, it seems probable that the results obtained by point measurement err in excess, that no additional error is introduced by a moderate inclination of the axis, or by slow integration, but that rapid integration causes error These, however, are only probabilities The real lesson to be derived from Stearns's investigations is that rating effected by steady motion in still water may be erroneous when applied to running streams, especially with rapid integration, and that additional tests should be adopted To move a meter obliquely or with an irregular velocity would be troublesome, and would not produce the conditions existing in streams It is best to place the meter in a running stream just below the surface, and to find the velocity by floats submerged to the same depth as the screw blades If a sufficient range of velocities cannot be obtained the meter can be moved upstream or downstream with a known velocity. This plan can be combined with ordinary rating The instrument can also be moved through still water while giving it a movement as in integration A comparison of discharges obtained by the meter, with results known to be correct, affords a further test An immense saving of labour is obviously effected by rating a number of meters together

When it is necessary to rely on ordinary rating rapid integration should be avoided The error, if any, will probably be less as the velocity is higher For ordinary velocities the relative error is probably nearly constant, so that the results will be consistent with one another, and sometimes that is all that is required

Section V —Pressure Instruments

14 Pitot's Tube—This instrument usually consists of two vertical glass tubes open at the ends placed side by side, one the 'pressure tube,' straight, and one the 'impact tube,' with its lower end bent at right angles and pointing upstream The water level in the pressure tube is nearly the same as that of the stream

in which the instrument is immersed, but that in the impact tube is higher by a quantity which is equal to $K\frac{V^2}{2g}$, V being the velocity of the stream at the end of the tube, and K a co-efficient whose value has to be found by experiment

The chief objections to this instrument were originally the fluctuation of the water level in the tubes, owing to the irregularity of the velocity, and the difficulty in observing the height of a small column very close to the water surface Darcy in his gauge tube reduces the fluctuations by making the diameter of the orifice only 1 5 millimetres, that of the tube being one centimetre The horizontal part of the tube tapers towards the point, and this minimises interference with the stream The difficulty in reading is surmounted by means of a cock near the lower end of the instrument, which can be closed by pulling a cord The instrument can then be raised and the reading taken To give strength and to carry the cock, the lower parts of the tubes are of copper and are in one piece For observations at small depths the heads of the water-columns are in the copper portion of the instrument, where they cannot be seen To get over this difficulty the tops of the tubes are connected by a brass fixing and a stop-cock to a flexible tube terminating in a mouthpiece By sucking the mouthpiece the air pressure in the tubes is reduced, and both columns rise by the amount due to the difference between the atmospheric pressure and that in the tubes, but the difference in the levels of the two columns is unaltered The upper cock being closed and the mouthpiece released, the reading can be taken For reading the instrument a brass scale with verniers is fixed alongside the tubes The instrument is attached to a vertical rod, to which it can be clamped at any height, and it can be turned in a horizontal plane, so that the horizontal part of the impact tube points upstream To get rid of the effect of the fluctuations in the tube several readings, say three maximum and three minimum, can be taken in succession The co-efficient K is nearly equal to unity, and it does not vary appreciably, if at all, with the velocity

The Pitot tube has been improved by interposing a flexible hose between the nozzles and the gauge The rod carrying the nozzles is thus more handy and the fluctuations of the water-column can be watched

In the Detroit pipe experiments mentioned in chapter x (art 4) various developments of the Pitot tube were used The

tubes were inserted in the pipes through stuffing boxes without interfering with the flow The diameters of the orifices both impact and pressure were usually $\frac{1}{3}$ inch The tubes were connected by rubber hose 10 feet long to a differential gauge (art 7), where the readings were taken When the impact tube was made to point at an angle with the axis of the stream the reading decreased When the angle was a little over 45° negative readings occurred up to an angle of 180°, the greatest negative reading being for an angle of 90° In one kind of tube the pressure orifices, instead of opening into the stream, opened into a ring or annular space outside the pipe and connected with the pipe by four holes $\frac{1}{16}$ inch in diameter, but this was not adopted to any considerable extent for use in the experiments

The Pitot tube is well adapted for observations in depths ranging up to 5 or 6 feet It has the great advantage of requiring no time observations It has never been used in large bodies of water, but there does not seem to be any reason why it should not be, if suitably constructed and strengthened It would be exceedingly useful for measuring velocities close to the border

15 Rating of Pitot Tubes—This was effected by Williams, Hubbell, and Fenkell (*a*) by moving the tubes through still water with velocities of 6 3 feet to 7 0 feet per second, and (*b*) by placing them in a 2 inch pipe at various points in a cross section and *finding V*, the mean velocity in the pipe (which varied from 73 feet to 4 6 feet per second), by weighing the water discharged The average results were as follows —

(Reference Number of Tubes)	No 3	No 6	No 5
Co efficients from still water ratings	926	950	869
Co efficients from pipe rating,	890	840	760
Difference,	036	110	109

For the still water rating the plotted curve of the co efficients of tube No 5 was sinuous (the others being straight), and this was attributed to a wave effect Tube No 5 was the bluntest and No 3 was the finest, and should have the highest co efficient The pipe ratings were accepted, but it does not seem to be proved that they would be correct for any pipe or any velocity The above co efficients are the means In the individual rating experiments the results differed from the means in still water by 10 to 17 per cent, and in the 2 inch pipe by 2 to 8 per cent In Bazin's experiments the differences in rating in moving water

whose velocity was known were about 4 per cent With the ring form of instrument the co efficient differed from that obtained with the same impact tube when used with the ordinary pressure tube The above figures seem to show that still water ratings are not at all reliable, and also that there are elements in the case that are somewhat doubtful and not thoroughly understood Probably further experience in ratings will place the instrument in a more satisfactory position

16 Other Pressure Instruments —

In Perrodil s Hydrodynamometer a vertical wire carries at its upper end a horizontal needle and at its lower end a horizontal arm, to the end of which is fixed a vertical disc The arm is connected with a graduated horizontal circle at the level of the needle When the arm points down stream the needle points to zero on the circle The needle is twisted round by hand till the arm is forced by the torsion of the wire to a position at right angles to the current The pressure of the water on the disc is proportional to the square of its velocity, and it is proportional to and measured by the angle of torsion of the wire as given by the position of the needle The disc oscillates owing to the unsteady motion of the stream, and the graduated circle oscillates with it, but the mean reading can be taken The instrument has not been much used but it is said to give good results and to register velocities as low as half an inch per second It interferes somewhat with the free movement of any stream in which it is placed

The Hydrometric Pendulum consists of a weight suspended from a string The pressure of the current causes the string to become inclined to the vertical, and the angle of inclination can be read on a graduated arc Except for observations near the surface the current pressure on the string must affect the reading Bruning s Tachometer also has an arm and disc, but the pressure of the water, instead of being measured by the torsion of a wire, is measured by a weight carried on the arm of a lever These two instruments have been little used, and it is not known *how far their results* can be relied on

CHAPTER IX

UNSTEADY FLOW

Section I.—Flow from Orifices

1. **Head uniformly varying.**—Let the head over an orifice during a time t vary from H_1 to H_2, and let the discharge in this time be Q. The mean head or equivalent head H is that which would, if maintained constant during the time t, give the discharge Q. Let the head H vary uniformly, that is, by equal amounts in equal times, as, for instance, in the case of an orifice in the side of an open stream, whose surface is falling or rising at a uniform rate. In this case $h=Ct$ where C is constant. Let A be the area of the orifice and c the co-efficient of discharge, which is supposed constant. The discharge in the short time dt under the head h is

$$dQ=ca\sqrt{2gh}\,dt=ca\sqrt{2gC}\,t^{\frac{1}{2}}\,dt$$

The discharge between the times T_1 and T_2 is

$$Q=\int_{T}^{T_1} ca\sqrt{2gC}\,t^{\frac{1}{2}}\,dt=\tfrac{2}{3}ca\sqrt{2gC}\,(T_1^{\frac{3}{2}}-T_2^{\frac{3}{2}})$$

$$=\tfrac{2}{3}\,ca\sqrt{2gC}\,\frac{H_1^{\frac{3}{2}}-H_2^{\frac{3}{2}}}{C^{\frac{3}{2}}}$$

Under a fixed head H

$$Q=ca\sqrt{2gH}\,(T_1-T_2)=ca\sqrt{2gH}\,\frac{H_1-H_2}{C}$$

Equating the two values of Q

$$\sqrt{H}=\tfrac{2}{3}\,\frac{H_1^{\frac{3}{2}}-H_2^{\frac{3}{2}}}{H_1-H_2} \qquad (83)$$

If $H_2=0$, that is, if the head varies uniformly from H_1 to O or from O to H_1,

$$\sqrt{H}=\tfrac{2}{3}\sqrt{H_1} \qquad (84),$$

or the equivalent head is $\dfrac{4H_1}{9}$.

2. **Filling or Emptying of Vessels.**—Let water flow from an

orifice in a prismatic or cylindrical vessel whose horizontal sectional area is A. The discharge in time dt is $d\,Q = A\ dh = ca\sqrt{2gh}\,dt$

$$dt = \frac{A\ dh}{ca\sqrt{2gh}} = \frac{A\ h^{-\frac{1}{2}}\,dh}{ca\sqrt{2g}}.$$

The time occupied in the fall of the surface from H_1 to H_2 is

$$t = \int_{H_2}^{H_1} \frac{A}{ca\sqrt{2g}}\,h^{-\frac{1}{2}}\,dh = \frac{2\,A}{ca\sqrt{2g}}(H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}})$$

Under a fixed head H

$$t = \frac{A\,(H_1 - H_2)}{ca\sqrt{2gH}}$$

Therefore $$\sqrt{H} = \frac{H_1 - H_2}{2\,(H_1^{\frac{1}{2}} - H_2^{\frac{1}{2}})} \qquad (85)$$

This is useful for canal locks

If $H_2 = O$, that is, if the vessel is emptied down to the level of the orifice,

$$\sqrt{H} = \frac{\sqrt{H_1}}{2} \qquad (86)$$

The following are the ratios of $\sqrt{H}$ to $\sqrt{H_1}$ for certain cases —

For a prism or cylinder,	$\frac{1}{2}$
For a sphere,	$\frac{5}{8}$
For a hemisphere concave downwards	$\frac{5}{7}$
For a hemisphere concave upwards	$\frac{5}{12}$
For a cone with apex downwards,	$\frac{5}{6}$
For a cone with apex upwards,	$\frac{5}{16}$
For a wedge with point downwards	$\frac{3}{4}$
For a wedge with point upwards,	$\frac{3}{8}$
For a vessel whose vertical section is a parabola with vertex downwards —	
When all vertical sections are the same (Paraboloid of revolution)	$\frac{3}{4}$
When the horizontal sections are rectangles (Two opposite sides of the vessel rectangles and two parabolas)	$\frac{2}{3}$

In the last case the surface falls at a uniform rate as in the case considered in art. 1

In all cases the times occupied in emptying the vessels are greater than with a constant head H, in the inverse ratios of the above fractions. If a vessel is filled, through an orifice in its bottom, from a tank in which the water remains level with the top of the vessel, the ratio of $\sqrt{H}$ to $\sqrt{H_1}$ is the same as for filling the vessel when *inverted*. Thus for a cylinder, prism or sphere the time for filling is the same as for emptying

If two prismatic vessels communicate by an orifice, and H_1 is the difference in the water levels of the vessels, and A_1 and A_2 their horizontal areas the time which elapses before the two heads become equal is

$$t=\frac{2A_1A_2\sqrt{H_1}}{ca\sqrt{2g}(A_1+A_2)} \qquad (87)$$

and is the same whichever is the discharging vessel This equation may be used for double locks

Section II—Flow in Open Channels

3 **Simple Waves** —Let ABG (Fig 147) represent the surface of a uniform stream in steady flow, the reach commencing from a fall

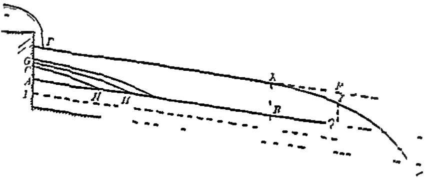


Fig 147

over which is introduced an additional steady supply q such that the surface will eventually be LF A wave is formed below A, the surface assuming successively the forms GH, $G H$, etc The point H travels downstream at first with a very high velocity —since the slope GH cannot remain steep for any but an extremely short time—but its velocity decreases as the slope at H becomes less The point G rises at a continually decreasing rate, because in equal times the volumes of water represented by $CG H H$, etc are equal Obviously the velocity of the point H is greater as q is greater, that is, it depends on the amount of the eventual rise It must not be supposed that the actual velocity of the stream even at its surface or velocity of 'translation,' is anything unusual As in other cases of wave motion it is the form of the surface which changes rapidly

When the surface has risen to F the wave advances only down stream and there is formed a reach FK, in which the flow is steady and uniform On consideration it will be seen that if the

channel is long enough, the elongation of the wave ceases, its profile KC becomes fixed, and it progresses at the same rate as the mean velocity in the risen stream EK. The motion of such a wave is uniform, and the mean velocity of the stream is the same at all cross sections. The proof given in chapter ii (art. 9) applies to any short portion of the wave. The pressure on the upstream end is greater than on the downstream end, but the surface slope is greater than the bed slope, and the equation comes out exactly the same, S being the surface slope. At different cross sections in the wave S is greater as R is less, so that V is the same everywhere. Obviously the wave is convex upwards. If at any cross section in the wave the slope were less than that required by the above consideration, the velocity there would be reduced, the upstream water would overtake it and increase the slope. If the slope at any cross section were too great, the velocity there would be increased, and the water would draw away from that upstream of it. Thus the wave is in a condition of stable equilibrium, and always tends to recover its form should this be accidentally disturbed. The curve KC produced to M and N gives the profile of the wave, supposing the original water surface to have been DM, or the channel to have been dry.

Thus the flood wave has two distinct characters according as its profile is forming or formed. The forming wave rises as well as progresses, its velocity is at first very high, and it depends on the amount of the rise that is on the height AF. The formed wave progresses at a uniform rate and its velocity depends only on that of the risen stream, and not on the amount of the rise. The surface is in all cases convex upwards. Since any change in the form of the wave occurring at either end would be communicated to the whole of it, it is probable that, in ordinary cases, the moment of time when the point H commences to move with a uniform velocity coincides nearly with the moment when the point G ceases to rise or the wave becomes formed.

As to the form of the curve KC the case is analogous to that of the surface curve in variable steady flow (chap. vii art. 13). The slope at J is such as will with uniform flow and depth JP give the same velocity as the depth KI with slope IK. Thus the surface slope corresponding to any depth is known and tangents to the curve can be drawn, but the distance between two points where the depths are given is not known. In a case of steady flow, with a drawing down KB, the surface slope at L must be greater than in the wave now under consideration because in that case V is greater than at K instead of being the same, and also because V is continually increasing and work being stored.

In the case of a reduced steady supply at S (Fig 148) the surface assumes the forms ST, ST, etc, the point T travelling with a

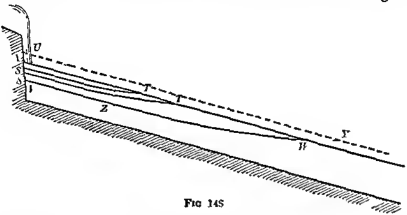


Fig 148

decreasing velocity and S falling with a decreasing velocity The surface eventually assumes the fixed form VZW, the portion VZ being in uniform flow, and ZW travelling with a velocity the same as the mean velocity of the fallen stream VZ If the original surface is UY the curve is ZWY

Ordinarily the curve of a wave is of great length, and the convexity or concavity slight If the point L is such that the volumes AFL and LQC are equal, the time at which this point in the wave will reach any place, after the wave is formed, is found by dividing the distance of the place from E by the velocity of the user stream

If the additional supply introduced, or the supply abstracted, instead of being steady, is supposed to change gradually as would be the case if it were caused by a wave coming down the upper reach or by the opening or closing of gates or shutters, the wave below A or X does not at its commencement travel with such rapidity, and it more quickly assumes its fixed form, unless the water is introduced or abstracted too slowly to allow it to do so

The form of a flood wave may be observed by means of a number of gauges, but the wave, except when it is first formed—and even then if the change in the supply is not made with great abruptness—is of great length and its form or even the times of passage of its downstream end, can be accurately found only by very exact gauge readings Slight changes in the supply, owing to rainfall or similar causes, are sufficient to vitiate the observations Absorption of water by the channel especially in the case of a wave travelling down a channel previously dry, may also

greatly affect the movement and form of the wave On the Western Jumna Canal in India, with a mean depth of water of about 7 feet, and a velocity of about 3 5 feet per second, a rise or fall in the surface of 25 foot to 55 foot, caused by the manipulation of regulating apparatus, and occupying in each case less than an hour, was found to occupy 5 or 6 hours at a point 12 miles downstream, and 6 to 7 hours at a point 40 miles downstream Attempts made to observe the form of the wave failed owing to the causes just mentioned

4 Complex Cases —If a fall is succeeded by a rise to the original level the fall travels with velocity due to the fallen stream, but the rise with velocity due to the risen stream At first it seems as if the rise must overtake the fall and fill up the hollow, which would result in places a long way down the stream being unaffected by the temporary diminution of supply This is impossible It would imply that the supply passing such a place was the same as if no temporary diminution had occurred What really happens is that the convex wave, as soon as it overtakes the other, begins to rise on it, suffers a decrease of slope, and is checked while the front wave receives an increase of slope and is accelerated The hollow lengthens and becomes shallower, and this goes on indefinitely Similarly, if a rise is succeeded by a fall, the wave lengthens out indefinitely while its height decreases At places a long way down the effect of fluctuations in the supply are slight in amount but long in duration

Given the height of a flood at A (Fig 147), the full effect of the flood will be felt at any place K only when the height at A is maintained for a sufficiently long period If this period is prolonged indefinitely the rise at K will not be increased, except in so far as may be due to the cessation of absorption by the flooded soil, but if the period is shortened the rise at K may be greatly reduced Empirical formulæ intended to give the height of a flood at any place, in terms of the heights in some reach upstream of it, must include the time as a factor, or, what is probably a better plan, must include gauge readings at several places upstream, and not at one place only This plan has been adopted on various rivers, the places selected being generally those where tributaries enter Sometimes it is sufficient merely to add together the different readings and take a given proportion

If the channel is not uniform the form of the wave, even if it has once become fixed, changes At a reduction of slope the wave assumes a more elongated, and, at an increase of slope, a

more compact form At an increase of surface width, supposing the mean velocity to be unaltered the wave is checked because additional space has to be filled up At a decrease of width the velocity of the wave increases

When an additional supply is introduced or abstracted at a place where there is not a fall, the water surface upstream is headed up or drawn down, and the form which it eventually assumes may be found by the methods explained in chapter vii (art 13) The volume of water eventually added to the stream upstream of the point of change can thus be found, but the time in which it is added cannot easily be found, because it is not known how much of the supply passes downstream The commonest case of the kind is that of the tide at the mouth of a river When the tide begins to rise the water in the river is headed up and its velocity reduced As the rise of the tide becomes more rapid the discharge of the river is insufficient to keep the channel filled up so as to keep pace with the rise of the tide the water in the mouth of the river becomes first still and level, and then takes a slope away from the sea and flows landwards At a place some way inland the water surface forms a hollow and water flows in from both directions This may obviously continue for some time after the tide has turned and high water then occurs later at the inland place than at the mouth of the river, a fact which is sometimes unnecessarily ascribed to 'momentum' A sudden and high flood in the Indus once caused a backward flow up the Cabul River where it joins the Indus

If in a long reach of a river the flood water way is reduced (say by embankments which prevent flood spill, or by training walls which cause the channel inside them to silt up) a flood of any kind will, in most of that reach, rise higher and travel more quickly than before The same effect will be produced but to a less degree at places further downstream When the rise is followed by a fall the wave will not flatten out to the same extent as before In the case of a permanent rise, except in so far as there will have been less absorption than before in the flooded area matters will be as before

5 Remarks —Sometimes a wave motion is seen in a stream when the supply seems to be quite uniform The cause may be at some abrupt change where air, becoming imprisoned, escapes at intervals (Cf unstable conditions at weir chap ix arts 10 and 13) It is believed that in a falling stream the surface is

slightly concave across, and in a rising stream convex, but the curvature is extremely small

The action of an unsteady stream on its channel is, no doubt, subject to the same laws as in a steady stream. At the front end of a rising wave the relation of V to D is exceptionally high, and scour is likely to occur. At the advancing end of a falling wave the reverse is the case, and hence a falling flood frequently causes deposits. In discussions on the training of estuaries the idea has often been put forward as a general law that it is wrong to diminish the flow of tidal water. No doubt it is the tidal water which has made the estuary. If only the upland water flowed through it the size would be far too great for the volume. The salt water may enter an estuary comparatively clear and return to sea silt-laden. But if training walls are made so as to reduce the volume of tidal water entering the estuary, the width to be kept open is also reduced. No such sweeping law as that above stated can be upheld. The Thames embankments in London contracted the channel and to some extent interfered with the tidal flow, but the channel was scoured and improved.

If a stream is temporarily obstructed by gates and the water headed up the silt deposited, if any, is removed again when the gates are opened. The same is true of obstruction caused by the rise of tides. If a given volume of water is available for the flushing of a sewer, it can probably be utilised best by introducing it intermittently, suddenly, and in considerable volumes at various points in the course of the sewer, commencing from near the tail and proceeding upwards. If there are any falls or gates it is clearly best to introduce it just below a fall or below a closed gate.

Ordinarily in a rising or falling stream the relative velocities at different points in a cross section are normal but where the fresh water of a river meets the sea the relations are apt to be much disturbed especially near the turns of the tide. The fresh water, being lighter, may rise on the salt water which may have a movement landwards while the fresh water above it is moving seawards. Such a landward current is obviously not the result of the surface slope and must be due to momentum and hence temporary.

CHAPTER X

DYNAMIC EFFECT OF FLOWING WATER

Section I.—General Information

1 **Preliminary Remarks**.—Hitherto we have been concerned almost entirely with questions relating to velocities, discharges and water levels. In this chapter will be considered questions relating to the Dynamic Effects of Flowing Water. In all cases the effect of friction will be neglected.

By dynamic pressure is meant the pressure produced by a stream of water when its velocity or its direction of motion is altered. This is, of course, entirely different from static pressure. Let V, A, and Q be the velocity, sectional area, and discharge of a stream, and W the weight of one cubic foot of the liquid. The volume discharged per second is AV, and its momentum is $WA\frac{V^2}{g}$. The force which, acting for one second, will produce or destroy this momentum is $F=WA\frac{V^2}{g}$. On this principle the pressures developed in various practical cases can be ascertained. Before proceeding to them it will be convenient to give two theorems regarding currents, though these do not strictly fall under the heading of this chapter, and might have been given in chapter ii if they had been required sooner.

2 **Radiating and Circular Currents**.—Suppose water to be supplied by the pipe AB (Fig. 149), and then to flow out radially between two parallel horizontal surfaces CD and EF, whose distance apart is d. Of radii R_1, R_2, let R_2 be the greater, and let the velocities be V_1, V_2, and the pressures P_1, P_2. Since the discharges past all vertical cylindrical sections are equal therefore $\frac{R_1}{R_2}=\frac{V_2}{V_1}$. Also since by Bernoulli's theorem the hydrostatic head is

$$H=\frac{P_1}{W}+\frac{V_1^2}{2g}=\frac{P_2}{W}+\frac{V_2^2}{2g}=\frac{P_2}{W}+\frac{V_1^2}{V_2^2}=\frac{V_1^2}{2g}$$

Therefore $$\frac{P_1}{W} = H - \frac{V_1^2}{2g}$$

And $$\frac{P_2}{W} = H - \frac{V_1^2}{2g}\cdot\frac{R_1^2}{R_2^2},$$

or the heights in pressure columns increase from the centre outwards and tend to reach, though never reaching, the value H. If

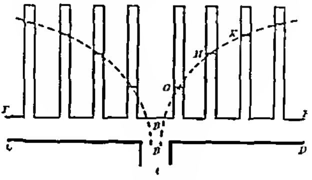


FIG 149

the water flows inwards and passes away by the pipe the law is the same. A curve through the points G, H, K, etc, is known as Barlow's curve.

In a vessel (Fig 150) which, with its contents, is revolving about a vertical axis with angular velocity a, the forces acting on a particle A whose velocity is u are its weight w or AC, acting vertically, and a horizontal centrifugal force $w\frac{u^2}{gx}$ or $w\frac{a^2}{g}x$ or AB. The water surface takes a form normal to the resultant AD of the above, that is, the angle DAC is $\tan^{-1}\frac{a^2}{g}x$. Hence $\frac{dy}{dx} = \frac{a^2}{g}x$.

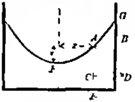


FIG 150

Integrating, $y = \frac{a^2}{2g}x^2$, *or the curve* EA *is a parabola with apex at* E. Since $u = ax$, therefore $y = \frac{u^2}{2g}$, or the elevation of any point above E is the head due to its velocity of revolution. The theoretical velocity of efflux from an orifice at F or L is that due to a head AF or GB.

A similar condition occurs in a mass of water driven round by radiating paddles. In either case the condition is termed a 'forced vortex.' Questions connected with the pressure in a radiating

current or in a forced vortex enter, though not to a very important degree, into the theories of certain hydraulic machines. In a centrifugal pump the pressures in the pump wheel follow the law of the radiating current, while those in the whirling chamber outside the wheel depend on the law of the forced vortex.

SECTION II.—REACTION AND IMPACT

3. **Reaction.**—Let a jet issue without contraction from an orifice A (Fig. 151) in the side of a tank. The force F causing the flow is the pressure on B. This force is called the reaction of the jet. It tends to move the tank in the direction AL. It is equal to $WA\frac{V^2}{g}$, or to $2WAH$ where H is the head due to V. If the tank is supposed to move with velocity v in the direction AL, the absolute velocity of the issuing jet is $V-v$, but the quantity issuing is still AV. Hence the momentum of the discharge per second is $WA\frac{V(V-v)}{g}$.

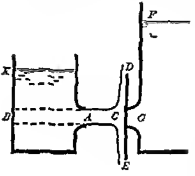


FIG. 151.

The principle of reaction has been utilised in driving a ship, water being pumped into the ship and driven out again sternwards. The energy of the water just after leaving the ship is $WAV\frac{(V-v)^2}{2g}$.

The work done on the ship is

$$FV = WA\frac{V(V-v)v}{g} \qquad (88)$$

The total work done on the water is the sum of the above or

$$WAV\frac{V^2-v^2}{2g} \qquad (89)$$

The efficiency of the machine is the ratio of (88) to (89) or $\frac{2v}{V+v}$.

The nearer v approaches V the nearer the efficiency is to 1.0, but the less the actual work done on the ship. If $V=v$ the efficiency is 1.0, but the work done is nil. In the *Waterwitch* V was $2v$, so that the efficiency was

The principle of reaction has also been applied in driving a

'Reaction Wheel' or 'Barker's Mill' (Fig 152) The preceding formulæ and remarks apply to this case, v being the velocity of the rotating orifices. If AC is the head in the shaft the head over the orifice D is BD, AB being an imaginary water surface found by the principles of article 2. If $AC=H$ the velocity of efflux at D is $\sqrt{2gH+v^2}$.

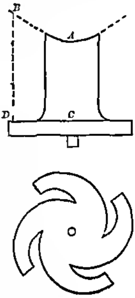


Fig 152

4 Impact.—When a jet of water (Fig 153) meets a solid surface which is at rest, it spreads out over the surface. There is not, strictly speaking, any shock, but there is loss of head owing to abrupt change. If the surface is horizontal and a jet strikes it vertically, it spreads out equally in all directions. In other cases the amount and directions of spreading depend on the circumstances. In all cases, without exception, the velocity of the jet relatively to the surface is the same after impact as before. The flow after impact is along the surface which, being smooth, cannot alter the velocity of the water, but only force it to change its direction. The pressure between the fluid and the surface in any direction is equal to the change of momentum in that direction of so much fluid as reaches the surface in one second.

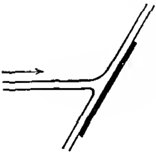


Fig 153

Let a jet AC (Fig 151) meet a fixed plane surface at right angles. The momentum in the direction AC is wholly destroyed and the pressure on the plane is $WA\frac{V^2}{g}$, or the same as the pressure (reaction) on A or twice the pressure due to the hydrostatic head which produces V. Thus the pressure on DE will balance the pressure due to the head FG where FG is twice KB. In the case shown in Fig 97 (p 135) the two heads are equal. In that case the head HG has to be produced, the discharge rising through GH. In the present case the head FG has merely to be maintained.

If the plane is moving with velocity v in the same direction as the jet the discharge meeting the plane per second is $A(V-v)$ and

the pressure is $WA\frac{(V-v)}{g}$ The work done on the plane per second is $WA\frac{(V-v)}{g}v$ The total energy of the water before impact is $WAV\frac{V^2}{2g}$ The efficiency is $\frac{2(V-v)v}{V^2}$ This is a maximum when $V=3v$ and the efficiency is then $\frac{8}{27}$

If for the vane there is substituted a series of vanes as in the case of a jet directed against a series of radial vanes of a large wheel the discharge reaching the vanes per second is AV and the whole pressure is $WAV(V-v)$ The work done per second is $WAV\frac{(V-v)v}{g}$ and the efficiency is $\frac{2V(V-v)v}{V^3}$ or $2v\frac{V-v}{V^2}$ It is a maximum when $v=\frac{V}{2}$ and is then $\frac{1}{2}$

If the vane is cup-shaped (Fig 154) so that the water leaving the vane is reversed in direction, the velocity of the water leaving the vane has relatively to the vane a velocity $V-v$ in a backward

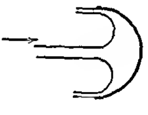
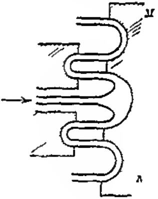


Fig 154



[illegible]

direction and an absolute velocity $v-V+v$ or $2v-V$ The change of momentum per second is $WA\frac{(V-v)}{g}\{V-(2v-V)\}$ or $2WA\frac{(V-v)}{g}$, and the pressure on the cup is double that on the plane considered above The work done on the cup is $2WA\frac{(V-v)^2}{g}v$ The efficiency is It is a maximum when $V=2v$, and is then 1 In th uted by Fig the pressure on the solid MN is t due to a si cup

If there is a series of cups the discharge per second reaching them is AV the whole pressure is $WA\frac{V}{g}\{V-(2v-V)\}$ or $2WA\frac{V(V-v)}{g}$ The efficiency is $\frac{4V(V-v)v}{V^3}$ It is a maximum when $V=2v$, and is then 1 0

The preceding cases illustrate the great principle to be adopted in the design of water motors such as turbines and Poncelet wheels, namely, that the water shall leave the machines deprived, as far as possible, of its absolute velocity If it has on departure any velocity it carries away work with it In the last case it had no velocity and the efficiency is 1 0

Another principle is that the water shall impinge on the vane so as to create as little disturbance as possible—that is, as nearly as possible tangentially to the vane—and thus minimise loss of energy by shock When the jet strikes tangentially it has no tendency to spread out laterally, but slides along the vane In practice an exact tangential direction is impracticable, but the vanes are provided with raised edges which prevent lateral spread and cause the water to be deflected entirely in one plane

A third principle is that all passages for water shall as far as possible, be free from abrupt changes in section or direction, so that loss of head from shock shall be avoided

Let AA (Fig 156) be a surface or vane moving in the direction

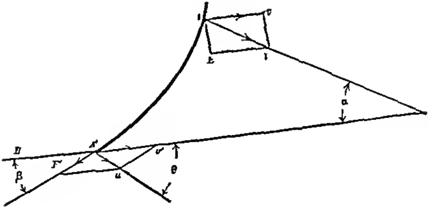


FIG 156

and with the velocity v, represented by Av, and let AV represent the direction and velocity V of a jet impinging on the vane Let

α be the angle between the two lines. The line vV represents the velocity V of the jet relatively to the vane at A. Let it be assumed that the jet is deviated entirely in planes parallel to the figure. The jet leaves the vane at A with the velocity V, represented by the line AE. Draw Av' equal and parallel to Ai. Then Au represents the absolute velocity of the water leaving the vane. Let the angle $vAu=\theta$ and $BAE=\beta$. If the quantity of water reaching the vane per second is w, the original and final momenta of the water resolved in a direction parallel to Av are $\frac{w}{g}V\cos\alpha$ and $\frac{w}{g}V\cos\theta$. The change of momentum or pressure in the direction Av is $\frac{w}{g}(V\cos\alpha-V\cos\theta)$ or $\frac{w}{g}(V\cos\alpha-v+V\cos\beta)$. These are general expressions covering all cases, and the preceding ones can be derived from them.[1]

When a jet impinges on a plane, as in Fig 157, the issuing velocity of the jet is theoretically $\sqrt{2gH_1}$, but on reaching the plane the velocity V is about $\sqrt{2gH}$. The outer streams at A press on the inner by reason of centrifugal force, and the intensity of pressure increases towards the centre of the jet. It cannot exceed the amount due to $\frac{V^2}{2g}$ or H, because otherwise the direction of flow would be reversed. Experiments made by Beresford[2] with jets ·175 inch to 1·95 inch in diameter falling on a brass plate show

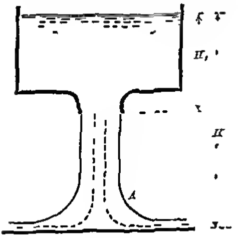


Fig 157

[1] Some machines which illustrate the principles of dynamic pressure have been referred to above. There are many machines such as water meters, modules, rams, presses, pumps, water wheels and water pressure engines which, though water passes through them, illustrate no principle of hydraulics, the questions involved in their design being engineering and dynamical. In fact, the principles involved in the above formulæ regarding vanes are dynamical, and are given here to bridge over a gap between hydraulics and another science. The same remark applies to parts of the succeeding article.

[2] Professional Papers on Indian Engineering No cccxii

that at the axis of the jet, the pressure is very nearly that due to H and the pressure becomes negligible at a distance from the axis equal to about twice the diameter of the jet. The pressure is thus distributed over an area of about four times that of the section of the jet. The pressures were measured by means of a water-column communicating with a small hole in the plate whose position could be altered.

5. Miscellaneous Cases.—When water flows round a bend in a channel the dynamic pressure produced on the channel is the same as if the channel was a curved vane. At bends in large pipes anchors are sometimes required to hold the pipe.

When a mass of water flowing in a pipe is abruptly brought to rest by the closure of a gate or valve the pressure produced is $f = \frac{v}{L}\frac{VTm}{2rm + VT}$ where l is the length of the pipe affected by the pulsation, m and V the moduli of elasticity for water and for the material of the pipe in pounds per square inch, T the thickness of the pipe in inches, r the radius of the pipe in feet, and v the velocity of the water in feet per second, f being in pounds per square inch over and above the static pressure.[1]

When a thin plate (Fig. 158) is moved normally through still water with velocity V, a mass of water in front of the plate is put in motion and those portions of it which flow off at the sides of the plate cannot turn sharp round and fill up the space behind the plate. Instead of doing this they penetrate into the rest of the water and so communicate forward momentum to it, while other portions of still water have to be set in motion to fill up the space behind. Thus there is produced a resistance which is independent of friction or viscosity. Practically it is found that the resistance is $kW\frac{V^2}{2g}$ where k is 1.2 to 1.8, the best results giving 1.3 to 1.6. The resistance is less than that caused by the impinging on a fixed plane of a jet of the same section as the area of the plate with a velocity V.

Fig. 158.

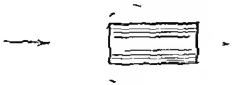


Fig. 159.

[1] Min. Proc. Inst. C.E., vol. cxxx.

If for the plate there is substituted a cylinder (Fig 159) whose length is not more than about three diameters, the resistance is less than in the case of the plate It is further reduced if the downstream end of the cylinder is pointed [1]

In the above cases, if the plane or cylinder is fixed and the water moving, the pressures are the same

The following statement shows the approximate results of some experiments made by Hagen to show the position assumed by a rectangular plane surface when pivoted (Fig 160) and placed in flowing water —

$\frac{x}{y}=10$	9	8	7	6	5	4	3	2
$\phi=90°$	74°	59°	46°	27°	13°	7°	6°	1°

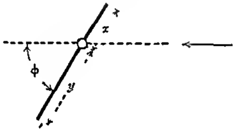


FIG 160

When a thin sharpened plate or a spindle shaped or ship shaped body is moved endways through still water the resistance is almost wholly frictional and is nearly as V^n, but if the body is only partly submerged waves are produced, and when V exceeds a certain limit (which bears a relation to the size of the body) the wave resistance increases and the total resistance increases faster than V^2 If the body, though sharp at both ends, tapers more rapidly at one end than at the other, it probably causes least resistance when the blunter end is forward

In experiments made by *Froude by towing boards through still water, it* was found that the power of the velocity to which the friction is proportional varies for different surfaces, being sometimes less than 2 and sometimes more Also that for long boards f (chap II art 9) is much less than for short ones, the reason being that the forward part of a long board communicates motion to the water, and the succeeding portion thus experiences less resistance

[1] For results of some recent experiments on cylinders with square and pointed ends see *Min Proc Inst C E*, vol cxvii

APPENDIX A

Calculation of m and n

(Chap ii. arts. 5 and 8)

The following is a specimen of the method of calculating:—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Height of Weir	Head.	M (observed).	$M^2\frac{H^2}{(s+H)^2}$	m (assumed).	$\frac{M}{m}$ or $1+\frac{3}{2}nM^2\frac{H^2}{(s+H)^2}$	$\frac{3}{2}nM^2\frac{H^2}{(s+H)^2}$	$\frac{3}{2}n$ or col. 7 ÷ col. 4.	n
Metres	Metres							
1·135	·15	·4254	·00258	·4230	1·0056	·0056	2·18	1·45
·75	Do	·4310	·00518		1·0130	·0130	2·50	1·67
·50	Do	·4359	·0100		1·0225	·0225	2·25	1·50

Height of Weir	Head	M as observed	Three assumed sets of values for m and for each the corresponding value of n					
(1)	(2)	(3)	(4)		(5)		(6)	
Feet	Feet		m	n	m	n	m	n
3 72	49	4284	4250	1 45	4270	87	4284	
2 46	Do	4316	Do	1 67	Do	1 36	Do	1
1 64	Do	4359	Do	1 52	Do	1 37	Do	1 14
1 15	Do	4424	4273	1 29	4283	1 19	4297	1 08
79	Do	4522	4303	89	4313	86	4327	84
Mean	Do			1 36		1 12		86
3 72	1 31	4286	4185	1 28	4200	1 09	4286	
2 46	Do	4430	4207	1 42	4221	1 30	4308	85
1 64	Do	4595	4245	1 20	4280	1 15	4346	96
1 15	Do	4794	4305	1 04	4320	1 02	4406	87
79	Do	5034	4395	86	4410	85	4500	75
Mean	Do			1 16		1 08		97
3 31[1]	1 44	4310	4167	1 32	4200	1 02	4214	1
2 46	Do	4452	4178	1 61	4233	1 28	4275	1 07
Mean	Do			1 47		1 15		1 04
3 31[2]	1 80	4334	4100	1 51	4190	98	4211	89

[1] Length of weir reduced to 3 8 feet
[2] Length of weir reduced to 1 64 feet

It will be noticed that slight changes in m cause great changes in n Obviously m cannot rise to the values shown in column 6, as it would then equal M for the highest weirs If reduced much below the value of column 4 it would make n very high The values of m and n which seem most suitable are those of column 5, the mean value of n being 1 1

APPENDIX B

NOTE REGARDING VALUE OF C IN VARIABLE FLOW

(Chap. II, arts. 10 and 12)

It may be said that no proof has been given that C is the same as in uniform flow. The case is somewhat analogous to the fourth proposition in Euclid's book I. Regarding this it has been remarked that it would be sufficient to state that the two triangles are equal because there is no reason why they should be different. Consider a portion of a uniform stream 200 feet long with $W=50$ feet, $D=5$ feet and $V=2.5$ feet per second. Now let W' be 52 feet at one end and 48 feet at the other, so that V_1 and V' are, respectively, 2.4 and 2.6 feet per second. There is no reason why any appreciable alteration should occur either in the total loss of head from the resistance of the border, or in the velocity curves of the central section, or the average of the velocity curves of the whole length considered. Consequently there is no reason why C should be altered.

APPENDIX C

VALUE OF N IN THE RAVI AT SIDHNAI

(Chap. VI, art. 13)

The river is straight for five miles upstream of the discharge site and one mile downstream, a reach unique, perhaps, among the rivers of the world, but its great length can hardly be the cause of the low value of N. The silt is caused by a dam a mile below the discharge site. In floods the dam is removed and the silt then scours out. Thus the bed is probably roughest for the greatest depths of water. In spite of this, N is very much the same for all the depths from 6 feet to 10 feet, and C somewhere about 200, whereas Bazin's highest figure is 152.